Homework Set #3.

Due Date: Thursday November 13, 2008

Solve the following 5 exercises:

1. Consider a matrix element that has $(1 - i\gamma_5)$ appearing instead of $\gamma_\mu(1-\gamma_5)$ for $\mu^- \to e^-\bar{\nu}_e\nu_\mu$. Calculate the e^- spectrum in the μ^- rest frame for this hypothetical matrix element, neglecting m_e, and compare to the standard case.

2. Suppose that there exists a fourth generation lepton doublet made of a nearly massless charged lepton ψ^- and a heavy neutrino ν_ψ. The ν_ψ decay mode would then be

$$\nu_\psi \to \psi^- \; l^+ \; \nu_l.$$

   (2.1) Neglecting m_ψ and m_l compared to m_{ν_ψ}, find the l^+ spectrum in the ν_ψ rest frame.

   (2.2) How does the shape of the spectrum compare to the e^+ spectrum in μ^+ decay?

   (2.3) Calculate the ν_ψ decay width for the decay mode above, as a function of m_{ν_ψ}.

3. Consider the decays $D_s^- \to \tau^- \; \bar{\nu}_\tau$ and $D_s^- \to \mu^- \; \bar{\nu}_\mu$. These can be calculated in terms of one constant, f_{D_s}, analogous to the pion decay constant f_π. Calculate the ratio of the two decay amplitudes. Using the measured D_s lifetime calculate the expected branching ratio to $\tau^- \; \bar{\nu}_\tau$, assuming $f_{D_s} = 280$ MeV.

4. Show explicitly that the CKM matrix is unitary using the following parametrization (same as in Seiden's Eq. (8.17))

$$V_{\rm CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix},$$

   where $c_{ij} \equiv \cos(\theta_{ij})$, $s_{ij} \equiv \sin(\theta_{ij})$ and $\theta_{ij},\ \delta \in \mathbf{R}$.

5. A "narrowband" neutrino beam is produced by bombarding a Be target with 400 GeV protons and forming a "pencil" secondary beam with a small spread in momentum centered at 200 GeV/c. This beam contains charged pions and kaons with one sign of charge and it traverses

an evacuated decay tunnel 300 m long, where a fraction of the mesons decay to muons and neutrinos. This is followed by an absorber consisting of 200 m of steel and 150 m of rock. A cylindrical detector of radius 2 m is placed 400 m beyond the end of the decay tunnel, and aligned with the beam axis.

(4.1) Find a relation between the laboratory energy of a neutrino and its angle relative to the axis, for neutrinos from pion and from kaon decays.

(4.2) What are the maximum and minimum energies of the neutrinos produced in the two cases?

(4.3) Above what neutrino energy do all neutrinos from kaon decay traverse the detector?

(4.4) If 10^{10} pions per burst enter the decay tunnel, how many neutrinos from pion decay traverse the detector?

(4.5) If the detector has mass 10^5 kg, how many neutrinos from pion decay interact in it per burst, if the cross-section per nucleon at energy E is $\sigma_{\nu N} \approx 0.6(E/\text{GeV}) \times 10^{-38}$ cm^2?

[$m_\pi \simeq 139$ MeV, $m_K \simeq 494$ MeV, $m_\mu \simeq 106$ MeV]