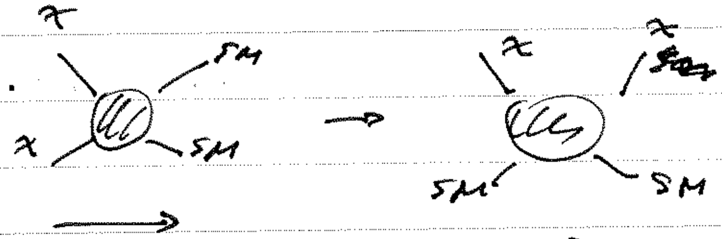
LECTURE # 2

ON THE MENU:

1. UNDERSTAND $\langle \sigma v \rangle$

? ?? v_{REL} ? BUT NOT LORENTZ INV...

2.



DECOUPLING → DIRECT DETECTION

3. ... BUT SAME CONJUGATED PROCESS, IN EARLY UNIVERSE: KINETIC DECOUPLING

SETS SMALL-SCALE CUT-OFF TO MATTER POWER SPECTRUM

[PROBABLY NEXT LECTURE]

1. $\langle \sigma v \rangle$ STARTING POINT: BOLTZMANN EQUATION

$$\hat{L}[f] = \hat{C}[f]$$

$f = f(\vec{x}, \vec{p}, t)$ PHASE-SPACE DENSITY

$\hat{L}$ → LIOUVILLE OP.: CHANGES IN PHASE SPACE DENSITY

$\hat{C}$ → COLLISION OP.: PARTICLES # DENSITY CHANGES PER PHASE-SPACE VOLUME

$$\hat{L}_{NR} = \frac{d}{dt} + \frac{dx}{dt}\vec{\nabla}_x + \frac{d\vec{v}}{dt}\vec{\nabla}_{\vec{v}}$$

$$\hat{L}_{COVARIANT}: p^\alpha \frac{\partial}{\partial x^\alpha} - \Gamma^\alpha_{\beta\gamma} p^\beta p^\gamma \frac{\partial}{\partial p^\alpha}$$

IN FRW COSMOLOGIES (HOMOGENEOUS, ISOTROPIC) $f = f(E, t)$

WE WANT TO CALCULATE NUMBER DENSITIES:

$$n(t) = \sum_{spin} \int \frac{d^3p}{(2\pi)^3} f(E, t)$$

... INTERESTING DERIVATION OF RESULTING EQ FOR $n(t)$:

GONDOLO + GELMINI NPB $\underline{360}$ (1991) 145

$$\begin{matrix} 1+2 & \longleftrightarrow & \underbrace{3+4}_{\text{IN TH. EQ.}} \\ \chi\ \chi & & \end{matrix}$$

$$\int L(f) \frac{d^3p}{(2\pi)^3} \to \frac{dn}{dt} + 3 \underbrace{H}_{\dot{a}/a} n$$

$$//$$

THE $\int C[f] \frac{d^3p}{(2\pi)^3} \longrightarrow -\langle \sigma v_{Møl} \rangle \left(n_1 n_2 - n_1^{eq} n_2^{eq} \right)$

(INTEGRATED OVER MOMENTUM)

WHERE:

- $\sigma \equiv \sum_f \sigma_{12 \to f}$ TOTAL $1+2 \to$ ANYTHING INVARIANT ANNIHIL. CROSS SECTION
- $v_{Møl} \equiv \dfrac{\sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2}}{E_1 E_2}$

SO THAT (a). $v_{Møl} n_1 n_2$ IS LORENTZ-INV.

(b). $v_{Møl} \to v_{Rel} = |\vec{v}_1 - \vec{v}_2|$

IN REST FRAME OF 1 (OR 2)

$\vec{v}_1 \equiv \vec{p}_1 / E_1$

- THERMAL AVERAGE

$$\langle \sigma v \rangle = \frac{\int \sigma v \, e^{-\frac{E_1}{T}} e^{-\frac{E_2}{T}} d^3p_1 d^3p_2}{\int e^{-\frac{E_1}{T}} e^{-\frac{E_2}{T}} d^3p_1 d^3p_2}$$

