Midterm

Wednesday November 9, 2016

Solve two of the following three problems
(extra points if you attempt/solve a third problem)

1. A particle of mass m moves under the action of a harmonic oscillator force with potential energy $\frac{1}{2}kr^2$. Initially, it is moving in a circle of radius a.

   (a) Find the orbital speed v.

   The particle is then given a blow of impulse mv in a direction making an angle α with its original velocity.

   (b) Use the conservation laws to determine the minimum and maximum distances from the origin during the subsequent motion.

   (c) Explain your results physically for the limiting cases $\alpha = 0$ and $\alpha = \pi$.

2. A comet moves toward the sun with initial velocity v_0. The mass of the sun is M and its radius R. Find the total cross section σ for striking the sun. Take the sun to be at rest and ignore all other bodies.

3. A massless spring of rest length l_0 (with no tension) has a point mass m connected to one end and the other end fixed so the spring hangs in the gravity field. The motion of the system is only in one vertical plane.

   (a) Write down the Lagrangian for the system using variables θ, the angle of the spring with the vertical, and $\lambda = (r - r_0)/r_0$, where r_0 is the rest length (hanging with mass m).

   (b) Calculate Lagrange's equations. Use $\omega_s^2 = k/m$ and $\omega_p^2 = g/r_0$.

   (c) Discuss the lowest order approximation to the motion when λ and θ are small with the initial conditions $\theta = 0$, $\dot{\theta} = \omega_p B$, $\lambda = A$, $\dot{\lambda} = 0$ at $t = 0$. A and B are constants.