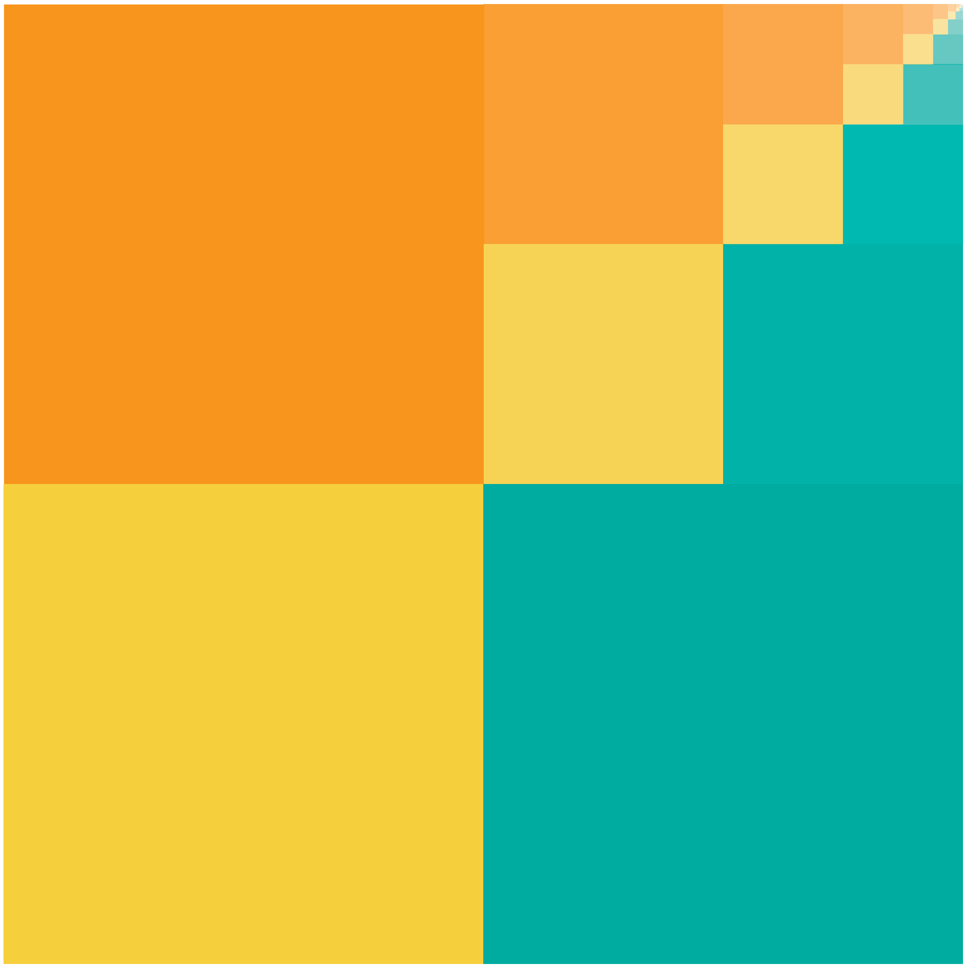

The IRS Uses Geometric Series?

Michelle Ghrist

While researching tax code relating to a church raffle, I discovered that the Internal Revenue Service had used an infinite geometric series to determine the quoted tax withholding rate if an organization chooses to cover the tax on the item being raffled. I subsequently turned this real-world application of infinite series into a writing assignment for my integral calculus class.

About fifteen years ago, I was the payroll and tax specialist on a small church's finance team. The pastor decided to hold a raffle to raise money for the church's building fund. The raffle prize was a donated ATV with a fair market value (FMV) of about \$4000. Speculating that people were hesitant to buy tickets because of the large tax bill if they won, the pastor decided that the church would pay the tax withholdings for the winner; he announced this before discussing the tax implications with anyone, so I began researching how much this generous offer was going to cost the church.

I learned that the Internal Revenue Service requires that Forms W-2G and 5754 be filed whenever gambling winnings exceed a certain amount. Form W-2G is filed by the organization giving the winnings (termed the "payer"), and the winner (termed the "payee") must include Form 5754 when filing their taxes. At that time, the IRS's *Instructions for Forms W-2G and 5354* said under the heading "Noncash Payments" that winnings below a certain level are subject to a 25% regular gambling withholding tax on the FMV of the prize (bit.ly/2ZFVGcD).

My initial thought was that the church would need to pay $0.25 \times \$4000 = \$1000$ to the IRS. However, I then wondered if this extra \$1000 payment would then be considered part of the prize and therefore also subject to 25% withholding, requiring the church to give 0.25 × \$1000 = \$250 more to the IRS. But then this \$250 would also be part of the prize and subject to withholding, with this process continuing forever.

I got quite excited about the possibility of an infinite geometric series being necessary to implement IRS tax code. By my calculations, the amount that church would owe was a convergent infinite geometric series with first term $a_0 =$ \$1000 and common ratio $r = 0.25$:

$$\$4000\left(0.25+(0.25)^2+(0.25)^3+\cdots\right)=\$4000\times\frac{0.25}{1-0.25}$$
$$=\frac{1}{3}(\$4000),$$

giving an effective tax rate of $33\,{}^{1}/_{3}$%.

I then read more of the instructions (bit.ly/2ZFVGcD), which clarified that if the payer pays the withholding tax for the payee, "the withholding is 33.33% of the FMV of the noncash payment minus the amount of the wager." It was satisfying to discover the behind-the-scenes math leading to that number.

Thus, the church actually owed 0.3333 × \$4000 = \$1333.20 for the pastor's generous offer. While we saved 13 cents due to the IRS rounding down the effective tax rate, we still owed \$333.20 more than we initially thought.

I recently turned this discovery into one of a series of application-focused writing assignments for my integral calculus course. I provided students with all of the above information other than the equations, the effective tax rate of $33\,{}^{1}/_{3}$%, and the term "geometric series." Students were asked to explain how the 25% withholding rate (if paid by the payee) reconciles with the 33.33% withholding rate (if paid by the payer) and to tell me how much money the church had to pay to the IRS; I also expected them to comment on the rounding of the effective withholding rate.

This exercise proved challenging for many students as they struggled to apply the appropriate mathematics to this scenario and then to find a clear way to explain their reasoning. However, many students commented afterwards that they really appreciated seeing a real-world application of infinite series, even though this exercise stretched them outside their comfort zones (which I thought was a positive outcome).

Note: Starting in January 2018, the payee withholding rate was reduced from 25% to 24% (bit.ly/2WRFy5L). Thus, the payer withholding rate should have changed to

$$\left(0.24+(0.25)^2+(0.24)^3+\cdots\right)=\frac{0.24}{1-0.24}=\frac{6}{19}\approx 0.315789.$$

The IRS's actual payer withholding rate is currently 31.58% (bit.ly/2L5khmQ), meaning that the IRS is now rounding up the effective tax rate rather than down. While these percentages are a little more complicated for students to work with than the pre-2018 percentages, this may actually make it easier to figure out how to reconcile the two different numbers, as I found that some students got tripped up on how ${}^{1}/_{4}$ turned into ${}^{1}/_{3}$.

In any event, I am glad to know that the IRS can properly apply infinite geometric series. ■

Michelle Ghrist joined Gonzaga University in 2017 after teaching at the U.S. Air Force Academy for fifteen years. She enjoys music, hiking, reading, and spending time with her family and dog.