Compton Scattering in Scalar QED

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1 Calculating Matrix Elements

There are three Feynman diagrams to consider when calculating the matrix elements for $\phi\gamma \to \phi\gamma$ in Scalar QED:

1.1 S-Channel

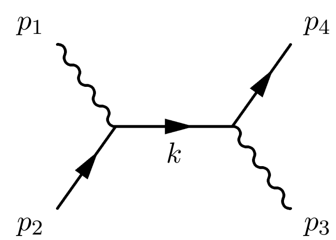

Figure 1: S-channel diagram

The matrix element can be found by following the Feynman rules:

$$iM = (-ie)(p_2^\mu + k^\mu)\epsilon_\mu^1[\frac{i}{k^2 - m^2}](-ie)(k^\nu + p_3^\nu)\epsilon_\nu^{4\star} \tag{1}$$

Kinematics requires that $k^\mu = p_1^\mu + p_2^\mu = p_3^\mu + p_4^\mu$, therefore we can replace the ks above:

$$iM = \frac{-ie^2}{k^2 - m^2}(2p_2^\mu + p_1^\mu)\epsilon_\mu^1(2p_2^\nu + p_4^\nu)\epsilon_{\nu\star}^4 \tag{2}$$

But $p_i^\mu \epsilon_{i\mu} = 0$ so:

$$M = \frac{-e^2}{k^2 - m^2}(2p_2 \cdot \epsilon_1 + p_1 \cdot \epsilon_1)(2p_3 \cdot \epsilon_4 + p_4 \cdot \epsilon_4) = \frac{-e^2}{k^2 - m^2}(2p_2 \cdot \epsilon_1)(2p_3 \cdot \epsilon_4) \tag{3}$$

However, $\vec{p_1} = -\vec{p_2}$ and $\vec{p_3} = -\vec{p_4}$, and the polarization vectors ϵ_i are purely transverse. So $p_2 \cdot \epsilon_1 = p_1 \cdot \epsilon_1 = 0$ and $p_3 \cdot \epsilon_4 = p_4 \cdot \epsilon_4 = 0$. Therefore the S-channel does not contribute to the scattering amplitude:

$$M = 0 \tag{4}$$

1.2 T-Channel

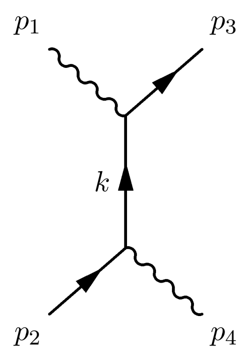

Figure 2: T-channel diagram

Again, we calculate the amplitude from the Feynman rules:

$$iM = \epsilon^i_\mu(-ie)(k^\mu + p_3^\mu)[\frac{i}{k^2 - m^2}](-ie)(p_2^\nu + k^\nu)\epsilon^{4\star}_\nu \tag{5}$$

Using kinematics to rewrite $k^\mu = p_3^\mu - p_1^\mu = p_2^\mu - p_4^\mu$:

$$M = \frac{e^2}{2p_2 \cdot p_4}(2p_3^\mu - p_1^\mu)(2p_2^\nu - p_4^\nu)\epsilon^1_\mu\epsilon^{4\star}_\nu \tag{6}$$

Simplifying with the fact that $p_i^\mu \epsilon_{i\mu} = 0$ yields:

$$M = \frac{e^2}{2p_2 \cdot p_4}(2p_3 \cdot \epsilon_1)(2p_2 \cdot \epsilon_4^\star) \tag{7}$$

1.3 Seagull Channel

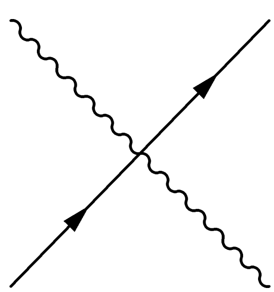

Figure 3: Seagull-channel diagram. For some reason the legs weren't labeled by LaTeX

The seagull diagram contributes a factor of:

$$M = 2e^2 g_{\mu\nu} \epsilon_1^{\mu} \epsilon_4^{\nu\star} = 2e^2 \epsilon_1 \cdot \epsilon_4^{\star} \tag{8}$$

Therefore the total amplitude at tree level is:

$$M = 2e^2 \left[\epsilon_1 \cdot \epsilon_4^{\star} + \frac{1}{p_2 \cdot p_4} (p_3 \cdot \epsilon_1)(p_2 \cdot \epsilon_4^{\star}) \right] \tag{9}$$

2 Calculating Cross Section

The differential cross section is given by:

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 E_{CM}^2} |M|^2 \tag{10}$$

Since we want $\frac{d\sigma}{d\cos\theta}$ instead of $\frac{d\sigma}{d\Omega}$, we multiply this by 2π. Then it's simply a matter of plugging everything in and simplifying:

$$\frac{d\sigma}{d\cos\theta} = \frac{e^4}{8\pi^2 E_{CM}^2} \left[\epsilon_1 \cdot \epsilon_4^{\star} + \frac{(p_3 \cdot \epsilon_1)(p_2 \cdot \epsilon_4^{\star})}{p_2 \cdot p_4} \right]^2 \tag{11}$$