Homework Set #3.

Due Date: Monday November 21, 2016

Solve the following three exercises:

1. The Lagrangian of an electron of mass m and charge $-e$ is

$$L = \frac{1}{2}mv^2 - \left(\frac{e}{c}\right)\vec{v}\cdot\vec{A},$$

in the absence of an electric field. In the case of a planar motion, we have $\vec{A} = (B/2)(-y, x)$.

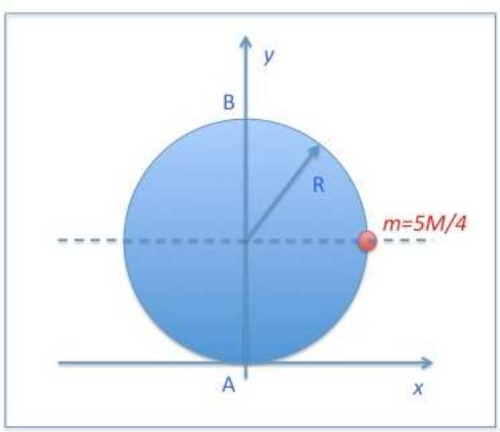
(a) Derive the Hamiltonian of the system in polar coordinates.

(b) Consider circular orbits: find the orbital radius r_0 and the angular velocity ω.

(c) Study the stability of circular orbits for a radial perturbation $r = r_0 + \rho$, where $\rho \ll r_0$, and determine the nature and the frequency of small oscillations of the radial motion.

2. A thin disk of radius R and mass M lying in the xy-plane has a point-mass $m = 5M/4$ attached on its edge. The moment of inertia of the disk about its center of mass (with the z-axis orthogonal to the disc) is

$$I = \frac{MR^2}{4}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}.$$



(a) Find the moment of inertia tensor of the combination of disk and point mass about point A in the coordinate system shown

(b) Find the principal moments and the principal axes about point A.

(c) The disk is constrained to rotate about the y-axis with angular velocity ω by pivots at A and B. Describe the angular momentum about A as a function of time and find the vector force applied at B, ignoring gravity.

3. A smooth uniform circular hoop of mass M and radius a swings in a vertical plane about a point O at which it is freely hinged to a fixed support. A bead B of mass m slides without friction on the hoop.

   (a) Define appropriate generalized coordinates and find the equations of motion for the system.

   (b) Find the characteristic frequencies and normal modes for small oscillations about the position of stable equilibrium.