Homework Set #7.

Due Date - Oral Presentation: Friday November 20, 2015
Due Date - Written Solutions: Monday November 30, 2015

1. **Compton Scattering in scalar QED**

   (a) Calculate the tree-level matrix elements for $(\gamma\phi \to \gamma\phi)$.

   (b) Calculate the cross section
   $$\frac{d\sigma}{d\cos\theta}$$
   for this process as a function of the incoming and outgoing polarizations, ϵ_μ^{in} and $\epsilon_\mu^{\text{out}}$, in the center of mass frame.

2. **Optional: Gordon Identity**
   Derive the Gordon identity
   $$\bar{u}(p')\gamma^\mu u(p) = \bar{u}(p')\left(\frac{p'^\mu + p^\mu}{2m} + \frac{i\sigma^{\mu\nu}q^\nu}{2m}\right)u(p),$$
   where $q = (p' - p)$.

3. **Optional: Supersymmetry**
   It is possible to write field theories with continuous symmetries linking fermions and bosons, called *supersymmetries*.

   (a) The simplest example of a supersymmetric field theory is the theory of a free complex boson and a free Weyl fermion, written as
   $$\mathcal{L} = \partial_\mu\phi^*\partial^\mu\phi + \chi^\dagger\bar{\sigma}\cdot\partial\chi + F^*F.$$

Here F is an *auxiliary* complex scalar field whose field equation is just $F = 0$. Show that this Lagrangian is invariant (up to a total divergence) under the infinitesimal transformation

$$\delta\phi = -i\epsilon^T \sigma^2 \chi,$$

$$\delta\chi = \epsilon F + \sigma \cdot \partial\phi \; \sigma^2 \epsilon^*,$$

$$\delta F = -i\epsilon^\dagger \bar{\sigma} \cdot \partial\chi,$$

where the parameter ϵ_a is a 2-component spinor (Grassmann numbers).

(b) Show that the term

$$\Delta\mathcal{L} = [m\phi F + \frac{1}{2} i m \chi^T \sigma^2 \chi] + [\text{complex conjugate}]$$

is also left invariant by the transformation given in part (a). Eliminate F from the complete Lagrangian $\mathcal{L} + \Delta\mathcal{L}$ by solving its field equation, and show that the fermion and boson fields ϕ and χ are given the same mass.