

# RG-Improved Predictions for Top-Quark Pair Production at Hadron Colliders

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# Based on:

- ♦ IR singularities of scattering amplitudes in non-abelian gauge theories

Thomas Becher, MN: 0901.0722 (PRL), 0903.1126 (JHEP), 0904.1021 (PRD)

Andrea Ferroglia, Ben Pecjak, MN, Li Lin Yang: 0907.4791 (PRL), 0908.3676 (JHEP)

- ♦ Threshold resummation for Higgs production

Valentin Ahrens, Thomas Becher, MN, Li Lin Yang: 0808.3008 (PRD), 0809.4283 (EPJC)

- ♦ Threshold resummation for top-quark pair production

Andrea Ferroglia, Ben Pecjak, MN, Li Lin Yang: 0912.3375 (PLB) & 1003.5827 (→JHEP)



# A tale of many scales

- ◆ Collider processes characterized by many scales:  $s$ ,  $s_{ij}$ ,  $M_i$ ,  $\Lambda_{\text{QCD}}$ , ...
- ◆ Large Sudakov logarithms arise, which need to be resummed (e.g. parton showers, mass effects, aspects of underlying event)
- ◆ Effective field theories provide modern, elegant approach to this problem based on scale separation (factorization theorems) and RG evolution (resummation)



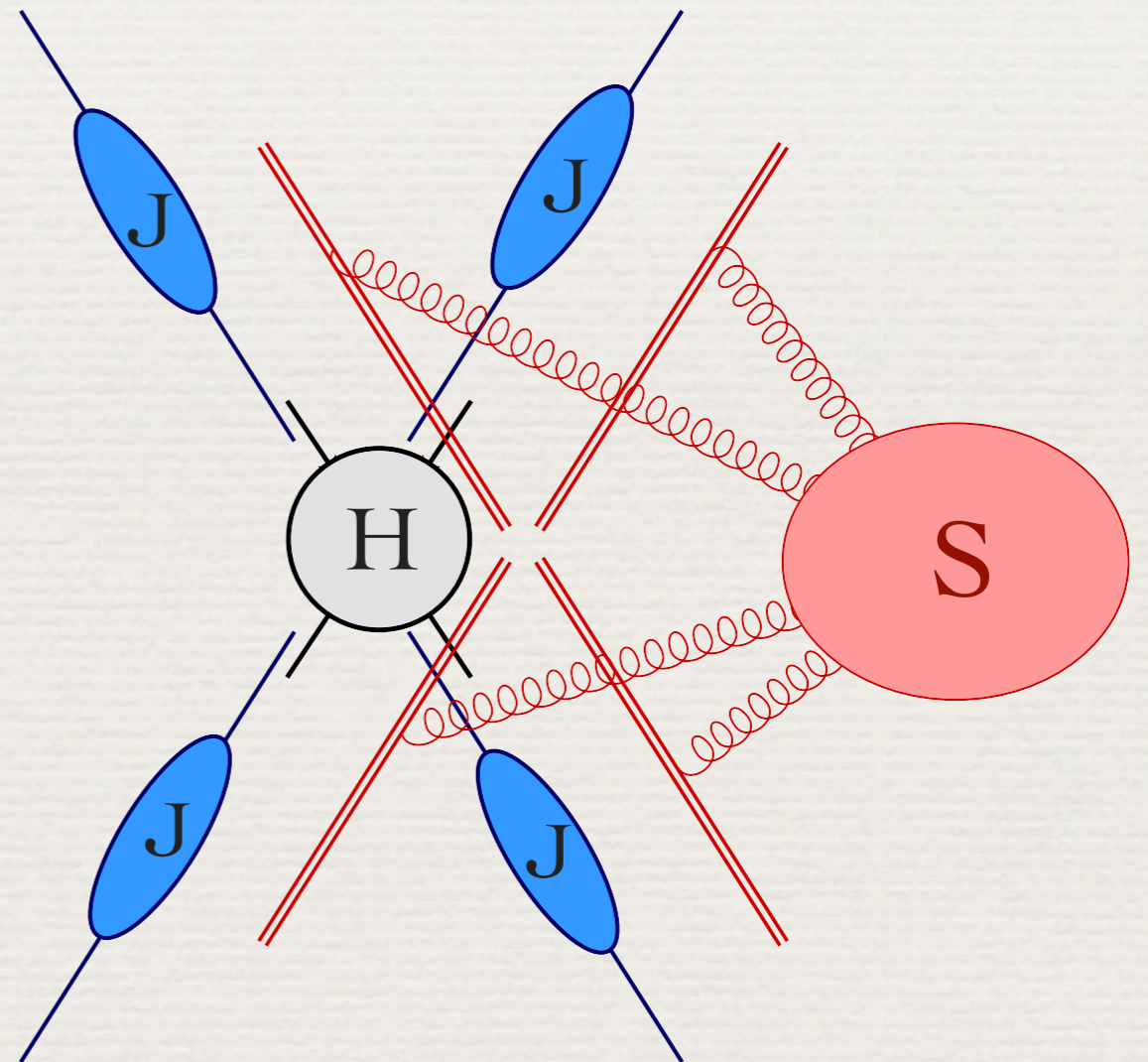
# Soft-collinear factorization

Sen 1983; Kidonakis, Oderda, Sterman 1998

- ◆ Factorize cross section:

$$d\sigma \sim H(\{s_{ij}\}, \mu) \prod_i J_i(M_i^2, \mu) \otimes S(\{\Lambda_{ij}^2\}, \mu)$$

- ◆ Define components in terms of field theory objects in SCET
- ◆ Resum large Sudakov logarithms directly in momentum space using RG equations





# Soft-collinear effective theory (SCET)

Bauer, Pirjol, Stewart et al. 2001, 2002; Beneke et al. 2002; ...

- Two-step matching procedure:



- Integrate out hard modes, describe collinear and soft modes by fields in SCET
- Integrate out collinear modes (if perturbative) and match onto a theory of Wilson lines

$$\Lambda_{ij}^2 = \frac{M_i^4}{S_{ij}} \frac{\text{hard}}{\text{collinear}} \frac{\text{collinear}}{\text{soft}}$$



# SCET for n-jet processes

- ♦ n different types of collinear quark and gluon fields (**jet functions  $J_i$** ), interacting only via soft gluons (**soft function  $S$** )
- ♦ Hard contributions ( $Q \sim \sqrt{s}$ ) are integrated out and absorbed into Wilson coefficients:

$$\mathcal{H}_n = \sum_i C_{n,i}(\mu) O_{n,i}^{\text{ren}}(\mu) \quad \text{Bauer, Schwartz 2006}$$

- ♦ Scale dependence controlled by RGE:

$$\frac{d}{d \ln \mu} |\mathcal{C}_n(\{\underline{p}\}, \mu)\rangle = \mathbf{\Gamma}(\mu, \{\underline{p}\}) |\mathcal{C}_n(\{\underline{p}\}, \mu)\rangle$$

anomalous-dimension matrix of n-jet SCET operators



# Goal: NLO+NNLL resummation

- ◆ Necessary ingredients:
  - ◆ **Hard functions:** from fixed-order results for on-shell amplitudes (but need amplitudes!)
  - ◆ **Jet functions:** from imaginary parts of two-point functions; needed at one-loop order (depend on cuts, jet definitions)
  - ◆ **Soft functions:** from matrix elements of Wilson-line operators
- ◆ Yields **jet cross sections** (not parton rates)
- ◆ Goes beyond **parton showers**, which are accurate only at LL order even after matching



# Evolution of hard functions

- ◆ Technically most challenging aspect besides the computation of the **hard functions** is their evolution, governed by **anomalous-dimension matrix of n-jet operators**:

$$\frac{d}{d \ln \mu} |\mathcal{C}_n(\{p\}, \mu)\rangle = \mathbf{\Gamma} |\mathcal{C}_n(\{p\}, \mu)\rangle$$

- ◆ We have obtained completely general, multi-loop expressions for the anomalous-dimension matrices for **generic n-jet processes with both massless and massive partons**



# Anomalous dimension to two loops

## ◆ General result:

extracted from:  
Korchinsky, Radyushkin 1987

$$\Gamma(\{\underline{p}\}, \{\underline{m}\}, \mu) = \sum_{(i,j)} \frac{\mathbf{T}_i \cdot \mathbf{T}_j}{2} \gamma_{\text{cusp}}(\alpha_s) \ln \frac{\mu^2}{-s_{ij}} + \sum_i \gamma^i(\alpha_s)$$

massless partons

$$- \sum_{(I,J)} \frac{\mathbf{T}_I \cdot \mathbf{T}_J}{2} \gamma_{\text{cusp}}(\beta_{IJ}, \alpha_s) + \sum_I \gamma^I(\alpha_s) + \sum_{I,j} \mathbf{T}_I \cdot \mathbf{T}_j \gamma_{\text{cusp}}(\alpha_s) \ln \frac{m_I \mu}{-s_{Ij}}$$

massive partons

$$+ \sum_{(I,J,K)} i f^{abc} \mathbf{T}_I^a \mathbf{T}_J^b \mathbf{T}_K^c F_1(\beta_{IJ}, \beta_{JK}, \beta_{KI})$$

new!

$$+ \sum_{(I,J)} \sum_k i f^{abc} \mathbf{T}_I^a \mathbf{T}_J^b \mathbf{T}_k^c f_2\left(\beta_{IJ}, \ln \frac{-\sigma_{Jk} v_J \cdot p_k}{-\sigma_{Ik} v_I \cdot p_k}\right) + \mathcal{O}(\alpha_s^3).$$

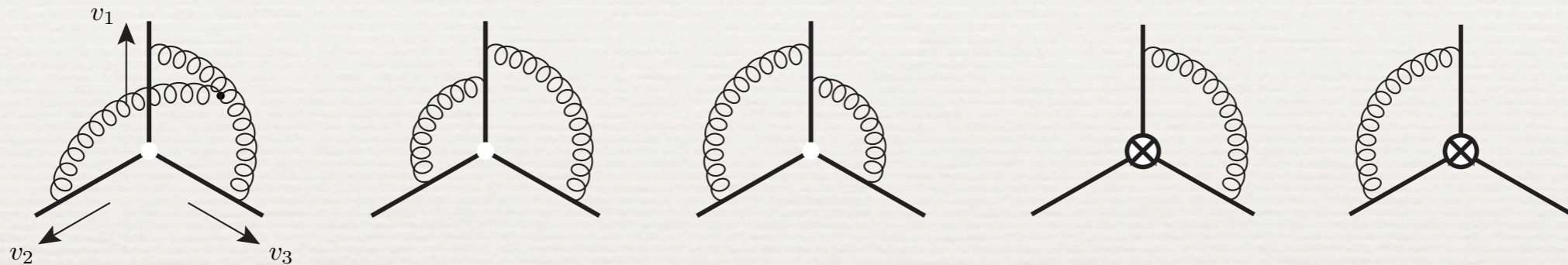
- ◆ Generalizes structure found for massless case
  - ◆ Novel three-parton terms appear at two-loop order
- Mitov, Sterman, Sung: 0903.3241; Becher, MN: 0904.1021



# Calculation of three-parton terms

Ferrogia, MN, Pecjak, Yang: 0907.4791, 0908.3676

- Relevant two-loop diagrams:



