Homework Set #4.

Due Date: Monday November 30, 2009

Solve the following exercises

1. Peskin & Schroeder, Problem 5.1 (Relativistic Coulomb Scattering)

2. **Proton Decay**. There are theoretical reasons to believe that the decay

$$p \to e^+ + \pi^0$$

might occur. The decay was never observed, and this has been turned into an experimental limit on the proton lifetime:

$$\tau_p = \frac{\hbar}{\Gamma} > 1.6 \times 10^{33} \text{ yr.}$$

The most general matrix element for the process, compatible with Lorentz invariance and momentum conservation, has the form:

$$\mathcal{M} = \bar{u}(\vec{p}_{e^+})\ T\ u(\vec{p}_p), \quad \text{with } \ T = A + B\gamma^5.$$

where A and B are constant.
(i) Which constraints would parity conservation impose on the constants A and B?
(ii) Calculate the decay width for the proton decay process as a function of A and B.
(iii) Calculate the constraints on A and B from the lower limit on the proton lifetime.
(iv) The π^0 is not directly observed, since it decays immediately into two photons. Knowing that the π^0 has spin zero, calculate the distribution of the angle the photons form with the direction of motion of the positron in the final state.

3. **The K^0_{l3} decay**. The decay process

$$K^0 \to \pi^- l^+ \nu_l$$

is well described by the Fermi interaction:

$$H = \frac{G}{\sqrt{2}} J^h_\mu \bar{\psi}_{\nu_l} \gamma^\mu (1 - \gamma^5) \psi_l$$

where the hadronic current J^h_μ has a matrix element

$$\langle \pi | J^h_\mu | K \rangle = f_1 P_\mu + f_2 q_\mu$$

with $f_{1,2}$ the form factors, which are functions of q^2, and $P_\mu = (p_K)_\mu + (p_\pi)_\mu$ and $q_\mu = (p_K)_\mu - (p_\pi)_\mu$.
(i) Calculate the matrix element squared for the process, summed over final state polarizations (take the neutrino as massless, but keep m_l)
(ii) Assuming the form factors are constant, calculate the differential decay width

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}E_l \mathrm{d}E_\pi}.$$

(iii) Neglect terms proportional to the charged lepton mass and integrate in the E_l and E_π variables over the appropriate kinematic regions to calculate the K^0_{l3} decay width.
(iv) Determine the numerical value of $|f_1|^2$ knowing that experimentally $\Gamma^{-1}_{K^0_{l3}} \simeq 1.3 \times 10^{-7}$ s.