Homework Set #8. (Last one!)

Due Date - Oral Presentation: Monday November 30, 2015
Due Date - Written Solutions: Monday December 7, 2015

1. **Searching for the Higgs Boson at LEP**

In the Fall of 2000, CERN decided to shut down LEP II – the second-stage Large Electron-Positron collider – to make room for the Large Hadron Collider (LHC). Somewhat ironically, just before its final shut-down, LEP II experiments reported a few events that could have been the first evidence of the Higgs particle being produced in e^+e^- collisions – but unfortunately not enough of these events were detected to statistically prove that they really involved the Higgs particle[1]. This exercise is a part of the theoretical calculation of the Higgs production rate at an electron-positron collider such as LEP (the same calculation holds for a future international Linear Collider). Specifically, consider the following process:

$$e^+ \quad + \quad e^- \quad \to \quad H^0 \quad + \quad Z^0, \tag{1}$$

in which an electron and a positron annihilate into the scalar neutral Higgs particle and the Z^0 neutral intermediate mass gauge boson, with spin 1. The Z^0 particle has mass $M_Z \simeq 91$ GeV, while the mass of the Higgs particle was unknown at that time, but indirectly known to be heavier – albeit likely not too much heavier – than M_Z; the initial electron and positron in this process are so ultra-relativistic that we may treat them as massless, $M_e \approx 0$.

The Standard Model of weak and electromagnetic forces yields a simple

[1] Of course, in retrospect, those events did not come from the Higgs anyways!

formula for the amplitude of the process of Eq. (1) to leading order in perturbation theory,

$$\mathcal{M}(e^+ + e^- \to H^0 + Z^0) = \frac{e^2 M_Z}{4\sin^2\theta_W} \frac{1}{s - M_Z^2} \times \epsilon_\mu(Z^0) \overline{v}(e^+) \left(4\sin^2\theta_W - 1 + \gamma^5\right) \gamma^\mu u(e^-) \tag{2}$$

where

$$s = (p_{e^+} + p_{e^-})^2 = (p_Z + p_H)^2 = E_{\text{c.m.}}^2,$$

$\epsilon_\mu(Z^0)$ is the polarization vector of the final Z_0 vector particle, $\overline{v}(e^+)$ and $u(e^-)$ are the polarization spinors of the initial positron and electron, θ_W is the electroweak mixing angle (the Weinberg angle), and where $\sin^2\theta_W \approx 0.233$, and e is the electric charge, which in natural units is such that $\alpha = e^2/(4\pi) \approx 1/137$.

Show that the sum over final spin states and average over initial spin states gives

$$\frac{1}{4} \sum_{\text{all spins}} \left| \mathcal{M}(e^+ + e^- \to H^0 + Z^0) \right|^2 = A \left((p_{e^+} \cdot p_{e^-}) + 2 \frac{(p_Z \cdot p_{e^+})(p_Z \cdot p_{e^-})}{M_Z^2} \right), \tag{3}$$

where

$$A = \frac{1 + (1 - 4\sin^2\theta_W)^2}{(4\sin^2\theta_W)^2} \left(\frac{e^2 M_Z}{s - M_Z^2} \right)^2$$

Note, as it should be, that the right hand side of the expression above is explicitly Lorentz-invariant.

You are asked to calculate and plot the total cross section for the process (1) as a function of the net center of mass energy $E_{\text{c.m.}}$ for $m_H = 125$ GeV, and the angular dependence of the differential cross section $\mathrm{d}\sigma/\mathrm{d}\Omega$ on the directions of the final Higgs and Z^0 particles, and plot it as a function of angle for a center of mass energy $E_{\text{c.m}}^{\text{LEP}} = 205$ GeV and $m_H = 113$ GeV, and for $E_{\text{c.m}}^{\text{ILC}} = 1000$ GeV and $m_H = 125$ GeV.

2. **Particle Decays**

Consider a particle ϕ decaying into an electron-positron pair.

(a) Show that the total rate is

$$\Gamma(\phi \to e^+e^-) = \frac{\sqrt{1-4x^2}}{8\pi m_\phi}|\mathcal{M}|^2, \quad x = \frac{m_e}{m_\phi}.$$

(b) Evaluate Γ if:

1. ϕ is a *scalar*, with interaction $g_S\phi\bar{\psi}\psi$;
2. ϕ is a *pseudoscalar*, with interaction $ig_P\phi\bar{\psi}\gamma_5\psi$;
3. ϕ is a *vector*, with interaction $g_V\phi_\mu\bar{\psi}\gamma^\mu\psi$;
4. ϕ is a *axial vector*, with interaction $ig_A\phi_\mu\bar{\psi}\gamma^\mu\gamma_5\psi$.

(c) Suppose a collider experiment reported evidence of a new particle that decays to leptons (τ, μ and e) whose mass is around 4 GeV. What spin and parity might this particle have if about 25% of the time it decays to $\tau^+\tau^-$?