- Surprisingly simple answer:

anti-symmetric in heavy-parton indices

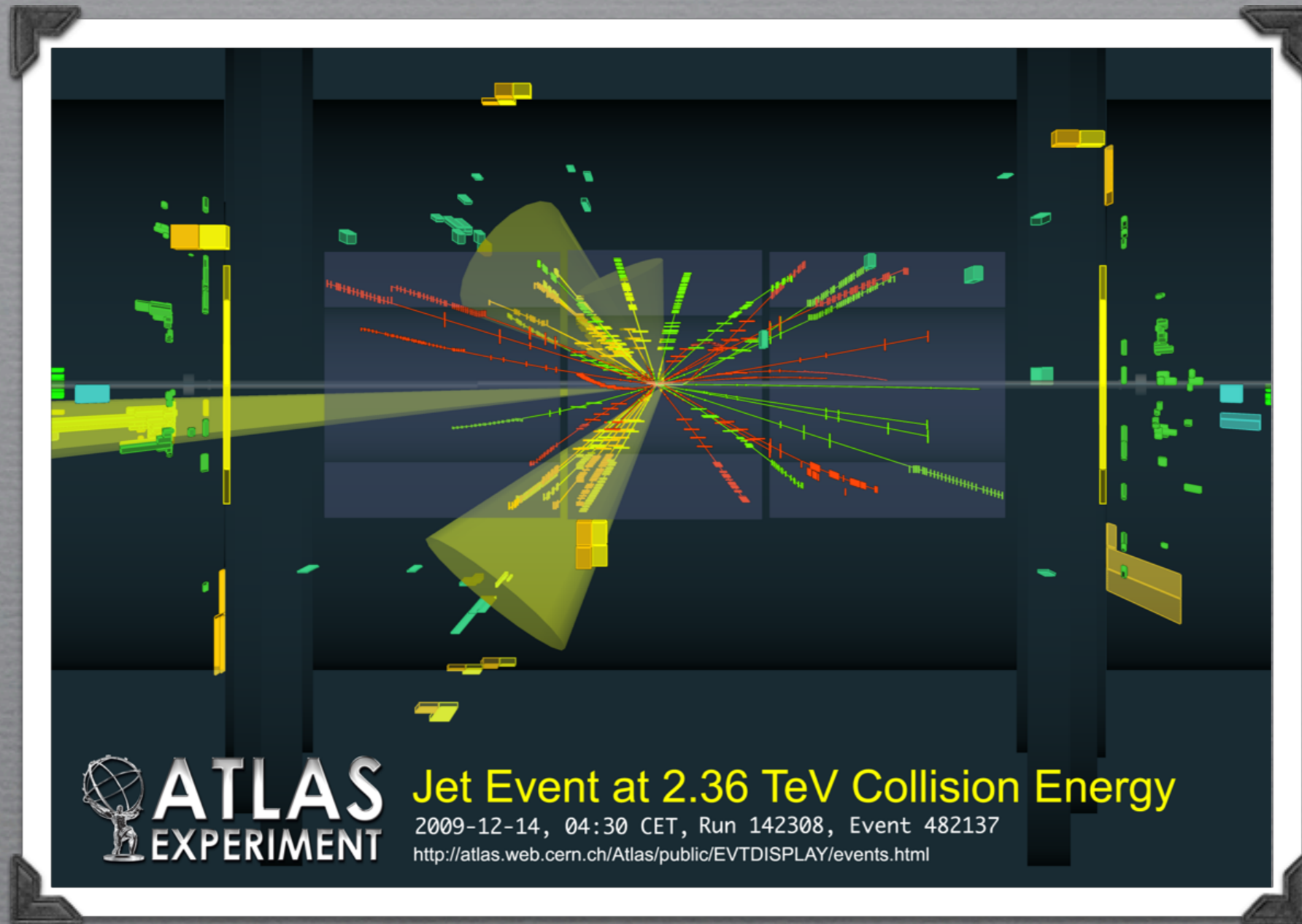
$$F_1(\beta_{12}, \beta_{23}, \beta_{31}) = \frac{1}{3} \sum_{I,J,K} \epsilon_{IJK} \frac{\alpha_s}{4\pi} g(\beta_{IJ}) \gamma_{\text{cusp}}(\beta_{KI}, \alpha_s)$$

$$f_2\left(\beta_{12}, \ln \frac{-\sigma_{23} v_2 \cdot p_3}{-\sigma_{13} v_1 \cdot p_3}\right) = -\frac{\alpha_s}{4\pi} g(\beta_{12}) \gamma_{\text{cusp}}(\alpha_s) \ln \frac{-\sigma_{23} v_2 \cdot p_3}{-\sigma_{13} v_1 \cdot p_3}$$

with:

$$g(\beta) = \coth \beta \left[ \beta^2 + 2\beta \ln(1 - e^{-2\beta}) - \text{Li}_2(e^{-2\beta}) + \frac{\pi^2}{6} \right] - \beta^2 - \frac{\pi^2}{6}$$





# EFT-based predictions for top-quark pair production at hadron colliders

Ahrens, Ferroglia, MN, Pecjak, Yang: 0912.3375 & [1003.5827](https://arxiv.org/abs/1003.5827) (v2 to appear)



# State of the art

- ◆ Fixed-order NLO calculations:
  - ◆ total cross section: Nason, Dawson, Ellis 1988  
Beenakker et al. 1989
  - ◆ differential: Nason, Dawson, Ellis 1989  
Mangano, Nason, Ridolfi 1992  
Frixione, Mangano, Nason, Ridolfi 1995
  - ◆  $A_{\text{FB}}^t$ : Kühn, Rodrigo 1998
- ◆ Fixed-order NNLO calculations:
  - ◆ **none exist!** (several pieces available)
  - ◆ “leading terms” (enhanced near threshold)  
for total cross section: Beneke, Falgari, Schwinn 2009  
Czakon, Mitov, Sterman 2009
  - ◆ “leading terms” for distributions,  $A_{\text{FB}}^t$  **this work!**



# State of the art

- ◆ Threshold resummation at NLL:

- ◆ total cross section: Bonciani, Catani, Mangano, Nason 1998  
Berger, Contopanagos 1995  
Kidonakis, Laenen, Moch, Vogt 2001
- ◆ distributions: Kidonakis, Vogt 2003; Banfi, Laenen 2005
- ◆  $A_{FB}^t$ : Almeida, Sterman, Vogelsang 2008

- ◆ Resummation at NNLL+NLO matching:

- ◆ total cross section: Beneke, Falgari, Schwinn 2009  
Czakon, Mitov, Sterman 2009 **& this work!**
- ◆ distributions: **this work!**



# Top-pair production at partial NNLO

Ferrogia, MN, Pecjak, Yang: 0908.3676

- ◆ Anomalous-dimension matrices in s-channel singlet-octet basis for  $q\bar{q}, gg \rightarrow t\bar{t}$  channels:

$$\begin{aligned}
 \mathbf{\Gamma}_{q\bar{q}} &= \left[ C_F \gamma_{\text{cusp}}(\alpha_s) \ln \frac{-s}{\mu^2} + C_F \gamma_{\text{cusp}}(\beta_{34}, \alpha_s) + 2\gamma^q(\alpha_s) + 2\gamma^Q(\alpha_s) \right] \mathbf{1} \\
 &+ \frac{N}{2} \left[ \gamma_{\text{cusp}}(\alpha_s) \ln \frac{(-s_{13})(-s_{24})}{(-s) m_t^2} - \gamma_{\text{cusp}}(\beta_{34}, \alpha_s) \right] \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \\
 &+ \gamma_{\text{cusp}}(\alpha_s) \ln \frac{(-s_{13})(-s_{24})}{(-s_{14})(-s_{23})} \left[ \begin{pmatrix} 0 & \frac{C_F}{2N} \\ 1 & -\frac{1}{N} \end{pmatrix} + \frac{\alpha_s}{4\pi} g(\beta_{34}) \begin{pmatrix} 0 & \frac{C_F}{2} \\ -N & 0 \end{pmatrix} \right] + \mathcal{O}(\alpha_s^3) \\
 \\
 \mathbf{\Gamma}_{gg} &= \left[ N \gamma_{\text{cusp}}(\alpha_s) \ln \frac{-s}{\mu^2} + C_F \gamma_{\text{cusp}}(\beta_{34}, \alpha_s) + 2\gamma^g(\alpha_s) + 2\gamma^Q(\alpha_s) \right] \mathbf{1} \\
 &+ \frac{N}{2} \left[ \gamma_{\text{cusp}}(\alpha_s) \ln \frac{(-s_{13})(-s_{24})}{(-s) m_t^2} - \gamma_{\text{cusp}}(\beta_{34}, \alpha_s) \right] \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
 &+ \gamma_{\text{cusp}}(\alpha_s) \ln \frac{(-s_{13})(-s_{24})}{(-s_{14})(-s_{23})} \left[ \begin{pmatrix} 0 & \frac{1}{2} & 0 \\ 1 & -\frac{N}{4} & \frac{N^2-4}{4N} \\ 0 & \frac{N}{4} & -\frac{N}{4} \end{pmatrix} + \frac{\alpha_s}{4\pi} g(\beta_{34}) \begin{pmatrix} 0 & \frac{N}{2} & 0 \\ -N & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right] + \mathcal{O}(\alpha_s^3).
 \end{aligned} \tag{55}$$



# Top-pair production at partial NNLO

- ♦ Can use these results to predict leading singular terms near partonic threshold  $z = M^2/\hat{s} \rightarrow 1$

- ♦ Obtain NNLO coefficients of distributions

$$P'_n(z) = \left[ \frac{1}{1-z} \ln^n \left( \frac{M^2(1-z)^2}{\mu^2 z} \right) \right]_+$$

and (partially) of  $\delta(1-z)$

- ♦ Yields **presently best estimate** of NNLO terms

- ♦ **Note:** includes some subleading terms  $\sim \ln(z)$

beyond distributions

$$P_n(z) = \left[ \frac{\ln^n(1-z)}{1-z} \right]_+$$



# Top-pair production at NNLL+NLO

Ahrens, Ferroglia, MN, Pecjak, Yang: 1003.5827

- ♦ Solving RG equations, leading singular terms can be resummed to all orders in perturbation theory with NNLL accuracy
- ♦ **Resummed hard-scattering coefficients** in momentum space: [Becher, MN 2006](#)

$$C(z, M, m_t, \cos \theta, \mu_f) = \exp [4a_{\gamma\phi}(\mu_s, \mu_f)] \\ \times \text{Tr} \left[ \mathbf{U}(M, m_t, \cos \theta, \mu_h, \mu_s) \mathbf{H}(M, m_t, \cos \theta, \mu_h) \mathbf{U}^\dagger(M, m_t, \cos \theta, \mu_h, \mu_s) \right] \\ \times \tilde{\mathbf{s}} \left( \ln \frac{M^2}{\mu_s^2} + \partial_\eta, M, m_t, \cos \theta, \mu_s \right) \left[ \frac{e^{-2\gamma_E \eta}}{\Gamma(2\eta)} \frac{z^{-\eta}}{(1-z)^{1-2\eta}} \right]$$

- ♦ Then match onto NLO fixed-order results



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$$C(z, M, m_t, \cos \theta, \mu_f) = \exp [4a_{\gamma\phi}(\mu_s, \mu_f)]$$

$$\times \text{Tr} \left[ \mathbf{U}(M, m_t, \cos \theta, \mu_h, \mu_s) \mathbf{H}(M, m_t, \cos \theta, \mu_h) \mathbf{U}^\dagger(M, m_t, \cos \theta, \mu_h, \mu_s) \right]$$

$$\times \tilde{\mathbf{s}} \left( \ln \frac{M^2}{\mu_s^2} + \partial_\eta, M, m_t, \cos \theta, \mu_s \right) \left[ \frac{e^{-2\gamma_E \eta}}{\Gamma(2\eta)} \frac{z^{-\eta}}{(1-z)^{1-2\eta}} \right]$$

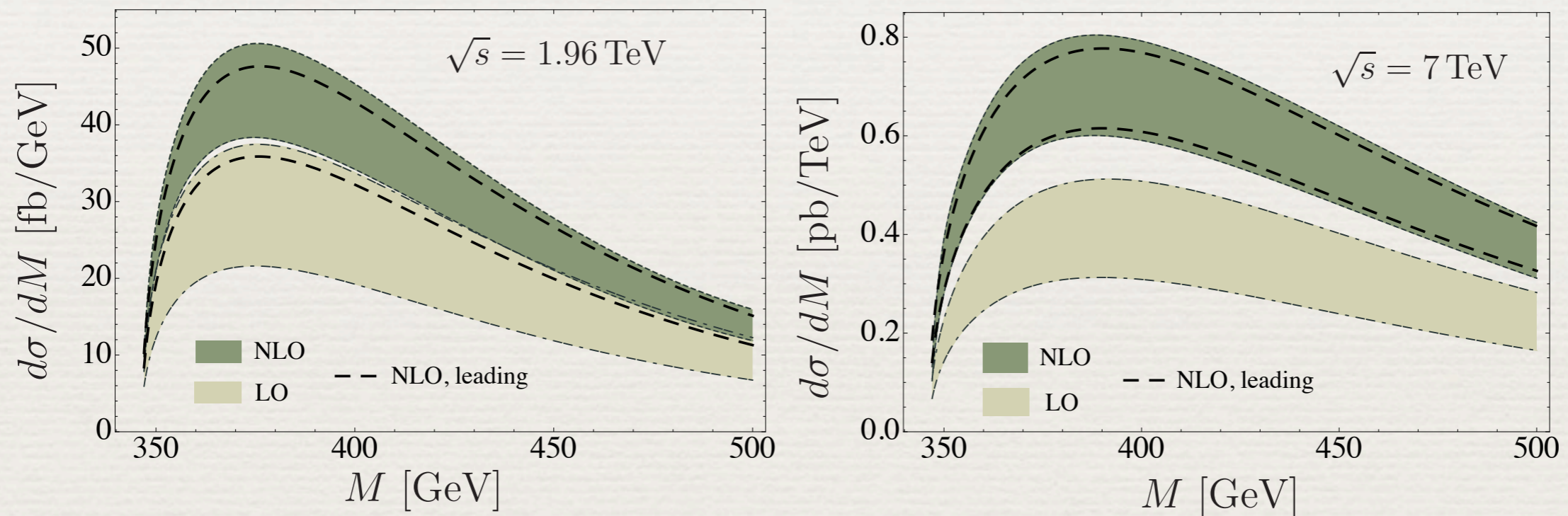
RG-impr. PT	log accuracy	$\Gamma_{\text{cusp}}$	$\gamma^h, \gamma^\phi$	$\mathbf{H}, \tilde{\mathbf{s}}$
LO	NLL	2-loop	1-loop	tree-level
NLO	NNLL	3-loop	2-loop	1-loop

