Homework Set #4.

Due Date: Tuesday November 25, 2008

Solve the following 6 exercises:

1. Consider a hypothetical T^+ meson $(t\bar{d})$. Given the structure of the CKM matrix, outline what is the most likely sequence of decays.

2. Show that the substitution of the Lagrangian

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - j^\mu A_\mu$$

into the Euler-Lagrange equation for A_μ gives the Maxwell equations

$$\partial_\mu F^{\mu\nu} = j^\nu,$$

where $F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu$. Hence, show that the current is conserved, i.e. $\partial_\nu j^\nu = 0$. With the addition of a mass term

$$\frac{1}{2}m^2 A_\mu A^\mu$$

show that the Lagrangian above leads to an equation of motion

$$\left(\Box^2 + m^2\right) A^\mu = j^\mu.$$

Argue that the mass term just introduced is not gauge invariant.

3. Recalling that a gauge transformation acts on the gluon gauge field as

$$G^a_\mu \to G^a_\mu - \frac{1}{g}\partial_\mu \alpha_a - f_{abc}\alpha_b G^c_\mu$$

show that the requirement of gauge invariance for the guage kinetic term

$$-\frac{1}{4}G^a_{\mu\nu}G^{\mu\nu}_a$$

implies the following form for the gluon tensor:

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g f_{abc} G^b_\mu G^c_\nu.$$

4. Associating the "vertex factor" for a given interaction to the coefficients multiplying the relevant fields in the interaction lagrangian, compute the (1) hW^+W^-, (2) hhW^+W^-, (3) hZZ and (4) $hhZZ$ vertex factors, using

$$\phi = \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}.$$

5. Calculate the $Z \to f\bar{f}$ decay rate (where f indicates any quark or lepton), neglecting fermion masses for the kinematically allowed decays. Assuming these are the dominant decay modes, find the branching ratio for each species of quark and lepton (remember that quarks come in three colors) and the Z lifetime. Knowing that, experimentally, the width to lepton pairs, e.g. $\Gamma_{Z\to e^+e^-}$, has been measured to be 84.0 ± 0.1 MeV, calculate $g^2/(4\pi)$ using the measured values of $M_Z = 91.1876(21)$ GeV and $\sin^2\theta_W = 0.23122(15)$. Compare this value to the value obtained using

$$\frac{g^2}{4\pi} = \frac{\alpha(M_Z)}{\sin^2\theta_W}, \quad \alpha(M_Z) \simeq 1/128.$$

6. Why is the contribution of the diagram $e^+e^- \to H \to W^+W^-$ too small to be appreciable when compared to the other diagrams in the electroweak theory leading to the same final state?