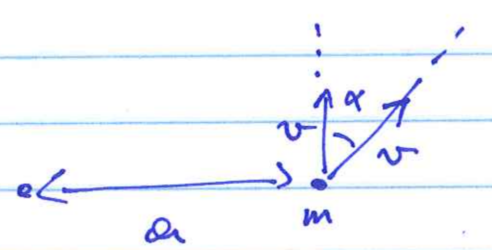
PHYSICS 210 FALL 2016

PROBLEM #1

(a) QUICKEST THING: $\vec{F} = m\vec{a}$

$$ka = \frac{mv^2}{a} \rightarrow v = \sqrt{\frac{k}{m}}\, a$$

(b)



$$L_{new} = mva\,(1 + \cos\alpha)$$

$$E_{new} = \frac{1}{2}ka^2 + \frac{1}{2}mv^2\left[(1+\cos\alpha)^2 + \sin^2\alpha\right] =$$

$$= \frac{1}{2}ka^2\left[1 + 1 + 2\cos\alpha + \cos^2\alpha + \sin^2\alpha\right] = \frac{ka^2}{2}(3 + 2\cos\alpha)$$

r_{max}, r_{min} CORRESPOND TO $\dot{r} = 0$ IN $\boxed{E_{new} = \frac{1}{2}m\dot{r}^2 + \frac{L_{new}^2}{2mr^2} + V(r)}$

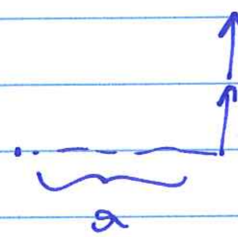
THUS:

$$\frac{ka^2}{2}(3 + 2\cos\alpha) = \frac{m^2 \frac{k}{m} a^2 (1+\cos\alpha)^2}{2mr^2} + \frac{1}{2}kr^2$$

$$r^2 = \frac{a^2}{2}\left[(3 + 2\cos\alpha) \pm \sqrt{9 + 4\cos^2\alpha + 12\cos\alpha - 4 - 4\cos^2\alpha - 8\cos\alpha}\right]$$

$$r = \frac{a}{\sqrt{2}}\left[(3 + 2\cos\alpha) \pm \sqrt{5 + 4\cos\alpha}\right]^{1/2}$$

(c) $\alpha = 0 \longrightarrow r = \frac{a}{\sqrt{2}}(5 \pm 3)^{1/2} = 2a,\ a$



ORBIT IS PERIODIC

KINETIC ENERGY IS X4
THUS WHEN r IS MAXIMUM
POT ENERGY MUST BE X4...

$\alpha = \pi \longrightarrow r = \frac{a}{\sqrt{2}}\left[1 \pm \sqrt{5-4}\right]^{1/2} = a,\ 0$

PERIODIC

MOTION IS
S.H.O.

PROBLEM #2

$$\frac{mv_0^2}{2} = \frac{mv^2}{2} - \frac{GMm}{r}$$ @ CLOSEST APPROACH

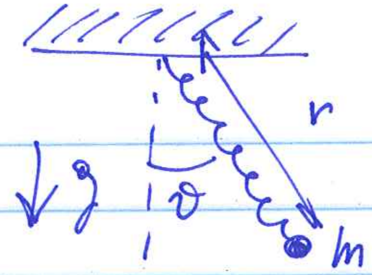
$$mbv_0 = mrv$$

THUS $$b = r\sqrt{1 + \frac{2GM}{v_0^2 r}}$$

TOTAL CROSS SECTION IS

$$\sigma = \pi b^2(R) = \pi R^2\left(1 + \frac{2GM}{v_0 R}\right)$$

PROBLEM #3



(a) $T = \frac{1}{2} m (\dot{r}^2 + r^2 \dot{\vartheta}^2)$

$V = -mgr\cos\vartheta + \frac{1}{2} k (r - l_0)^2$

$L = T - V = \frac{1}{2} m (\dot{r}^2 + r^2\dot{\vartheta}^2) + mgr\cos\vartheta - \frac{1}{2} k (r - l_0)^2$

(b) $$\begin{cases} m\ddot{r} - mr\dot{\vartheta}^2 - mg\cos\vartheta + k(r - l_0) = 0 \\ mr^2\ddot{\vartheta} + 2mr\dot{r}\dot{\vartheta} + mgr\sin\vartheta = 0 \end{cases}$$

$k(r_0 - l_0) = mg$, THUS WITH $\lambda = \frac{r - r_0}{r_0}$

$r = r_0(1+\lambda) \qquad \dot{r} = r_0\dot{\lambda} \qquad \ddot{r} = r_0\ddot{\lambda}$

THE EQ OF MOTION BECOME

$$\begin{cases} \ddot{\lambda} + \frac{k\lambda}{m} - (1+\lambda)\dot{\vartheta}^2 + \frac{g}{r_0}(1 - \cos\vartheta) = 0 \\ (1+\lambda)\ddot{\vartheta} + 2\dot{\lambda}\dot{\vartheta} + \frac{g}{r_0}\sin\vartheta = 0 \end{cases}$$

OR

$$\begin{cases} \ddot{\lambda} + (\omega_s^2 - \dot{\vartheta}^2)\lambda - \dot{\vartheta}^2 + \omega_p^2(1 - \cos\vartheta) = 0 \\ (1+\lambda)\ddot{\vartheta} + 2\dot{\lambda}\dot{\vartheta} + \omega_p^2\sin\vartheta = 0 \end{cases}$$

(c) NEGLECTING SECOND ORDER QUANTITIES IN $\lambda, \dot{\lambda}, \vartheta, \dot{\vartheta}$ WE HAVE

$$\begin{cases} \ddot{\lambda} + \omega_s^2 \lambda = 0 \\ \ddot{\vartheta} + \omega_p^2 \vartheta = 0 \end{cases}$$

THUS WITH THE INITIAL CONDITIONS GIVEN,

$$\begin{cases} \lambda = A \cos(\omega_s t) \\ \vartheta = B \sin(\omega_p t) \end{cases}$$

$\blacksquare$