- ♦ Then match onto NLO fixed-order results



# Dominance of threshold terms

- ◆ Fixed-order results for invariant mass distribution at Tevatron and LHC:

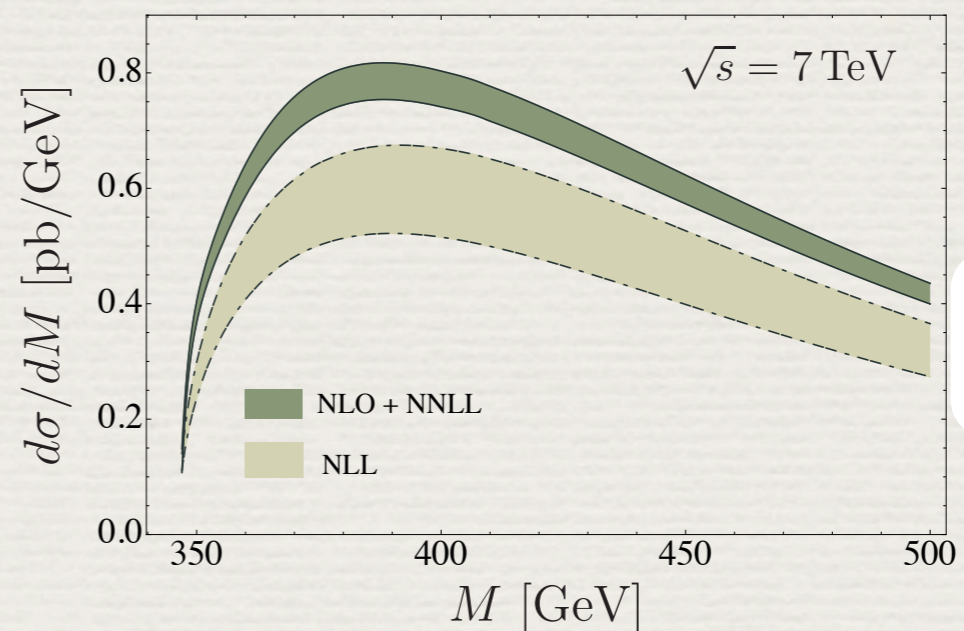
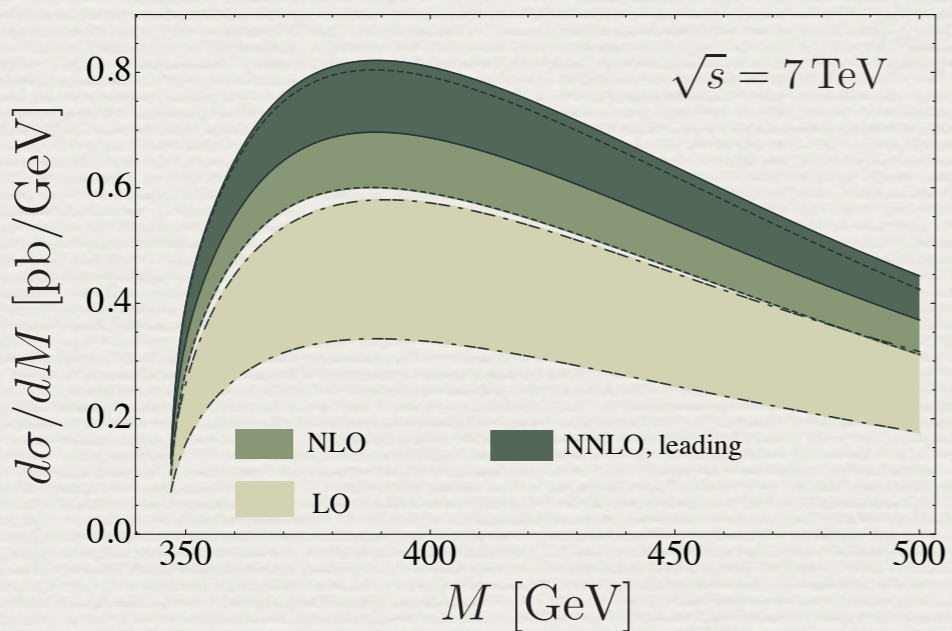
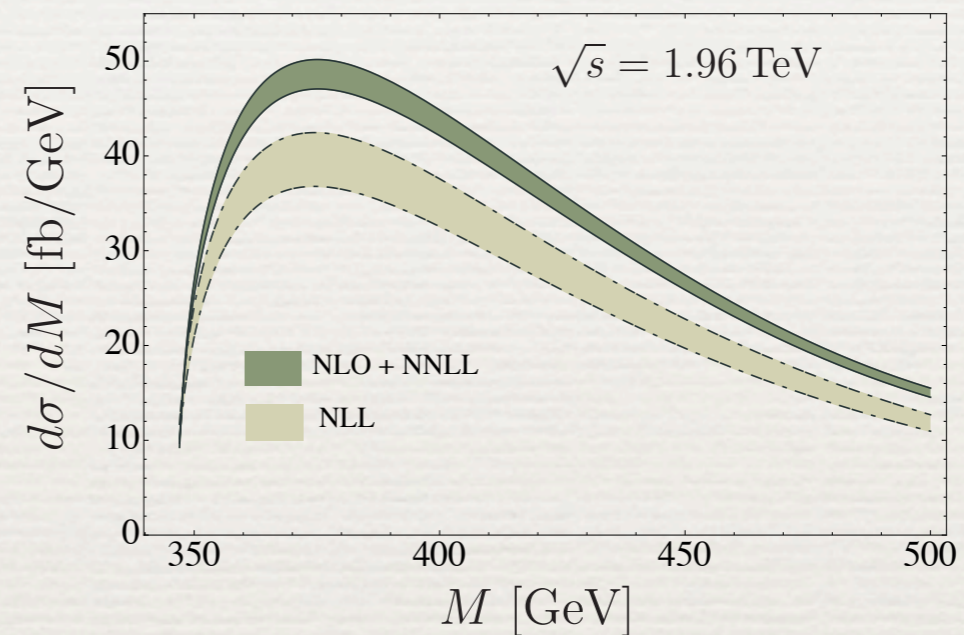
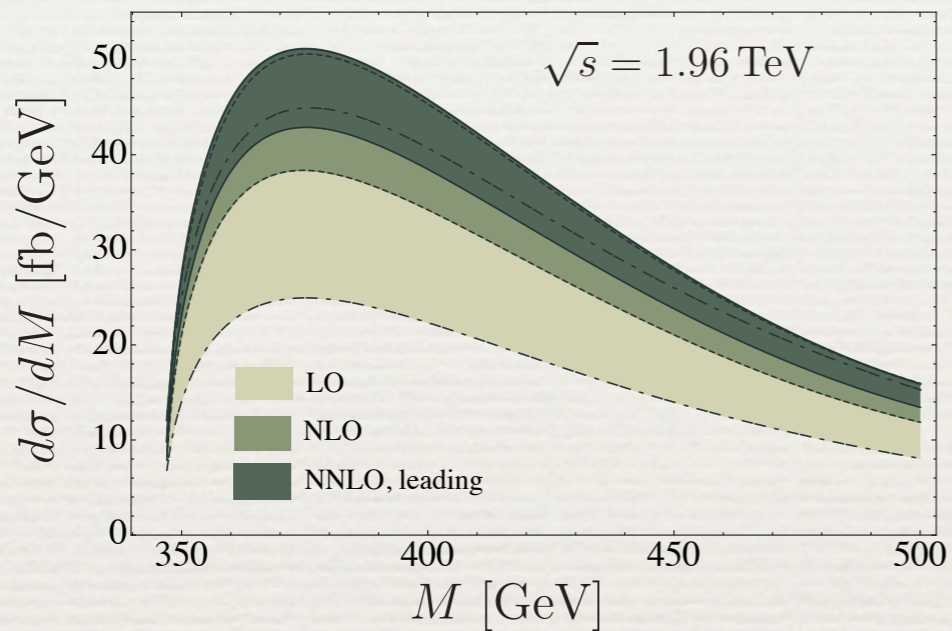


- ◆ Leading singular terms near partonic threshold  $z = M^2/\hat{s} \rightarrow 1$  give dominant contributions even at low and moderate  $M$  values



# Invariant mass distributions

- Fixed-order vs. resummed PT (matched to NLO):



NNLO  
(partial)

