# Homework Set #3.

**Due Date**: Monday November 16, 2009

*Solve the following exercises*

1. Peskin & Schroeder, Problem 4.2 (Decay of a scalar particle)

2. Peskin & Schroeder, Problem 4.4 (c) (Rutherford scattering)

3. (i) Consider the decay of an excited atomic state $A^*$ into a lower energy state $A$ with the emission of a photon,

$$A^* \to A + \gamma$$

Verify that the phase space can be written as:

$$d\Pi^{(2)} = \frac{\omega}{16\pi^2 M_A} d\Omega$$

where $\omega$ is the photon energy and $M_A$ the mass of the atom.
(ii) Verify that the decay width can be written as

$$d\Gamma = |\mathcal{M}_{fi}|^2 \frac{\omega}{8\pi^2} d\Omega,$$

with the normalization of one particle per unit volume for the atomic states.
(iii) Consider now the two-photon decay of an atomic state,

$$A^* \to A + \gamma_1 + \gamma_2.$$

Show that the decay width can be written as

$$\frac{d\Gamma}{d\omega_1} = \frac{\omega_1(\omega - \omega_1)}{8(2\pi)^5} \int d\Omega_1 \; d\Omega_2 \; |\mathcal{M}_{fi}|^2,$$

where $\omega_1$, $\omega_2$ are the energies of the two photons, $\omega = E_{A^*} - E_A = \omega_1 + \omega_2$, and $d\Omega_{1,2}$ are the solid angles of the two photons.

4. Prove the following three expressions for the 2- and 3-dimensional phase space $d\Pi^{(n)}$:

$$d\Pi^{(2)} = \frac{|\vec{p}_1|}{16\pi^2 E_{c.m.}} d\Omega \tag{1}$$

$$d\Pi^{(2)} = \frac{d\Omega}{32\pi^2 M^2} \left( M^4 + (m_1^2 - m_2^2)^2 - 2M^2(m_1^2 + m_2^2) \right)^{1/2} \tag{2}$$

(for a decay with particle masses $M \to m_1 + m_2$)

$$d\Pi^{(3)} = \frac{dp_1^0 \; dp_2^0}{4(2\pi)^3} = \frac{ds \; dt}{16 P_0^2 (2\pi)^3}, \tag{3}$$

where $s = (p_2 + p_3)^2 = (P - p_1)^2$ and $t = (p_1 + p_3)^2 = (P - p_2)^2$.