Homework Set #1.

Due Date: Friday October 14, 2016
(drop it by the end of the day in the Instructor's mailbox)

1. A particle starts at rest and moves along a *cycloid* whose equation is

$$x = \pm\left[a\cos^{-1}\left(\frac{a-y}{a}\right) + \sqrt{2ay - y^2}\right].$$

There is a gravitational field of strength g in the negative y direction. Obtain and solve the equations of motion. Show that no matter where on the cycloid the particle starts out at time $t = 0$, it will reach the bottom at the same time.

2. A point particle of mass m is constrained to move frictionlessly on the inside surface of a circular wire hoop of radius r, uniform density and mass M. The hoop is constrained to the xy-plane, it can roll on a fixed line (the x-axis), but it does not slide, nor can it lose contact with the x-axis.
The point particle is acted on by gravity exerting a force along the negative y-axis. At $t = 0$ suppose the hoop is at rest. At this time the particle is at the top of the hoop, and is given a velocity v_0 along the x-axis.
What is the velocity v_f, with respect to the fixed axis, when the particle comes to the bottom of the hoop? Simplify your answer in the limits $m/M \to 0$ and $M/m \to 0$.

3. A double plane pendulum consists of a simple pendulum (mass m_1, length l_1) with another simple pendulum (mass m_2, length l_2) suspended from m_1, both constrained to move in the same vertical plane.

   (a) Describe the configuration manifold $\mathcal{Q}$ of this dynamical system. Say what you can about $\mathbf{T}\mathcal{Q}$ (the tangent bundle of $\mathcal{Q}$).

   (b) Write down the Lagrangian of this system in suitable coordinates.

   (c) Derive Lagrange's equations.

4. A cartesian coordinate system with axes x, y, z is rotating relative to an inertial frame with constant angular velocity ω about the z-axis. A particle of mass m moves under a force whose potential is $V(x, y, z)$. Set up the Lagrange equations of motion in the coordinate system x, y, z. Show that these equations are the same as those for a particle in a fixed coordinate system acted on by the force $-\nabla V$ and a force derivable from a velocity-dependent potential U, and find U.