NLO

LO

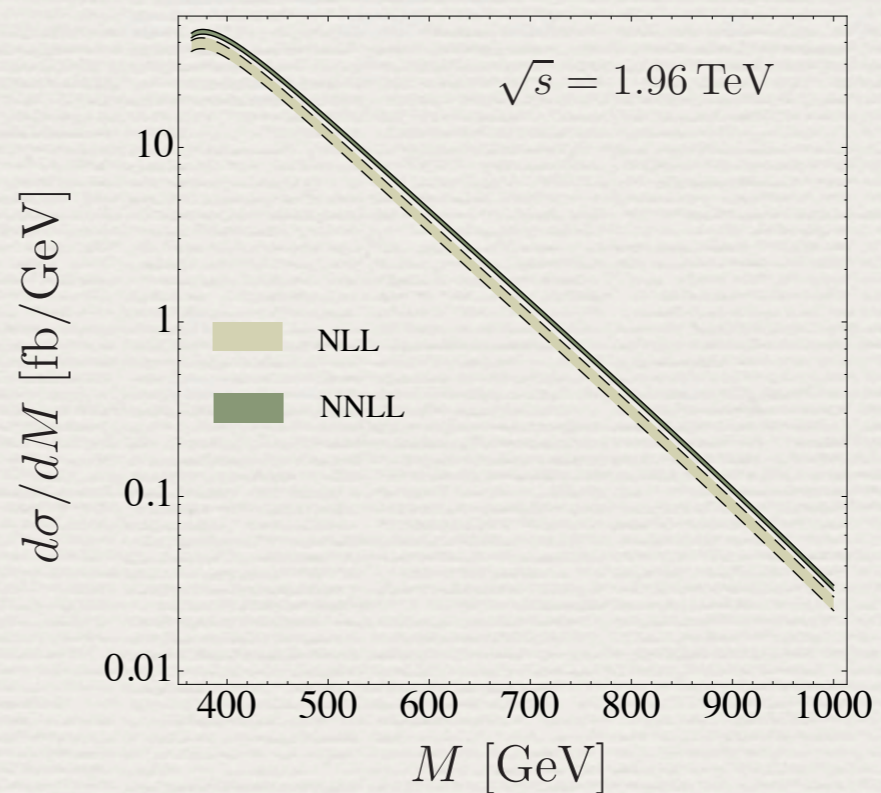
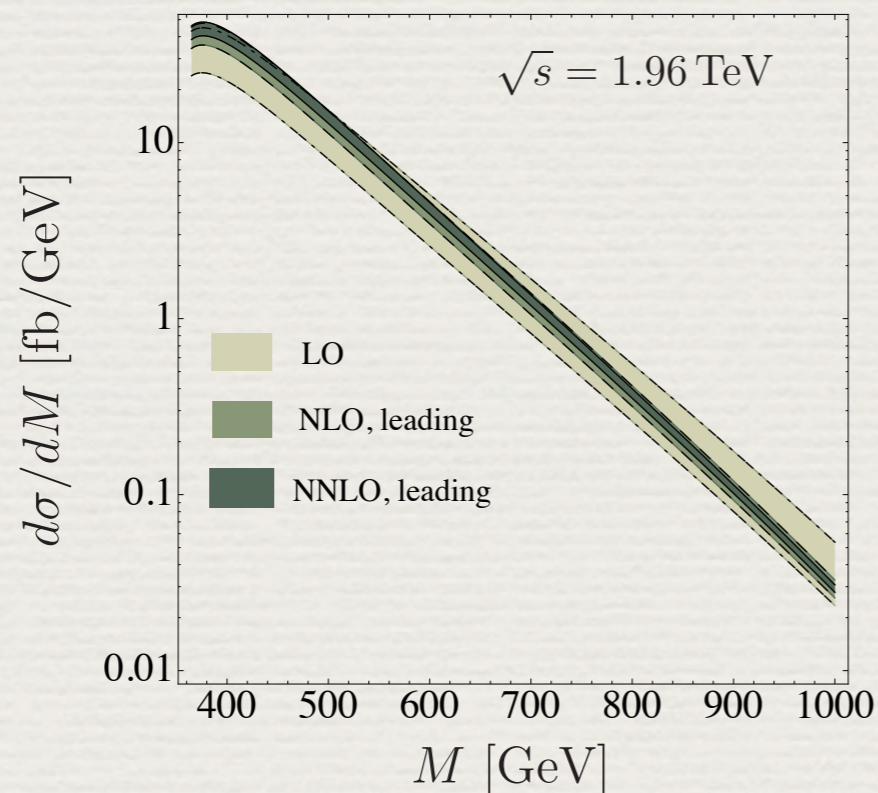
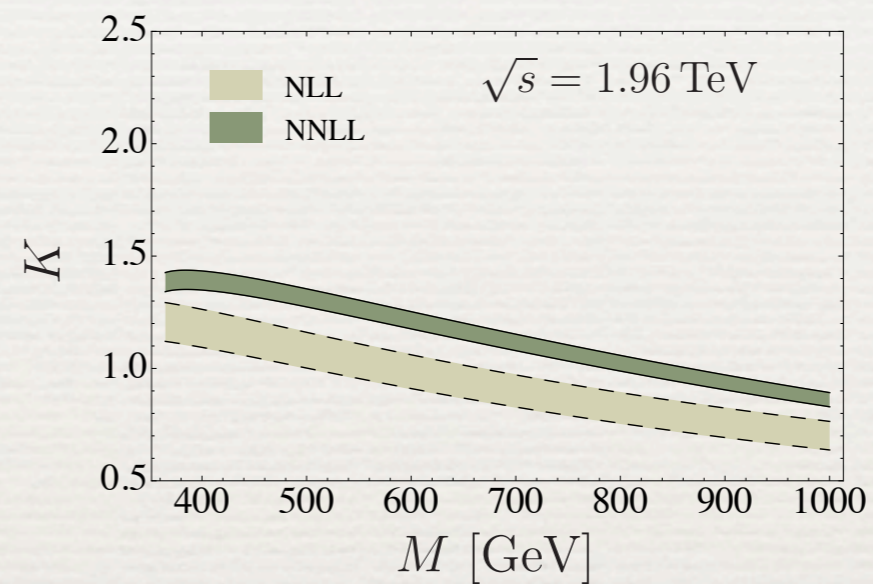
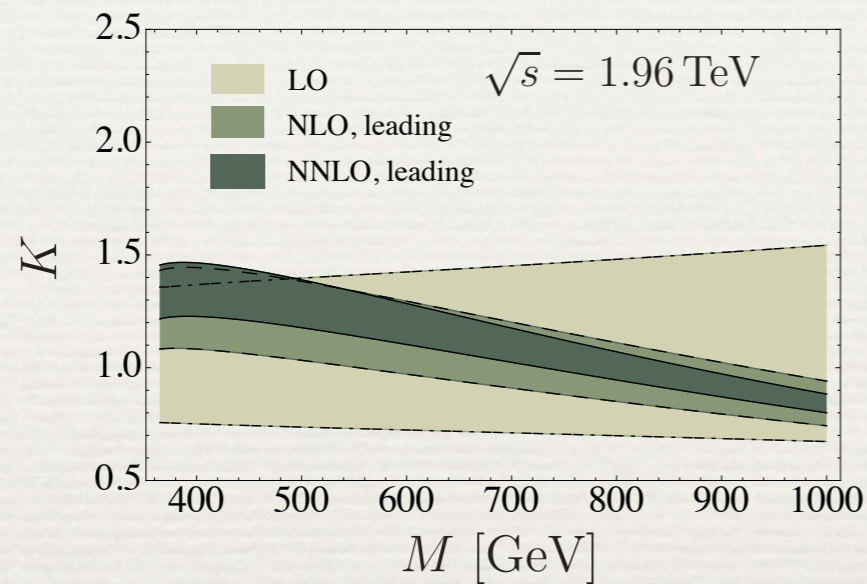
NNLL+NLO

NLL+LO



# Invariant mass distributions

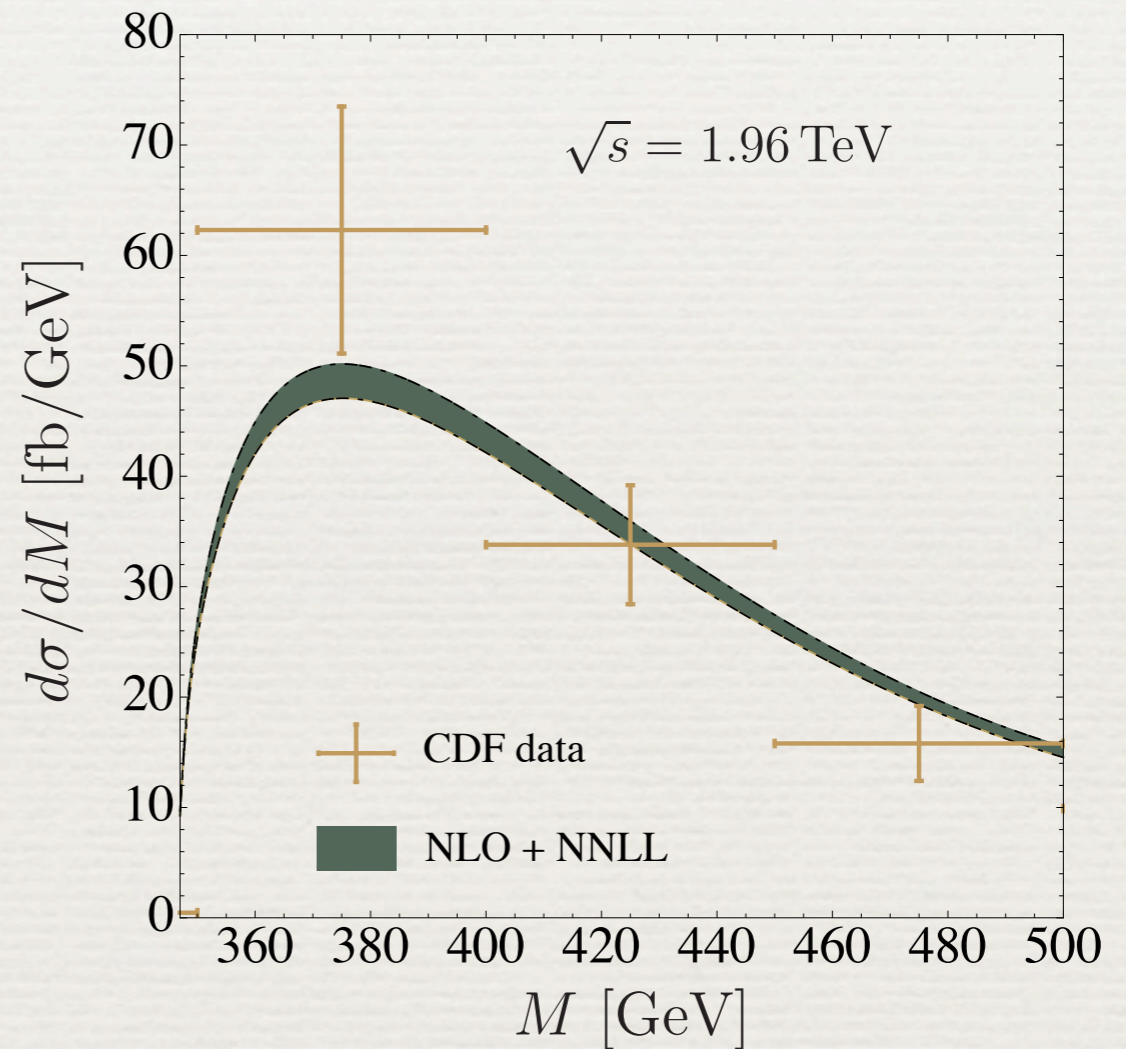
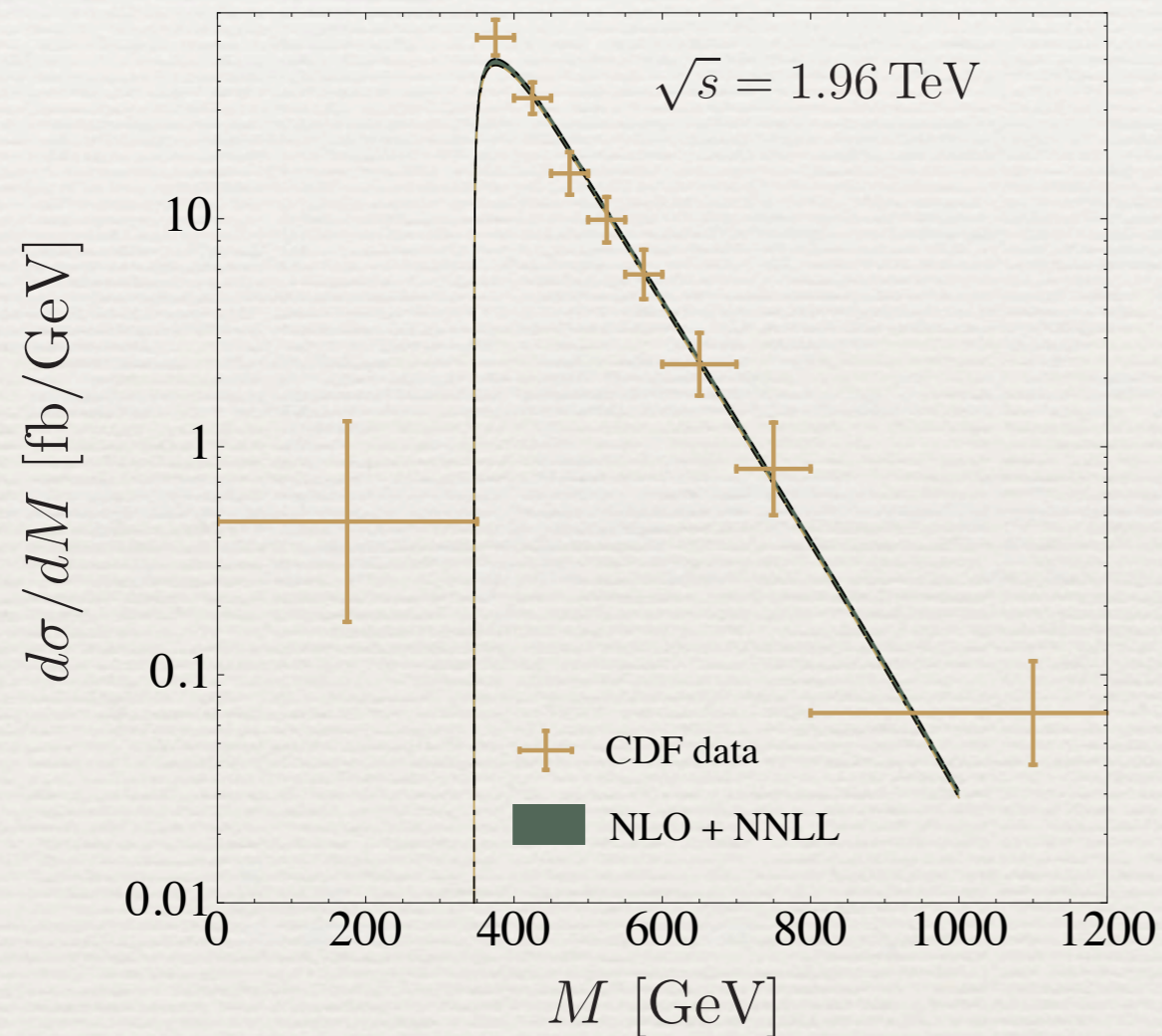
## ◆ High-mass region (Tevatron):





