FALL '08 — PHYSICS 221A

SOLUTIONS TO HW SET #2

#1 CONSIDER THE RELATIVISTIC INVARIANTS FOR THE FINAL STATE PARTICLES, IN THE CENTER OF MASS FRAME:

$$(1):\ E_1^2 = P^2 + m_1^2 \qquad (2):\ E_2^2 = P^2 + m_2$$

WHERE $\vec{P_1} = -\vec{P_2}$, $|\vec{P_1}| = |\vec{P_2}| = P$

CONSIDER ENERGY CONSERVATION, AGAIN IN THE C.O.M FRAME:

$$(E_1 + E_2)^2 = M_{IN}^2$$

$$\left(\sqrt{P^2+m_1^2} + \sqrt{P^2+m_2^2}\right)^2 = M_{IN}^2$$

$$2P^2 + m_1^2 + m_2^2 + 2\sqrt{(P^2+m_1^2)(P^2+m_2^2)} = M_{IN}^2$$

$$4\left(P^4 + (m_1^2+m_2^2)P^2 + m_1^2 m_2^2\right) =$$

$$= M_{IN}^4 + m_1^4 + m_2^4 + 4P^4 - 2M_{IN}^2 m_1^2 + 2m_1^2 m_2^2$$

$$- 2M_{IN}^2 m_2^2 - 4P^2 M_{IN}^2 + 4P^2 m_1^2 + 4P^2 m_2^2$$

WHENCE $$P^2 = \frac{1}{4M_{IN}^2}\left(M_{IN}^4 + (m_1^2 - m_2^2)^2 - 2M_{IN}^2(m_1^2 + m_2^2)\right)$$

SUBSTITUTING IN (1) AND (2) WE FIND E_1 AND E_2

#2

WITH THE CUSTOMARY KINEMATIC VARIABLES, AND NEGLECTING THE PROTON'S KINETIC ENERGY, WE HAVE

$$E_\nu = E_0 - E \quad , \quad \text{WITH} \quad E_\nu^2 = m_\nu^2 + q^2$$

HENCE:

$$m_\nu^2 + q^2 = E_0^2 - 2EE_0 + E^2$$

$$2q\,dq = 2E_0\,dE_0 - 2E\,dE_0$$

$$dq = \frac{(E_0 - E)}{q}\,dE_0$$

SO IN THE EXPRESSION FOR THE NEUTRINO PHASE SPACE,

$$q^2 dq = q(E_0 - E)\,dE_0 = (E_0 - E)^2\sqrt{1 - \frac{m_\nu^2}{(E_0-E)^2}}\,dE_0$$

THEREFORE

$$\frac{dN}{dE_0} = \frac{1}{4\pi^4\hbar^6}\,p^2(E_0 - E)^2\sqrt{1 - \frac{m_\nu}{(E_0-E)^2}}\,dp$$

RB

3

SARGENT'S RULE: $\tau \sim E_0^{-5} G^{-2}$

WHERE FOR μ DECAY: $E_0 = E_\mu^0 = 105.5$ MeV

n DECAY: $E_0 = E_n^0 = 1.3$ MeV

CONSIDERING THE LIFE-TIME RATIO:

$$\frac{\tau_n}{\tau_\mu} = \left(\frac{E^0_\mu}{E^0_n}\right)^5 \left(\frac{G_\mu}{G_n}\right)^2$$

WE FIND $\quad \dfrac{G_\mu}{G_n} = \sqrt{\left(\dfrac{\tau_n}{\tau_\mu}\right)\left(\dfrac{E_n}{E_\mu}\right)^5} \simeq 0.35$

#4

(a): DIRECT SUBSTITUTION YIELDS

$$\gamma_5 = \begin{pmatrix} 0 & 0 & i & 0 \\ 0 & 0 & 0 & -i \\ -i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & i\sigma_3 \\ -i\sigma_3 & 0 \end{pmatrix}$$

(b) AND (c) FOLLOW BY DIRECT SUBSTITUTION. PARTICULARLY EASY IF ONE CONSIDERS THE PAULI MATRIX ALGEBRA...

(d) $$\left(i\gamma_\mu \partial_\mu - m\right)\psi = 0$$

TAKE THE COMPLEX CONJUGATE:

$$\left(-i\gamma_\mu^* \partial_\mu - m\right)\psi^* = 0$$

$$\left(i\gamma_\mu \partial_\mu - m\right)\psi^* = 0 \qquad \gamma_\mu^* = -\gamma_\mu$$

WHICH MEANS THAT IN MAJORANA'S REPRESENTATION, THE COMPLEX CONJUGATE ~~OF~~ DIRAC SPINOR OF A SOLUTION OF DIRAC'S EQUATION IS ALSO A SOLUTION TO IT.

5

LET'S FIRST INTEGRATE OVER POLAR ANGLES AND X.

$$d\Gamma = \int_0^1 dx \int_0^{2\pi} d\varphi \, \frac{G_F^2 m_\mu^5}{192\pi^3} \left(2x^2(3-2x)\right)\left(1 + \left(\frac{1-2x}{3-2x}\right)\cos\vartheta_e\right)\frac{d\cos\vartheta}{4\pi}$$

$$d\Gamma = \frac{G_F^2 m_\mu^5}{192\pi^3}\left(\frac{1}{2} - \frac{1}{6}\cos\vartheta_e\right) d\cos\vartheta$$

SO
$$\frac{d\Gamma}{d\cos\vartheta} = \frac{G_F^2 m_\mu^5}{384\pi^3}\left(\frac{3 - \cos\vartheta_e}{3}\right)$$

6

TO HAVE AN EVENT RATE $R = \frac{1}{84.600} s^{-1}$

WE NEED $N_{^{71}Ga} \cdot \phi \cdot \sigma \cdot \epsilon = R$

SO $N_{^{71}GA} = \frac{R}{\phi \cdot \sigma \cdot \varepsilon} \simeq 5 \times 10^{28}$

THIS CORRESPONDS TO A NUMBER OF MOL: $N_{MOL} = \frac{N_{^{71}GA}}{N_A} \simeq 8.3 \times 10^4$

AND TO A MASS $M_{^{71}Ga} = N_{MOL} \times 71 \, g/mol \simeq 6 \, t$

THE NATURAL GALLIUM MASS IS THUS $M_{Nat\,Ga} = M_{^{71}Ga} / \alpha \simeq 15 \, t$