- MEANING OF VARIOUS TERMS

HW SHOW THE DENOMINATOR IS, FOR $m_1 = m_2 = m$

$$\Big(\underbrace{4\pi m^2 T\, K_2\!\left(\tfrac{m}{T}\right)}\Big)^2$$

MODIFIED BESSEL FUNCTION OF 2^{nd} ORDER

<u>NUMERATOR</u>:
$$\int_{4m^2}^{\infty} \underbrace{\sigma\,(s-4m^2)}_{\text{CROSS SECTION}}\ \underbrace{\sqrt{s}\,K_1\!\left(\frac{\sqrt{s}}{T}\right)}_{\text{THERMAL KERNEL}}\, ds$$

..KEY TO UNDERSTAND IMPORTANT CAVEATS TO, e.g. $\langle\sigma v\rangle \sim 3\times10^{-26}\,\frac{cm^3}{s}$

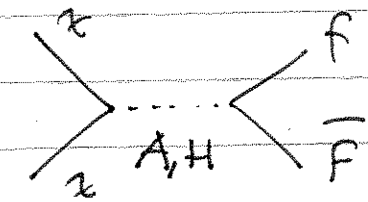
CLASSIC PAPER: GRIEST + SECKEL PRD <u>43</u> (1991) 3191

("3 EXCEPTIONS IN THE CALCULATION OF RELIC ABUNDANCES")

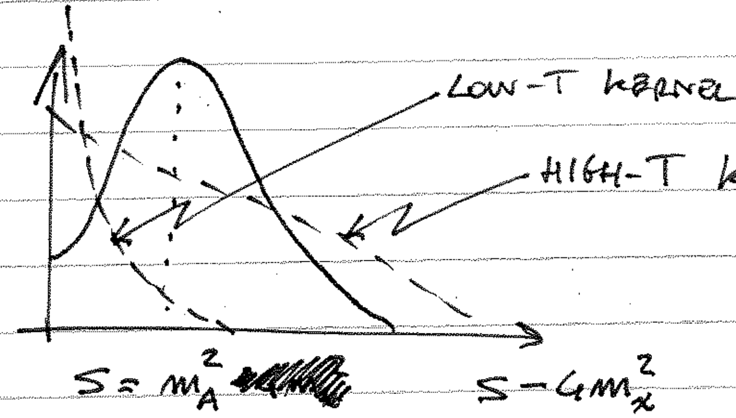
1. RESONANCES
2. THRESHOLDS
3. COANNIHILATION

— ALL 3 EXCEPTIONS RELEVANT e.g TO SUSY DM!

1. RESONANCES (MADE POPULAR BY "FUNNEL REGION" OF mSUGRA / CMSSM)

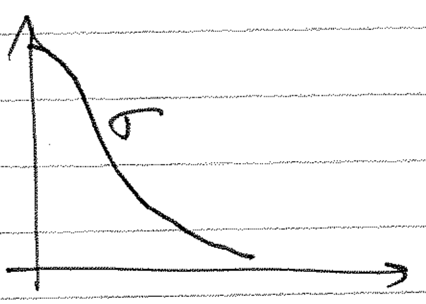


e.g.: $m_A^2 \gtrsim 4 m_\chi^2$



So $\langle \sigma v \rangle (T=0)$ RELEVANT FOR INDIRECT DETECTION CAN BE MUCH <u>SMALLER</u> THAN $\langle \sigma v \rangle (T = T_{f.o.})$

OPPOSITE CAN ALSO BE TRUE: SUPPOSE $m_A^2 \lesssim 4 m_\chi^2$



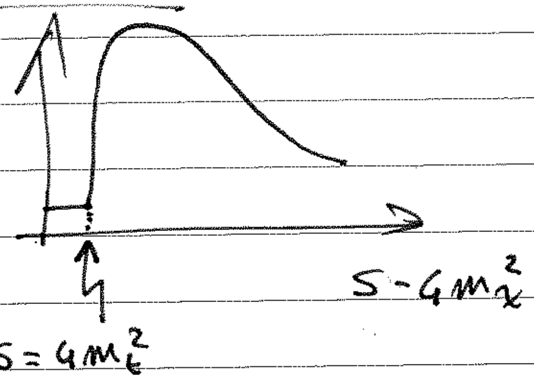
EFFECT OF RESONANCE CAN BE NEGLIGIBLE AT F.O. (ESPECIALLY IF $\Gamma_A \ll T_{f.o.}$!) BUT DRAMATIC AT $T = 0$!