# Comparison with CDF data

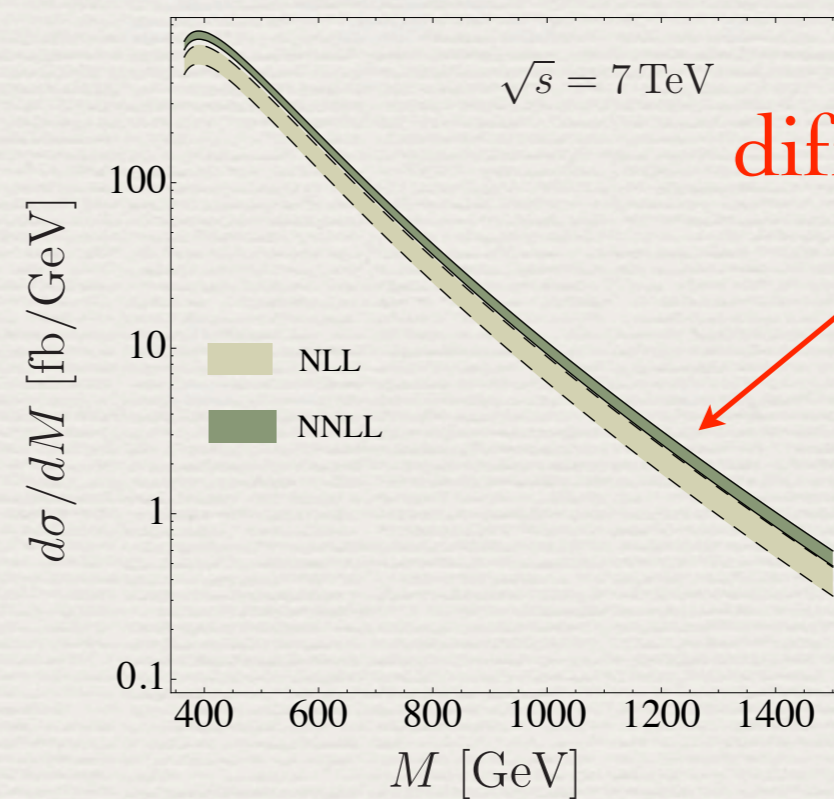
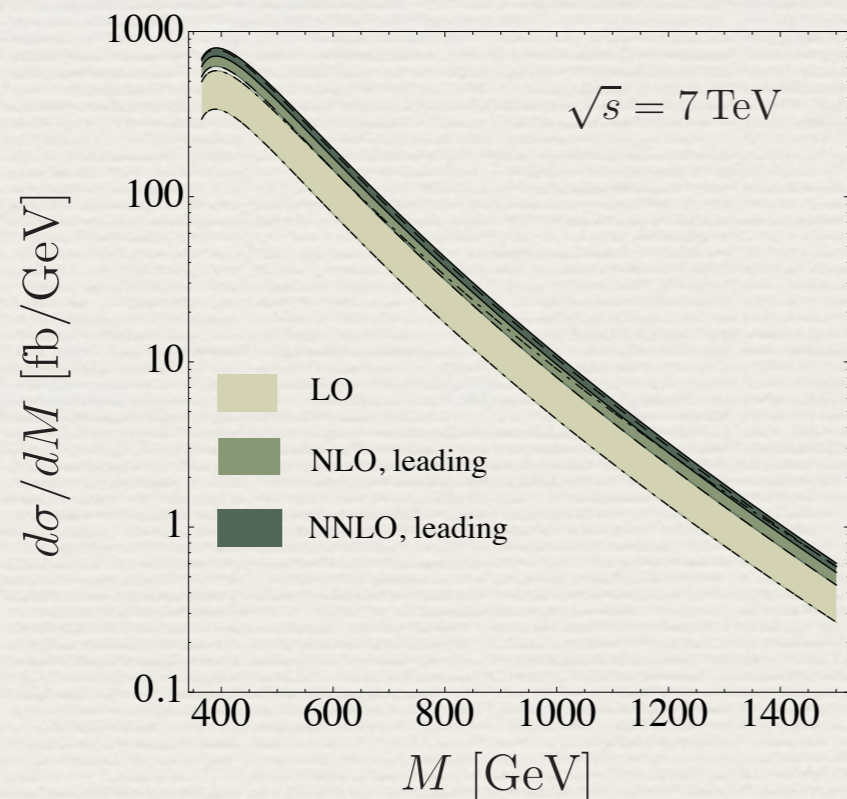
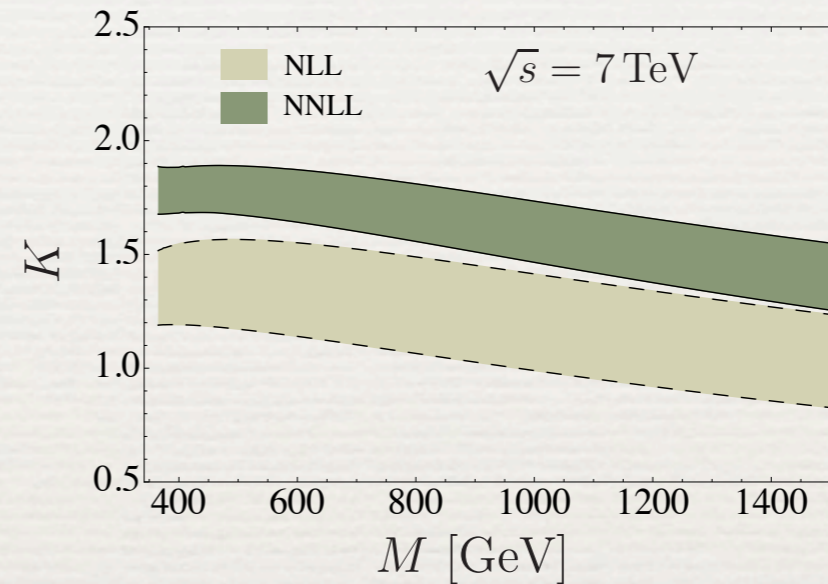
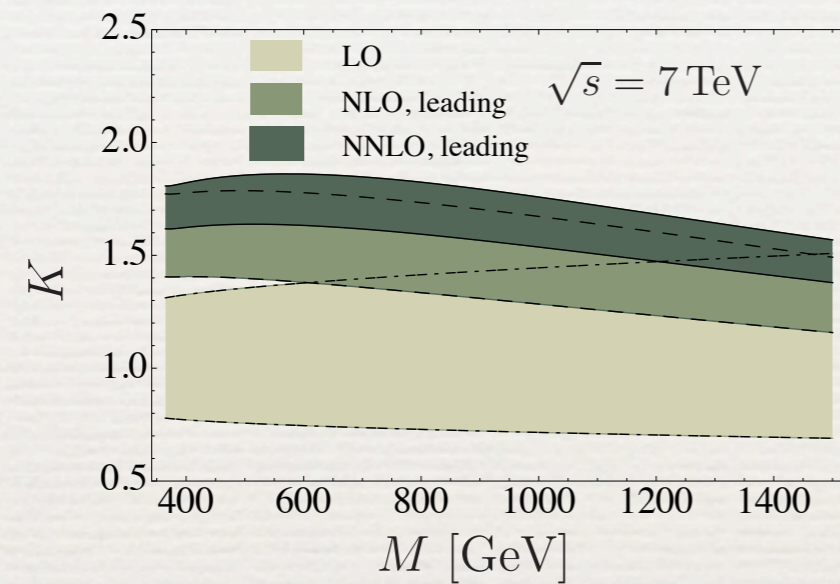
- ◆ Overlay (not a fit!) for  $m_t=173.1$  GeV:





# Invariant mass distributions

## ◆ High-mass region (LHC):

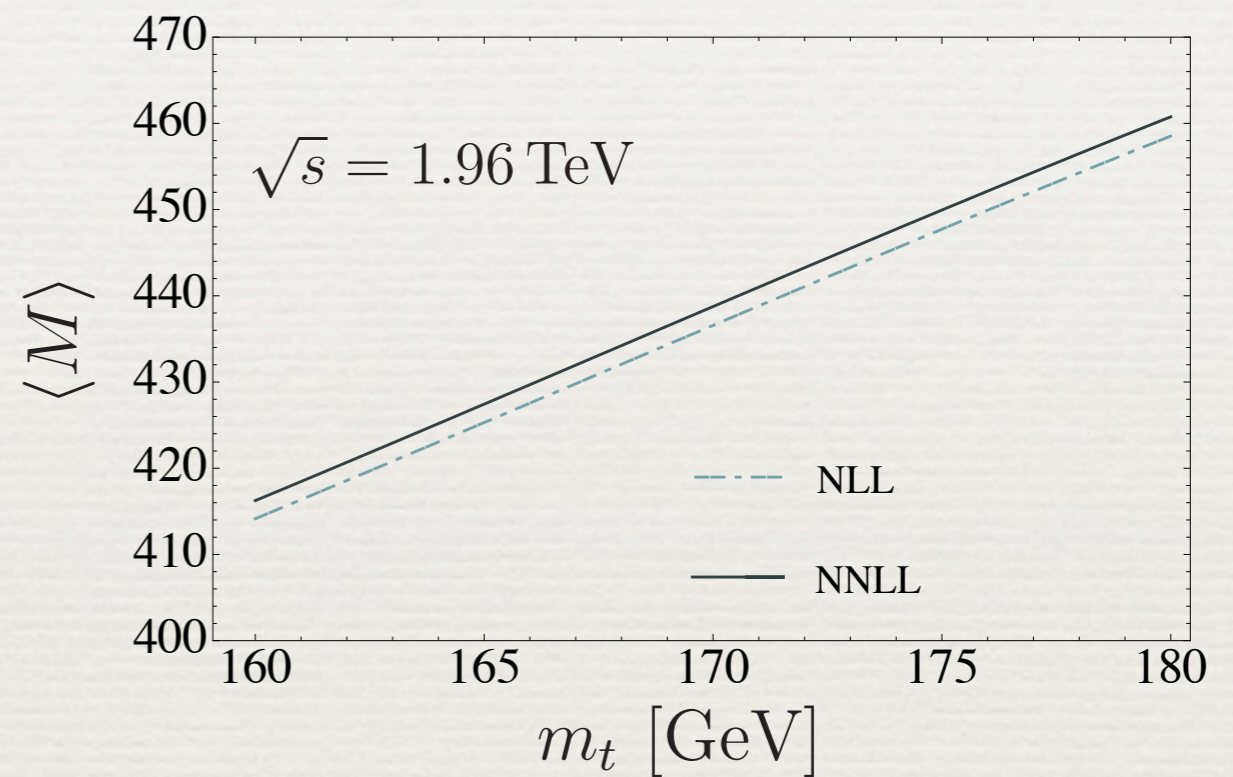
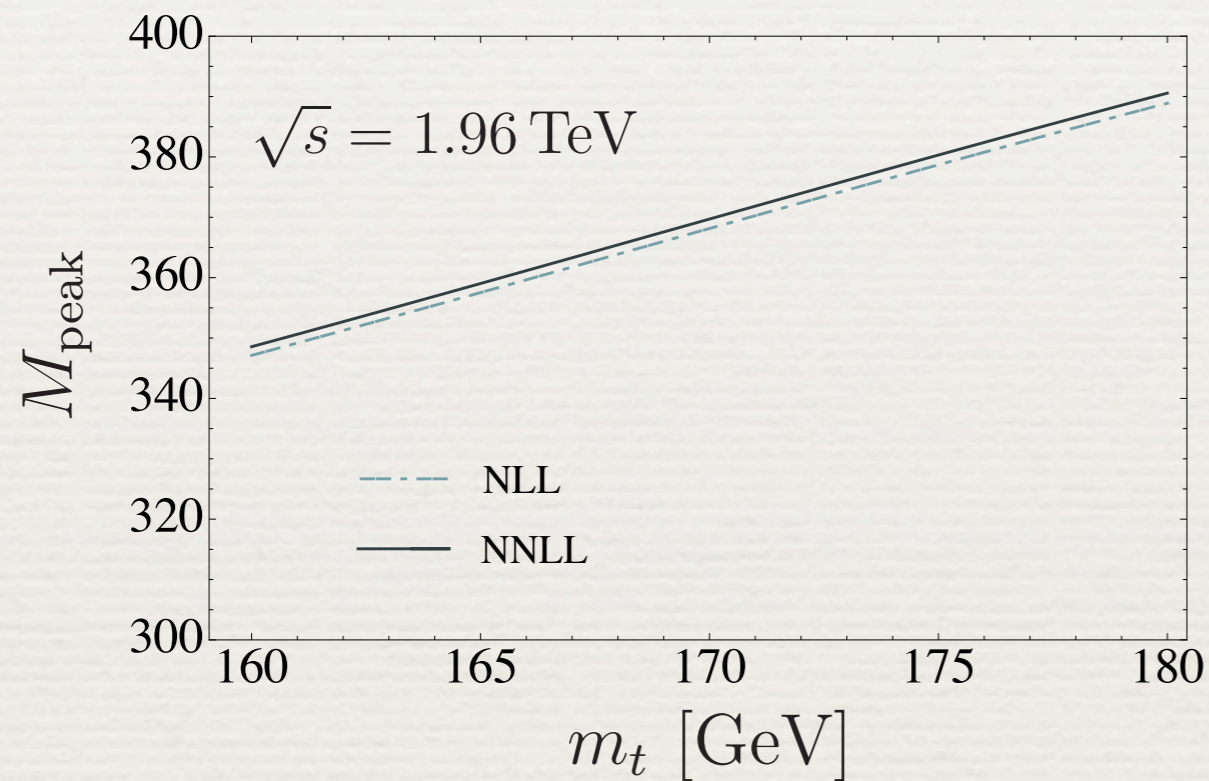


different shape!



