Homework Set #2.

Due Date: Friday October 28, 2016

Solve the following three exercises:

1. Consider a planet of mass m in orbit around a sun of mass M. Assume further that there is a uniform distribution of dust, of density ρ, throughout the space surrounding the sun and the planet.

   (a) Show that the effect of the dust is to add an additional attractive central force $F' = -mkr$, where $k = (4\pi\rho G/3)$, and G is the gravitational constant. You may neglect any drag force due to collisions with the dust particles.

   (b) Consider a circular orbit for the planet corresponding to angular momentum L. Give the equation satisfied by the radius of the orbit r_0, in terms of $L,\ G,\ M,\ m$ and k. You need not solve the equation.

   (c) Suppose that the sun moves at a constant velocity V through the dust cloud, and neglect any effect due to the planet; If the sun has a radius R, and if all the dust particles that collide with the sun are trapped by it, find the rate at which the mass of the sun increases.

   (d) Calculate the drag force on the sun, with the approximation that the thermal velocities of the dust particles are negligible relative to V and the interactions of atoms with each other can be neglected.

2. Show that the angle of scattering in the laboratory system, ϑ, is related to the energy before scattering E_0 and the energy after scattering E_1 according to the equation

$$\cos\vartheta = \frac{m_2 + m_1}{2m_1}\sqrt{\frac{E_1}{E_0}} - \frac{m_2 - m_1}{2m_1}\sqrt{\frac{E_0}{E_1}} - \frac{m_2 Q}{2m_1\sqrt{E_0 E_1}}.$$

3. Examine the scattering produced by a repulsive central force $f = kr^{-3}$. Show that the differential cross section is given by

$$\sigma(\Theta)\,\mathrm{d}\Theta = \frac{k}{2E}\frac{(1-x)\mathrm{d}x}{x^2(2-x)^2\sin\pi x},$$

   where $x = \Theta/\pi$ and E is the energy.