$\Rightarrow \langle \sigma v \rangle_{f.o.} \ll \langle \sigma v \rangle_0$

<u>BUT</u> ONE MORE CAVEAT: e.g. IN SUSY

$\tilde{\chi}_1^0$ ARE MAJORANA: AT $T=0$ (S WAVE) $\uparrow\downarrow$ AND CP-ODD! $\rightarrow$ $\chi\chi \xrightarrow{h} f\bar{f}$ VANISHES IF h IS CP EVEN!

2. THRESHOLDS: OBVIOUS EFFECT



e.g. $m_{\tilde{\chi}_1^0} \lesssim m_t$

ALWAYS PRODUCES $\langle \sigma v \rangle_{f.o.} > \langle \sigma v \rangle_0$

3. COANNIHILATIONS

RELEVANT FOR "COUPLED" DECOUPLING PROCESSES,

IF e.g. $\underbrace{m_2 - m_1}_{\Delta m_2} \lesssim T_{f.o.}$ (OTHERWISE 2 DIES BY BOLTZMANN SUPPRESSION)

$$\langle \sigma v \rangle \longrightarrow \langle \sigma_{eff} v \rangle = \frac{\sum'_{ij} \sigma_{ij} \, e^{-\frac{\Delta m_i + \Delta m_j}{T}}}{\sum_{i=1}^{N}{}' g_i \, e^{-\frac{\Delta m_i}{T}}}$$

COUNTS THE EFFECT OF THE ADDITIONAL DEGREE OF FREEDOM, SUITABLY WEIGHTED

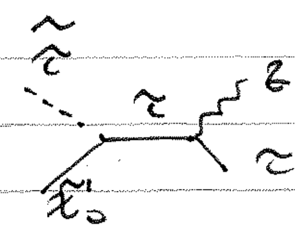
TWO SCENARIOS
1. "PARASITE" COANNIHILATION (DENOMINATOR DOMINATES)
2. "SYMBIOTIC" COANNIHILATION (NUMERATOR DOMINATES)

1: UED $\begin{matrix} \vdots \\ e^{(1)} \\ B^{(1)} \end{matrix}$ COMPRESSED SPECTRUM,

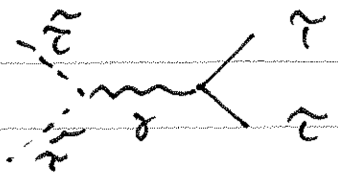
TON OF COUPLED D.O.F., OVERALL GIVES SUPPRESSION EFFECT

2: STAU / CHARGINO / STOP COANNIHILATION IN MSSM

e.g.: $\tilde{\tau}\tilde{\chi} \to \tau Z$



$\tilde{\tau}\tilde{\tau}^{(*)} \to \tau\tau^{(*)}$



LOW # EXTRA D.O.F., BUT EM INTERACTING!

SO FAR: CAVEATS TO LHS IN $\Gamma(= n \cdot \sigma) = H$

WHAT HAPPENS IF H IS CHANGED?

GOOD EXAMPLE: QUINTESSENCE FIELD (SALATI, 2003; PROFUMO+ULLIO, 2003)

ϕ — SPATIALLY HOMOGENEOUS SCALAR, R FIELD

$$\rho_\phi = \frac{1}{2}\left(\frac{d\phi}{dt}\right)^2 + V(\phi) \longrightarrow \text{e.g.: } M_p^4 \exp\left(-\frac{\lambda\phi}{M_p}\right)$$

$$P_\phi = \frac{1}{2}\left(\frac{d\phi}{dt}\right)^2 - V(\phi)$$

SO $P_\phi = w\rho_\phi$ YIELDS $w = \begin{cases} +1 & \text{"KINATION"} \\ \downarrow & \\ -1 & \text{"COSMOLOGICAL CONSTANT"} \end{cases}$

A CROSS-SYMMETRIC PROCESS TO PAIR-ANNIHILATION IS SCATTERING OFF OF SM PARTICLES

• IN "LATE UNIVERSE": $\chi \underbrace{q}_{} \to \chi q$

e.g. QUARK IN LOW-BCKG ENV.