# Features of inv. mass distribution

- ♦ Mass dependence (pole scheme):

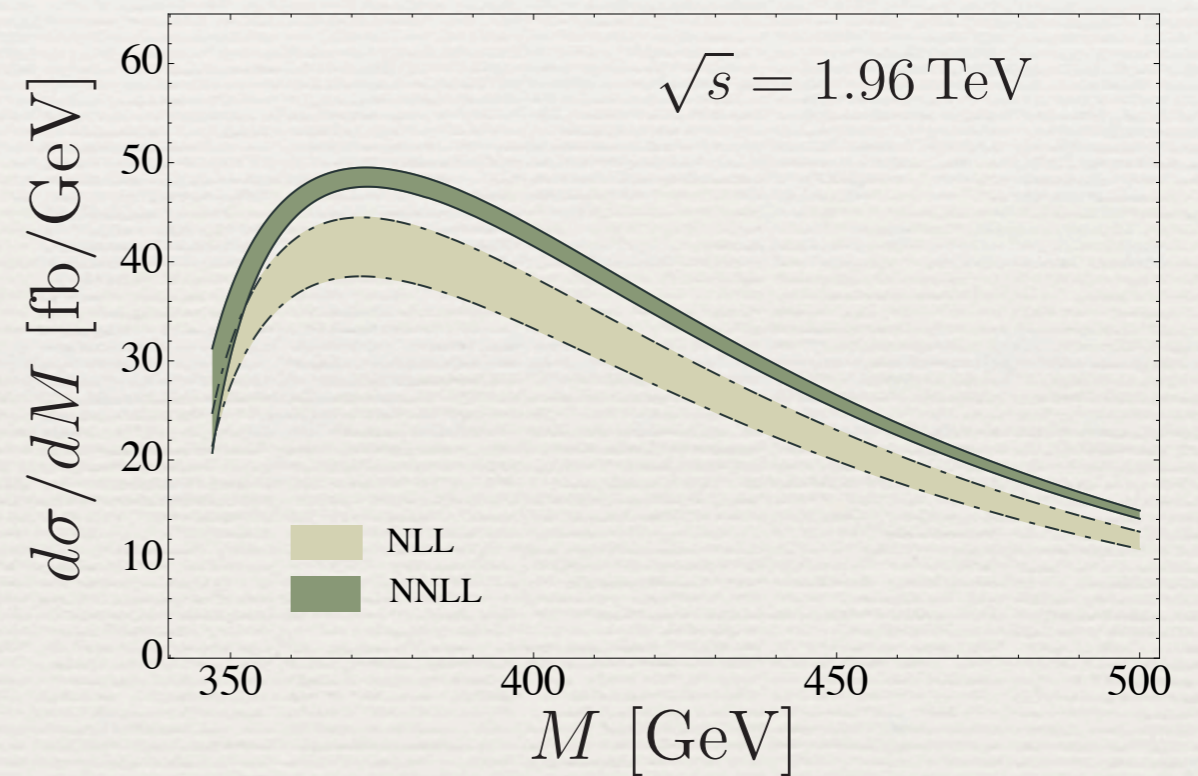
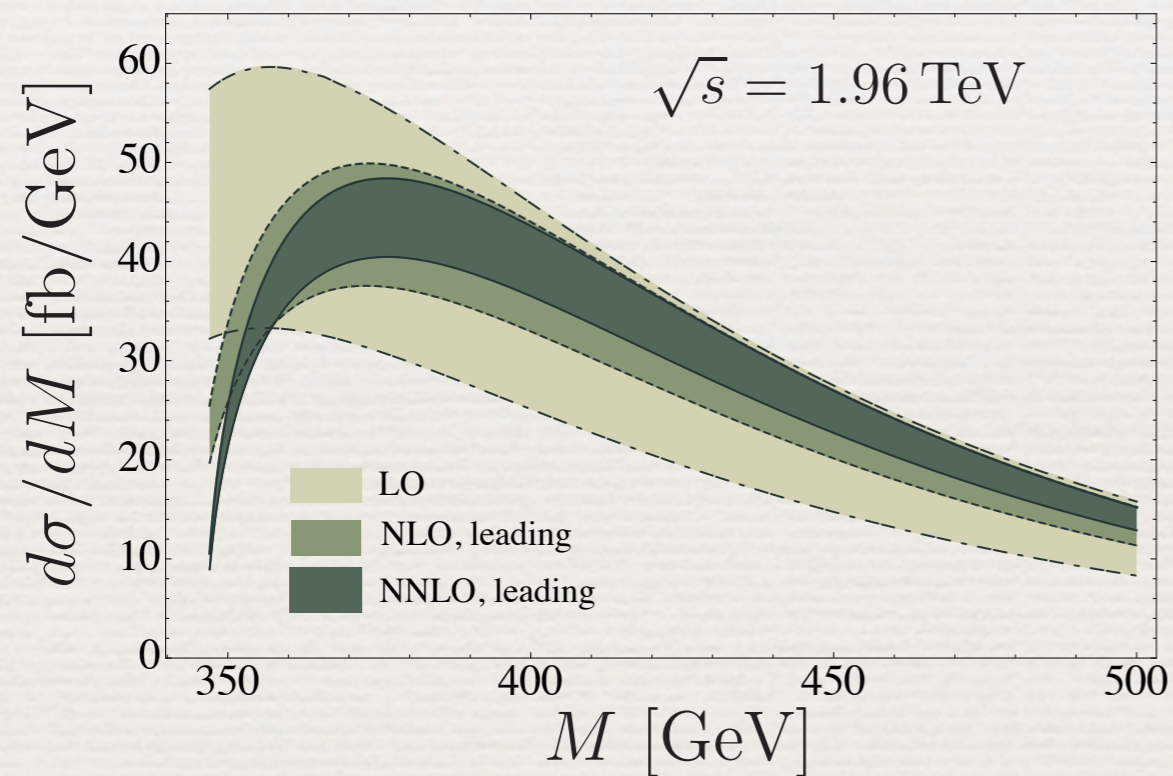


- ♦ In future, this may provide **high-precision determination** of top-quark mass



# Features of inv. mass distribution

- ◆ Spectrum predictions in  $\overline{\text{MS}}$  scheme, obtained with  $\overline{m}_t(\overline{m}_t) = 164.0 \text{ GeV}$ :

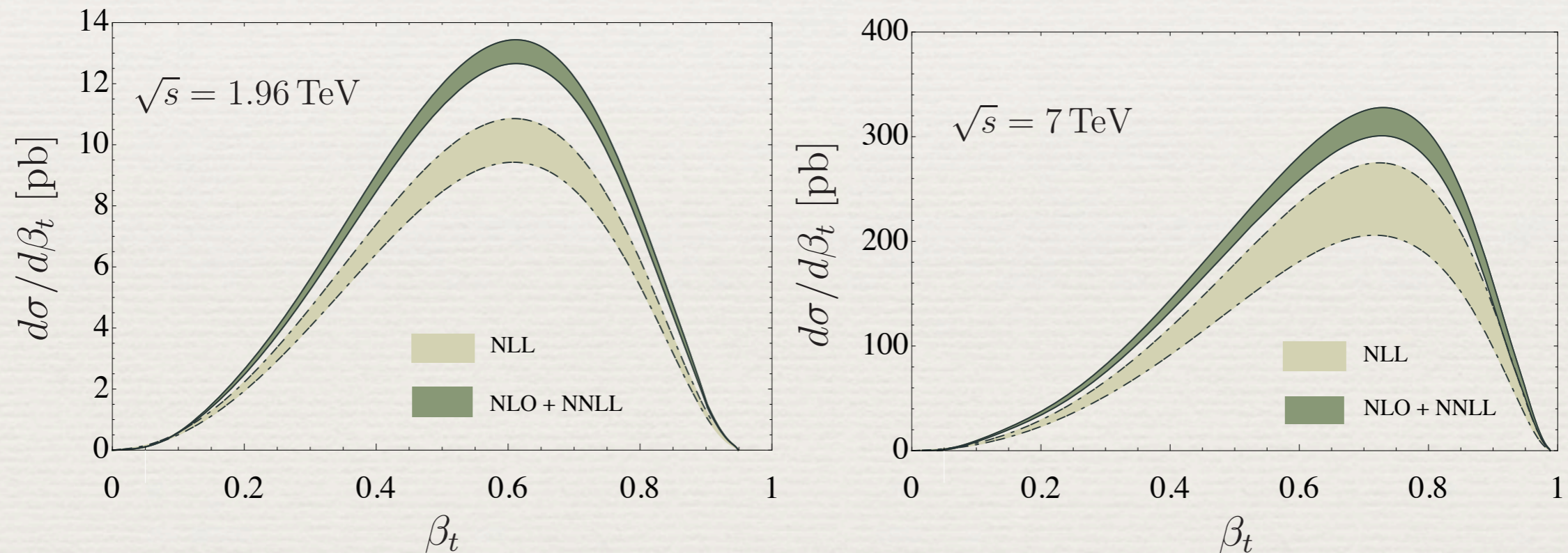


- ◆ Improved convergence [see also: Langenfeld, Moch, Uwer 2009](#)



# Velocity distribution

- ◆ Transform to relative 3-velocity of top quarks in  $t\bar{t}$  rest frame:  $\beta_t = \sqrt{1 - \frac{4m_t^2}{M^2}}$

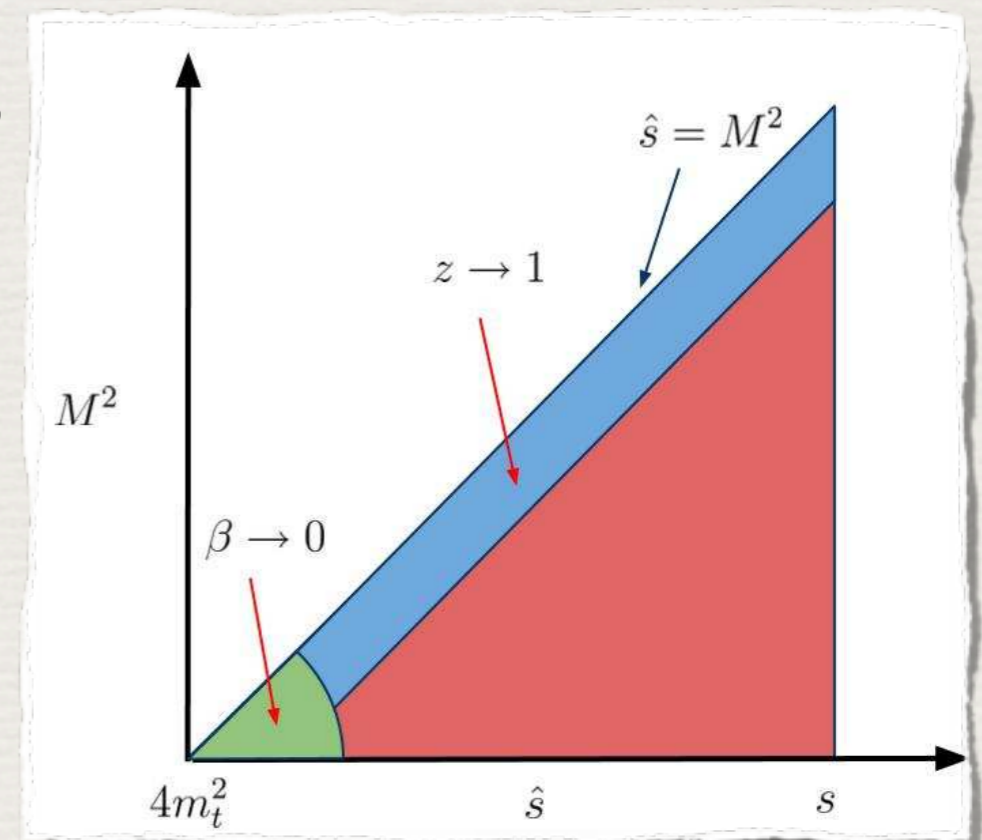


- ◆ Top quarks are predominantly relativistic,  $\beta_t \sim 0.4-0.9$  not small



