Homework Set #3.

Due Date - Oral Presentation: Wednesday October 21, 2015
Due Date - Written Solutions: Wednesday October 28, 2015

1. **Muon decay in the Fermi theory**

   A muon decays to an electron, an electron (anti)neutrino and a muon neutrino,

   $$\mu^- \to e^- \nu_\mu \bar{\nu}_e.$$

   In the so-called Fermi theory, the matrix element for this process, ignoring the electron and neutrinos masses, is given by

   $$|\mathcal{M}|^2 = 32 G_F^2 (m^2 - 2mE) mE,$$

   where m is the muon mass, E is the energy of the outgoing electron antineutrino, and $G_F = 1.166 \times 10^{-5}$ GeV^{-2} is the Fermi constant.

   (a) Perform the integral over $d\Pi_{\rm LIPS}$ and show that the decay rate reads:

   $$\Gamma = \frac{G_F^2 m^5}{192\pi^3};$$

   (b) Convert from natural units to inverse seconds, using $m = 106$ MeV, and compare your result to the observed value $\tau = \Gamma^{-1} = 2.20$ μs. How big is the discrepancy as a percentage? What might account for the discrepancy?

2. **Mandelstam variables**

   We calculated that the $e^+e^- \to \mu^+\mu^-$ cross section had the form, in the CM frame,

   $$\frac{d\sigma}{d\Omega} = \frac{e^4}{64\pi^2 E_{\rm CM}^2}(1 + \cos^2\theta).$$

   (a) Work out the Lorentz-invariant quantities

   $$s = (p_{e^+} + p_{e^-})^2,\ t = (p_{\mu^-} - p_{e^-})^2,\ u = (p_{\mu^+} - p_{e^-})^2,$$

known as Mandelstam variables, in terms of E_{CM} and $\cos\theta$ (still assuming $m_\mu = m_e = 0$).

(b) Derive a relationship between s, t and u.

(c) Rewrite $\frac{d\sigma}{d\Omega}$ in terms of s, t and u.

(d) Now assume m_μ and m_e are non-zero. Derive a relationship between s, t and u and the masses.