↓

DIRECT DETECTION

... EFFECTIVELY: SCATTERING OFF OF A NUCLEUS

• IN "EARLY UNIVERSE", $\chi f \leftrightarrow \chi f$ IS PROCESS THAT KEEPS χ IN KINETIC EQUILIBRIUM

KINETIC DECOUPLING SIGNALS START OF (BOTTOM-UP) FORMATION OF STRUCTURE, i.e. CUT-OFF TO SMALLEST DARK MATTER HALOS ⟶ IMPORTANT FOR COSMO SIMULATIONS, ALSO INDIRECT DM DETECTION ("BOOST FACTOR")

START WITH DIRECT DETECTION: WHAT ARE ENERGY SCALES?

$$\bar{v}_\odot \sim 200 \frac{km}{s}$$

SO: DEEPLY NON RELATIVISTIC! $\frac{v}{c} \sim 10^{-3}$

$$\left[\cancel{\text{TYPICAL DM MOMENTUM} \sim \frac{\bar{v}_\odot}{c} m_\chi \sim 0.1\,GeV\left(\frac{m_\chi}{100\,GeV}\right)}\right]$$

TYPICAL MOMENTUM TRANSFER: STANDARD SCATTERING THEORY:

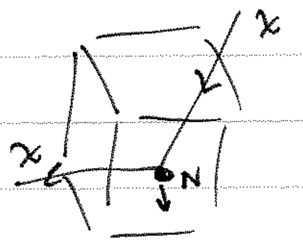
$$|\vec{q}|^2 = 2 m_{red}^2 v^2 (1-\cos\vartheta^*) \qquad m_{red} = \frac{m_\chi m_N}{m_\chi + m_N}$$

m_N → NUCLEUS MASS

TYPICAL ENERGY TRANSFER: $Q = |\vec{q}|^2/(2m_N) = \frac{m_{red}^2 v^2}{m_N}(1-\cos\vartheta^*)$

FOR WIMP IN 10 – 500 GeV RANGE HITTING A NUCLEUS WITH $m_N \sim 1 - 200$ GeV, $Q \sim 1 - 100$ keV

HOW DOES ONE DETECT THIS ENERGY DEPOSITION?



IONIZATION ON SOLIDS

XENON, ZEPLIN

CDMS, EDELWEISS

IONIZATION IN SCINTILLATORS (DETECTING γ'S)

CRESST II

TEMPERATURE INCREASE (DETECT PHONONS)

HOW DO WE CALCULATE EVENT RATE FOR A GIVEN DM THEORY?

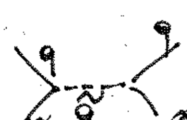
IT'S A MULTI-LAYERED PROBLEM

NUCLEONS

MATRIX ELEMENTS OF QUARK, GLUONS IN A NUCLEON STATE

QUARK - GLUON

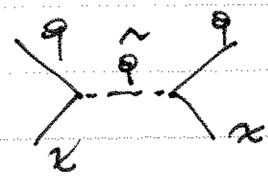
EFFECTIVE LAGRANGIAN, e.g.: $\mathcal{L} = f_q \chi \chi \bar{q} q + \dots$



$\boxed{1}$ QUARK - GLUON

EFFECTIVE NR LAGRANGIAN FROM MICROSCOPIC THEORY

e.g. $\mathcal{L} = f_q \, \bar{\chi}\chi \, \bar{q}q + \dots$

FROM  ... MORE LATER

$f_q \sim \frac{1}{m_{\tilde{q}}^2}$

$\boxed{2}$ NUCLEONS (p, n)

USE MATRIX ELEMENTS OF QUARKS AND GLUONS IN A NUCLEON STATE (FROM χPT, SCATTERING EXP. ...)

e.g.:

SCALAR INTERACTION $\langle n | m_q \bar{q} q | n \rangle = m_n \, f_{Tq}^{(n)}$ FROM MEAS. OF PION-NUCLEON SIGMA TERM USING χPT

AXIAL CURRENT: $\langle \underbrace{n}_{p,n} | \bar{q} \gamma_\mu \gamma_5 q | n \rangle = 2 \underbrace{S_\mu^{(n)}}_{\text{SPIN OF NUCLEON}} \underbrace{\Delta q^{(n)}}_{\text{FROM LEPTON-NUCLEON SCATTERING}}$

$\boxed{3}$ NUCLEUS

NUCLEAR WAVE FUNCTIONS $\longrightarrow$ ADD SPIN AND SCALAR COMP. OF NUCLEONS COHERENTLY TO GIVE WIMP- NUCLEUS

SANDWICH NUCLEON OP'S IN A NUCLEAR STATE

$\longrightarrow$ FORM FACTOR SUPPRESSION (COHERENCE LOSS)

SPIN-SPIN: LENGTHY...