# Total cross section

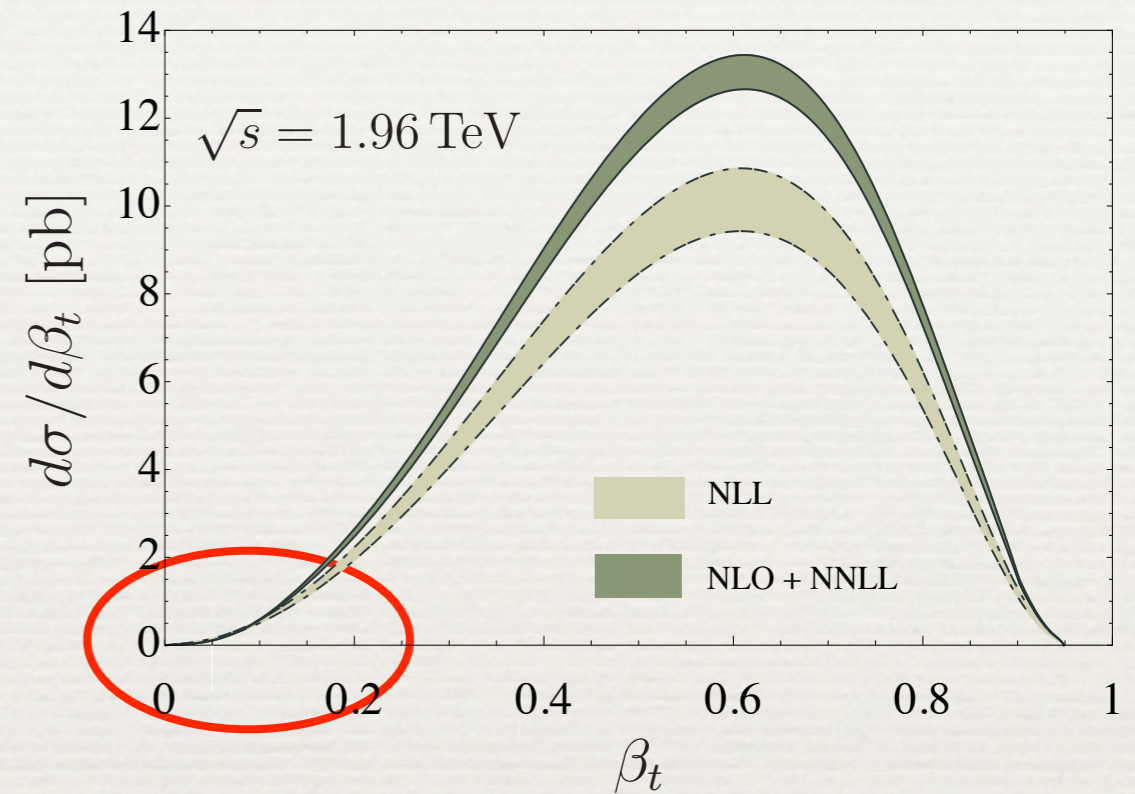
- ♦ Computed at NLO already in 1988
- ♦ Usually, resummation is done around absolute threshold at  $\hat{s}=4m_t^2$
- ♦ Mixed Coulomb and soft gluon singularities arise for  $\beta = \sqrt{1 - 4m_t^2/\hat{s}} \rightarrow 0$
- ♦ Obtain partial NNLO results based on small- $\beta$  expansion  
*Moch, Uwer 2008; Beneke et al. 2009*
- ♦ But this covers only a tiny of phase space!





# Total cross section

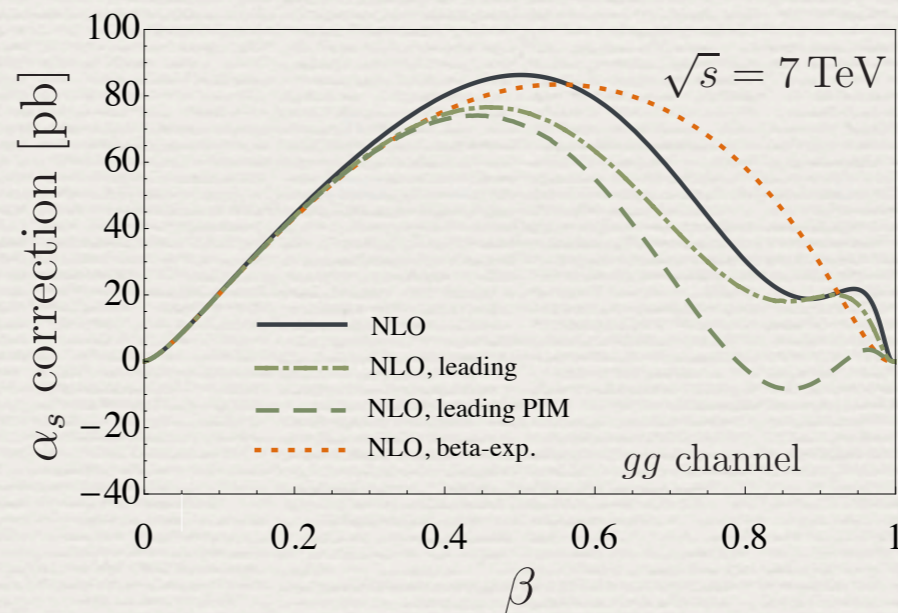
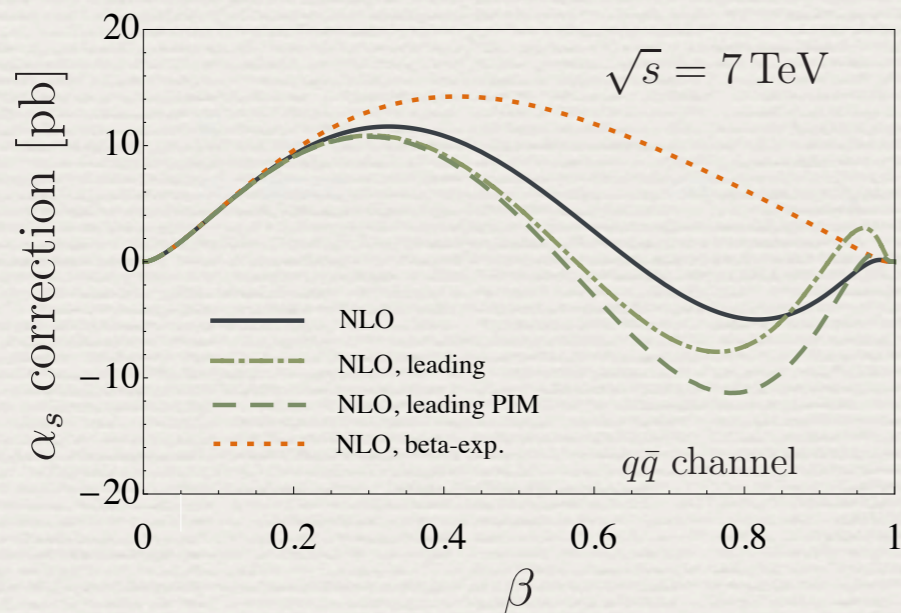
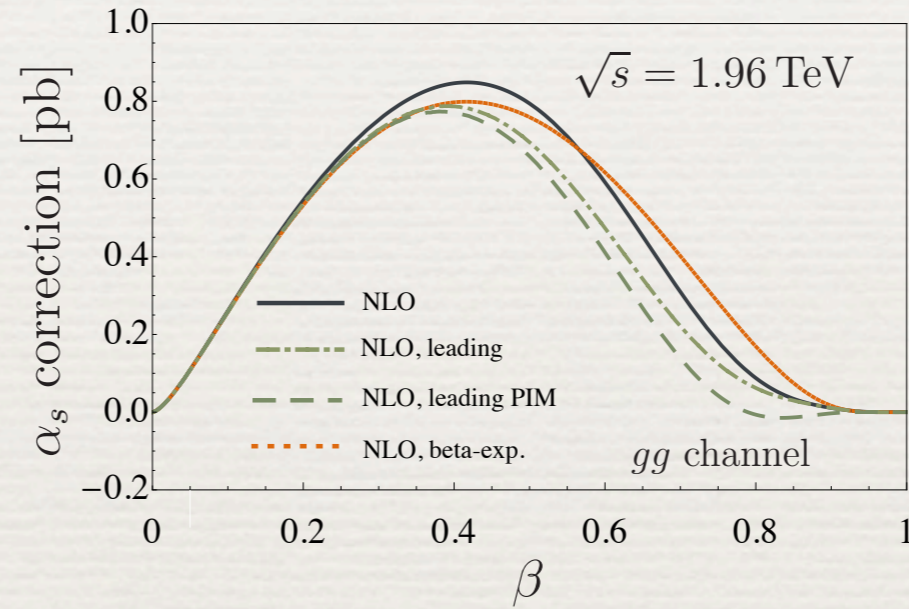
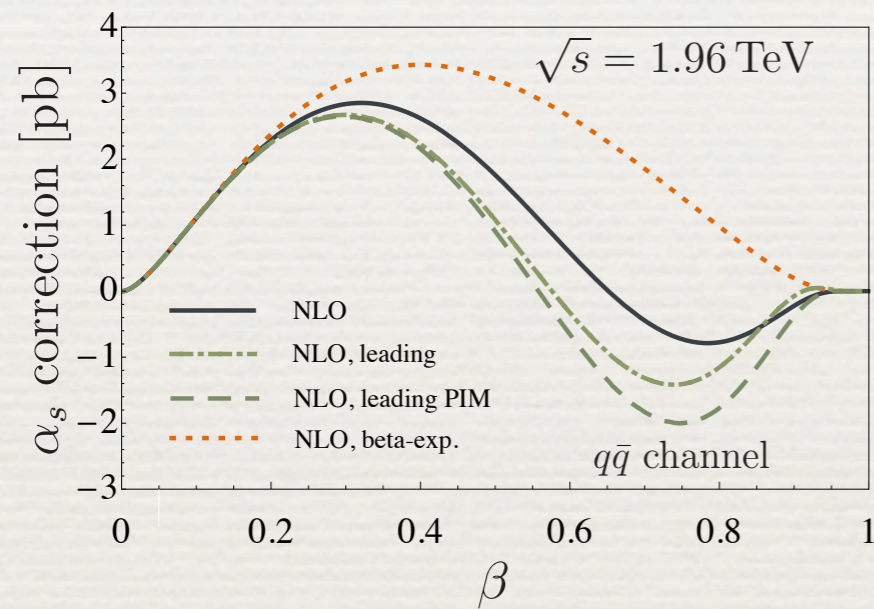
- ◆ Fact that  $\beta \geq \beta_t$  and shape of  $\beta_t$  distribution imply that **small- $\beta$  region is unimportant** for the total cross section
- ◆ In our approach, soft gluon effects are resummed also far above absolute threshold
- ◆ Different systematics & more accurate results!





# Total cross section

- Comparison of approximations to NLO corrections (including parton luminosities):




$$\beta = \sqrt{1 - \frac{4m_t^2}{\hat{s}}}$$



# Total cross section

- ◆ Detailed predictions for total cross sections:

Cross section (pb)	Tevatron	LHC (7 TeV)	LHC (10 TeV)	LHC (14 TeV)
$\sigma_{\text{LO}}$	$4.49^{+1.71+0.24}_{-1.15-0.19}$	$84^{+29+4}_{-20-5}$	$217^{+70+10}_{-49-11}$	$495^{+148+19}_{-107-24}$
$\sigma_{\text{NLL}}$	$5.07^{+0.37+0.28}_{-0.36-0.18}$	$112^{+18+5}_{-14-5}$	$276^{+47+10}_{-37-11}$	$598^{+108+19}_{-94-19}$
$\sigma_{\text{NLO, leading}}$	$5.49^{+0.78+0.31}_{-0.78-0.20}$	$134^{+16+7}_{-17-7}$	$341^{+34+14}_{-38-14}$	$761^{+64+25}_{-75-26}$
$\sigma_{\text{NLO}}$	$5.79^{+0.79+0.33}_{-0.80-0.22}$	$133^{+21+7}_{-19-7}$	$341^{+50+14}_{-46-15}$	$761^{+105+26}_{-101-27}$
$\sigma_{\text{NLO+NNLL}}$	$6.30^{+0.19+0.31}_{-0.19-0.23}$	$149^{+7+8}_{-7-8}$	$373^{+17+16}_{-15-16}$	$821^{+40+24}_{-42-31}$
$\sigma_{\text{NNLO, approx}}$ (scheme A)	$6.14^{+0.49+0.31}_{-0.53-0.23}$	$146^{+13+8}_{-12-8}$	$369^{+34+16}_{-30-16}$	$821^{+71+27}_{-65-29}$
$\sigma_{\text{NNLO, approx}}$ (scheme B)	$6.05^{+0.43+0.31}_{-0.50-0.23}$	$139^{+9+7}_{-9-7}$	$349^{+23+15}_{-23-15}$	$773^{+47+25}_{-50-27}$

scale uncertainty  PDF uncertainty


- ◆ Singular terms dominate NLO corrections
- ◆ Resummation stabilizes scale dependence



