PHYSICS 210 — FALL 2016

PROBLEM #1

(a) $[Q,Q] = [P,P] = 0$

$$[Q,P] = \frac{\partial Q}{\partial q}\frac{\partial P}{\partial p} - \frac{\partial P}{\partial q}\frac{\partial Q}{\partial p} =$$

$$= \frac{q^{-1/2}\cos p}{1+q^{\frac{1}{2}}\cos p}\left[-q\sin^2 p + \left(1+q^{\frac{1}{2}}\cos p\right)q^{\frac{1}{2}}\cos p\right]$$

$$+ \frac{q^{\frac{1}{2}}\sin^2 p}{1+q^{\frac{1}{2}}\cos p}\left[\cos p + \left(1+q^{\frac{1}{2}}\cos p\right)q^{-\frac{1}{2}}\right] = 1 \quad \checkmark$$

(b)

$$M = \begin{pmatrix} \frac{\partial Q}{\partial q} & \frac{\partial P}{\partial q} \\ \frac{\partial Q}{\partial p} & \frac{\partial P}{\partial p} \end{pmatrix} = \begin{pmatrix} \frac{\cos p}{2(\sqrt{q}+q\cos p)} & \sin p\left(\frac{1}{\sqrt{q}}+2\cos p\right) \\ \frac{-\sqrt{q}\sin p}{1+\sqrt{q}\cos p} & 2\left(\sqrt{q}\cos p + q\cos 2p\right) \end{pmatrix}$$

$$MJM^t = \ldots = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad \checkmark$$

(c) FIRST SOLVE TRANS. EQ:

$$q = \left(e^{Q}-1\right)^{2} \sec^{2} p$$

$$P = 2e^{Q}\left(e^{Q}-1\right) \tan p$$

IMMEDIATE TO VERIFY THAT

$$\frac{\partial F_3}{\partial Q} = P \quad \text{AND} \quad \frac{\partial F_3}{\partial p} = q$$

PROBLEM #2

(a) CONDITION FOR EQUILIBRIUM.

$$mg \sin\alpha - k(s_0 - d) = 0$$

SO $$s_0 = \frac{mg \sin\alpha}{k} + d$$

(b) LET h BE THE HEIGHT OF THE WEDGE THEN THE COORDINATES OF m ARE:

$$(x + s\cos\alpha,\ h - s\sin\alpha)$$

THUS $L = T - V =$

$$= \frac{1}{2} M\dot{x}^2 + \frac{1}{2} m\left[(\dot{x} + \dot{s}\cos\alpha)^2 + (\dot{s}\sin\alpha)^2\right] - \frac{1}{2}k(s-d)^2 - mg(h - s\sin\alpha)$$

$\mathcal{L}$ - EQUATIONS:

$$(*) \begin{cases} (m+M)\ddot{x} + m\ddot{s}\cos\alpha = 0 \\ m\ddot{x}\cos\alpha + m\ddot{s} + k(s - s_0) = 0 \end{cases}$$

(c) CONSIDER x AND $\sigma = s - s_0$

$$(*) \longrightarrow \begin{cases} (m+M)\ddot{x} + m\cos\alpha\,\ddot{\sigma} = 0 \\ m\cos\alpha\,\ddot{x} + m\ddot{\sigma} + k\sigma = 0 \end{cases}$$

SET $x = Ae^{i\omega t}$ $\quad \vartheta = Be^{i\omega t}$

THE SECULAR EQUATION BECOMES

$$\begin{vmatrix} -(m+M)\omega^2 & -m\omega^2\cos\alpha \\ -m\omega^2\cos\alpha & k - m\omega^2 \end{vmatrix} = 0$$

$\longrightarrow \quad \omega_1 = 0 \qquad$ (TRANSL. MODE)

$$\omega_2 = \sqrt{\frac{k(m+M)}{m(M+m\sin^2\alpha)}}$$

5

PROBLEM # 3

(a) : $I = 2\pi \int_0^R r\,dr\, \mu r^2 \qquad \mu = \frac{M}{\pi R^2}$

$= \cancel{2}\cancel{\pi}\left(\frac{M}{\cancel{\pi}\cancel{R^2}}\right)\frac{\cancel{R^4}^2}{\cancel{4}_2} = \frac{1}{2}MR^2$

(b) $J = I\dot{\vartheta} + mr^2\dot{\vartheta}$

$T + V = \frac{1}{2}I\dot{\vartheta}^2 + \frac{1}{2}(\dot{r}^2 + r^2\dot{\vartheta}^2) + \frac{1}{2}k(r-\ell)^2$

$= \frac{J^2}{2(I+mr^2)} + \frac{1}{2}m\dot{r}^2 + \frac{1}{2}k(r-\ell)^2$

(c) $m\ddot{r} - mr\dot{\vartheta}^2 + k(r-\ell) = 0$

$(I + mr^2)\dot{\vartheta} = J$ = CONSTANT

i.e. $m\ddot{r} - \frac{mrJ^2}{(I+mr^2)^2} + k(r-\ell) = 0 \qquad (*)$

(d) FROM $(*)$ FOR $\ddot{r} = 0$, USING $J = (I+mr_0^2)\Omega_0$

$\frac{mr_0 \cancel{(I+mr_0^2)}^2 \Omega_0^2}{\cancel{(I+mr_0^2)^2}} = k(r_0 - \ell) \rightarrow r_0 = \frac{k\ell}{k - m\Omega_0^2}$

(e) $J = (I + m r_0^2)\Omega_0$

EQ OF MOTION:

$$m\ddot{r} - \frac{m r J^2}{(I+mr^2)^2} + k(r-\ell) = 0 \qquad (**)$$

LET $r = r_0 + \rho$, $\rho \ll r_0$

$$\frac{m r J^2}{(I+mr^2)^2} \simeq \frac{m(r_0+\rho)J^2}{(I+mr_0^2+2mr_0\rho)^2}$$

$$\simeq \frac{m r_0 J^2}{(I+mr_0^2)^2}\left(1 + \frac{\rho}{r_0} - \frac{4mr_0}{I+mr_0^2}\rho\right)$$

$$\simeq \underbrace{\frac{m r_0 J^2}{(I+mr_0^2)^2}}_{k(r_0-\ell)}\left(1 - \left(\frac{3mr_0^2 - I}{I+mr_0^2}\right)\frac{\rho}{r_0}\right)$$

THUS $(**)$ BECOMES $\ddot{\rho} + \left[\frac{k}{m} + \left(\frac{3mr_0^2 - I}{I+mr_0^2}\right)\Omega_0^2\right]\rho = 0$

AND $\omega = \sqrt{\frac{k}{m} + \left(\frac{3mr_0^2 - I}{I+mr_0^2}\right)\Omega_0^2}$