~~AAAA~~ SCALAR: NUCLEON OP HAVE NO SPIN STRUCTURE
$\rightarrow$ ONLY $\propto$ NUCLEON NUMBER

FORM FACTOR
AT $Q \neq 0$, SIMPLY FOURIER TRANSFORM OF
(NON-ZERO MOM. TRANSFER) NUCLEON DENSITY

e.g.: $F(Q) = \exp\left(-Q/2Q_0\right)$

$$Q_0 = \frac{1.5}{m_N R_0^2} \quad \text{NUCLEAR COHERENCE ENERGY}$$

$$R_0 \sim 10^{-13}\,\text{cm}\left[0.3 + 0.91\left(\frac{m_N}{\text{GeV}}\right)^{1/3}\right]$$

NUCLEAR RADIUS

WHAT IS THE DETECTION RATE?

PER UNIT DETECTOR MASS, $R \approx \dfrac{n_\chi \cdot \sigma \cdot \langle v \rangle}{m_N}$

$$n_\chi = \frac{\rho_{DM}}{m_\chi}$$

$$dR = \frac{\rho_{DM}}{m_\chi m_N} \cdot v f(v) dv \left(\frac{d\sigma}{d|q|^2}\right) d|q|^2$$

CHANGING VARIABLES TO ENERGY DEPOSITION Q

$$\frac{dR}{dQ} = \frac{\sigma_0 \rho_{DM}}{m_\chi m_{red}^2} F^2(Q) \int_{v_{min}}^{\infty} \frac{f(v)}{v} dv$$

WHERE • $F(Q)$ ENCODES NUCLEAR PHYSICS FORM FACTOR

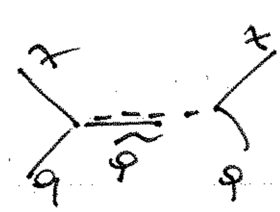
$$\bullet\ \sigma_0 \sim \left(Z f_p + (A-Z) f_n\right)^2$$

$$\frac{f_p}{m_p} = \sum_{q=u,d,s.} \frac{f_{T_q}}{m_q}\left[f_q + \dots \right] + \dots$$

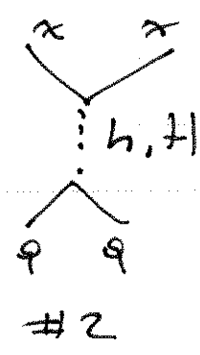
TWO REMARKS:

1) f_p USUALLY $\sim f_n$ (EXCEPTION: ISOSPIN VIOLATING D.M.!)

SO $\sigma_0 \sim A^2 \longrightarrow$ BIG NUCLEI!

2) f_q FOR SUSY NEUTRALINOS: 

#1

, 

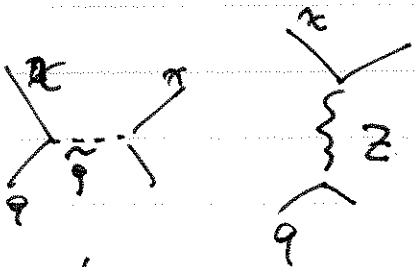
#2

USUALLY #1 << #2

• #2 DEPENDS ON GAUGINO-HIGGSINO MIXING (VANISHES FOR PURE HIGGSINOS)

h, H ADD COHERENTLY, $\mu < 0$ CANCELLATIONS

FOR SPIN DEPENDENT:



#1′ #2′

~~AND~~ AGAIN $\#1' \ll \#2'$

#2′ DEPENDS ONLY ON HIGGSINO CONTENT.

MANY TOOLS HAVE DEDICATED CALCULATIONS OF:

Ω_χ, σ_p etc e.g. DARKSUSY

MICROMEGAS (COMPATIBLE WITH GENERIC COMPHEP INPUT FILES)

NOW: WHAT HAPPENS FOR EARLY UNIVERSE SCATTERING?