# Total cross section

- Small- $\beta$  expansion misses important NLO effects:

Cross section (pb)	Tevatron	LHC (7 TeV)	LHC (10 TeV)	LHC (14 TeV)
$\sigma_{\text{NLO}}$	$5.79^{+0.79+0.33}_{-0.80-0.22}$	$133^{+21+7}_{-19-7}$	$341^{+50+14}_{-46-15}$	$761^{+105+26}_{-101-27}$
$\sigma_{\text{NLO, leading}}$	$5.49^{+0.78+0.31}_{-0.78-0.20}$	$134^{+16+7}_{-17-7}$	$341^{+34+14}_{-38-14}$	$761^{+64+25}_{-75-26}$
$\sigma_{\text{NLO, } \beta\text{-exp. v1}}$	$6.59^{+0.96+0.38}_{-0.95-0.25}$	$151^{+15+8}_{-18-8}$	$386^{+30+15}_{-39-16}$	$863^{+49+29}_{-73-30}$
$\sigma_{\text{NLO, } \beta\text{-exp. v2}}$	$8.22^{+0.54+0.49}_{-0.88-0.33}$	$157^{+12+8}_{-16-8}$	$395^{+24+14}_{-36-15}$	$877^{+49+29}_{-73-30}$
$\sigma_{\text{NLO+NNLL}}$	$6.30^{+0.19+0.31}_{-0.19-0.23}$	$149^{+7+8}_{-7-8}$	$373^{+17+16}_{-15-16}$	$821^{+40+24}_{-42-31}$
$\sigma_{\text{NNLO, } \beta\text{-exp. v1}}$	$6.98^{+0.17+0.37}_{-0.40-0.27}$	$156^{+2+8}_{-6-8}$	$394^{+2+16}_{-10-17}$	$871^{+0+29}_{-14-31}$
$\sigma_{\text{NNLO, } \beta\text{-exp.+potential v1}}$	$6.95^{+0.16+0.36}_{-0.39-0.26}$	$159^{+3+8}_{-7-8}$	$401^{+6+17}_{-12-17}$	$888^{+7+30}_{-19-32}$
$\sigma_{\text{NLO, } \beta\text{-exp. v2}}$	$8.22^{+0.54+0.49}_{-0.88-0.33}$	$157^{+12+8}_{-16-8}$	$395^{+24+14}_{-36-15}$	$877^{+49+29}_{-73-30}$
$\sigma_{\text{NNLO, } \beta\text{-exp.+potential v2}}$	$7.30^{+0.00+0.39}_{-0.18-0.28}$	$158^{+3+8}_{-6-8}$	$398^{+7+16}_{-13-17}$	$880^{+12+29}_{-22-31}$

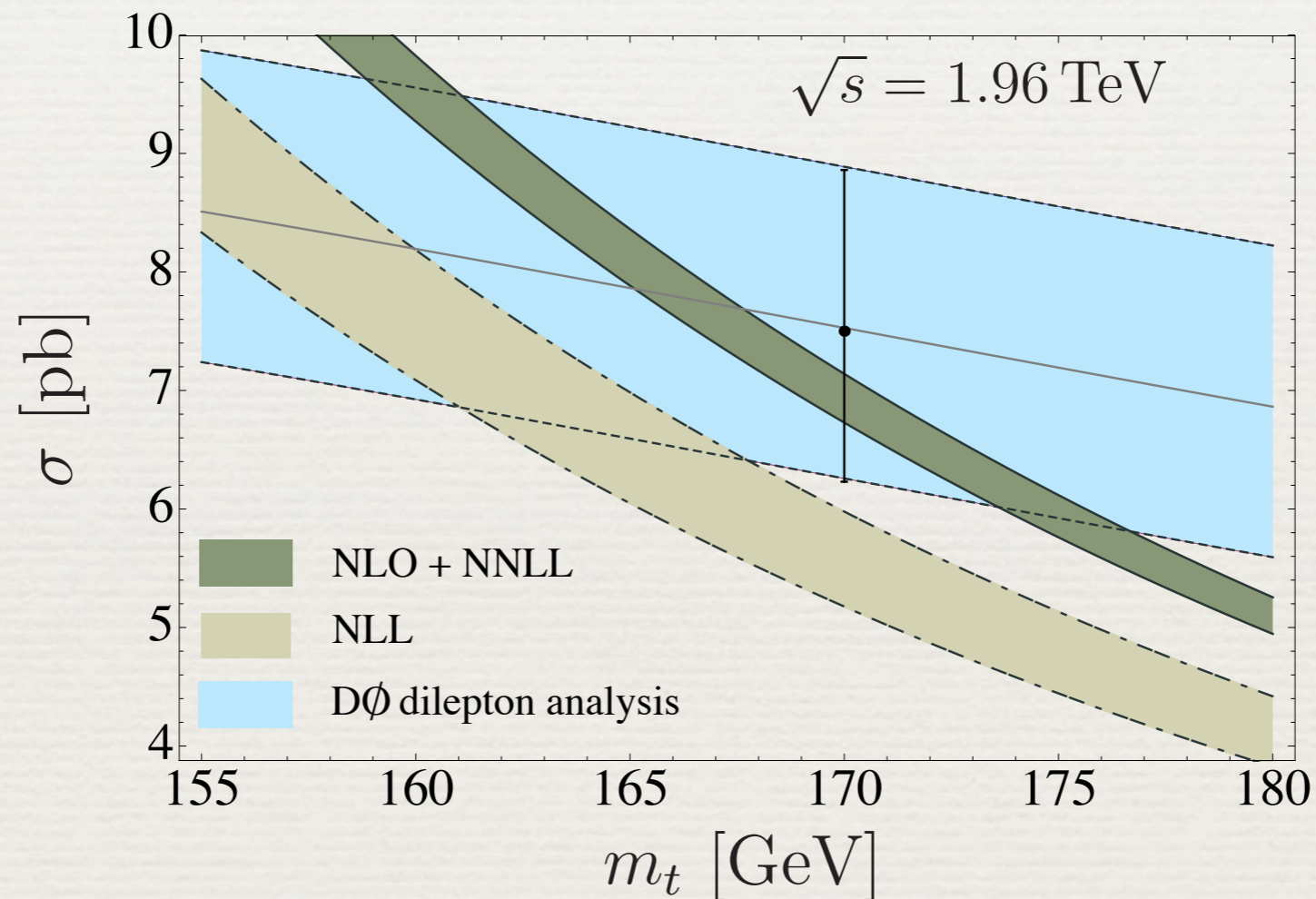
scale uncertainty  PDF uncertainty

- Likely that this remains true at NNLO



# Total cross section

- ♦ Mass dependence (pole scheme):

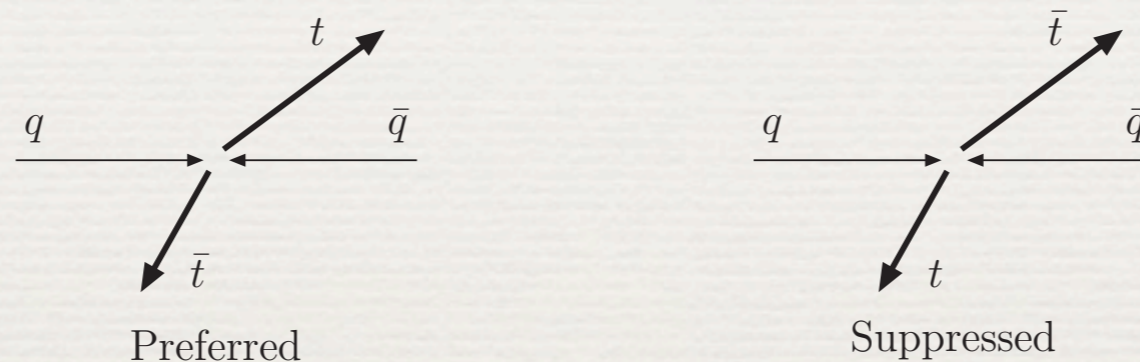


- ♦ Extract  $m_t = (166.4^{+10.3}_{-7.6})$  GeV, in agreement with world average  $m_t = (173.1 \pm 1.3)$  GeV



# Forward-backward asymmetry

- ♦ At Tevatron, top-quark are emitted preferably in direction of incoming quark



- ♦ Define inclusive asymmetry:

$$A_{\text{FB}}^t \equiv \frac{\int_{4m_t^2}^s dM \left( \int_0^1 d \cos \theta \frac{d^2 \sigma^{N_1 N_2 \rightarrow t \bar{t} X}}{dM d \cos \theta} - \int_{-1}^0 d \cos \theta \frac{d^2 \sigma^{N_1 N_2 \rightarrow t \bar{t} X}}{dM d \cos \theta} \right)}{\int_{4m_t^2}^s dM \left( \int_0^1 d \cos \theta \frac{d^2 \sigma^{N_1 N_2 \rightarrow t \bar{t} X}}{dM d \cos \theta} + \int_{-1}^0 d \cos \theta \frac{d^2 \sigma^{N_1 N_2 \rightarrow t \bar{t} X}}{dM d \cos \theta} \right)}$$

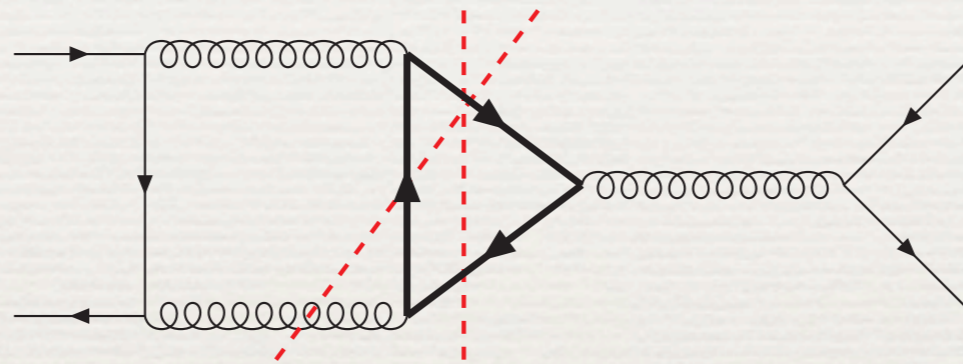
- ♦ Surprising result by CDF:

$$A_{\text{FB}}^t |_{\text{exp}} = (19.3 \pm 6.9)\%$$



# Forward-backward asymmetry

- Non-zero contributions arise first at one-loop order, from interference terms such as:



- Predictions:

	$0.2 < \mu_f/\text{TeV} < 0.8$		$m_t/2 < \mu_f < 2m_t$	
	$\Delta\sigma_{\text{FB}}$ [pb]	$A_{\text{FB}}^t$ [%]	$\Delta\sigma_{\text{FB}}$ [pb]	$A_{\text{FB}}^t$ [%]
NLL	$0.29^{+0.16}_{-0.16}$	$5.8^{+3.3}_{-3.2}$	$0.31^{+0.16}_{-0.17}$	$5.9^{+3.4}_{-3.3}$
NLO, leading	$0.19^{+0.09}_{-0.06}$	$5.2^{+0.4}_{-0.4}$	$0.31^{+0.16}_{-0.10}$	$5.7^{+0.5}_{-0.4}$
NLO	$0.25^{+0.12}_{-0.07}$	$6.7^{+0.6}_{-0.4}$	$0.40^{+0.21}_{-0.13}$	$7.4^{+0.7}_{-0.6}$
NLO+NNLL	$0.40^{+0.06}_{-0.06}$	$6.6^{+0.6}_{-0.5}$	$0.45^{+0.08}_{-0.07}$	$7.3^{+1.1}_{-0.7}$
NNLO, approx (scheme A)	$0.37^{+0.10}_{-0.08}$	$6.4^{+0.9}_{-0.7}$	$0.48^{+0.11}_{-0.10}$	$7.5^{+1.3}_{-0.9}$
NNLO, approx (scheme B)	$0.34^{+0.08}_{-0.07}$	$5.8^{+0.8}_{-0.6}$	$0.45^{+0.09}_{-0.09}$	$6.8^{+1.1}_{-0.8}$



# Conclusions

- ◆ Effective field theory provides **efficient tools** for addressing difficult collider-physics problems
- ◆ Systematic “derivation” of factorization theorems (known ones and ones to be discovered) and simple, transparent **resummation techniques**
- ◆ Detailed applications exist for Drell-Yan, Higgs, and top-quark pair production
- ◆ Longer-term goal is to understand resummation at NNLL+NLO order for **jet processes**, such as  $pp \rightarrow n \text{ jets} + V/H$  at LHC (with  $n \leq 3$ ,  $V = \gamma, Z, W$ )