ASTR 257 - Homework 4

Due: May 16

1. Run an initial feasibility study for your observing proposal. This should include 1) an estimate of the necessary exposure time, and 2) a search for existing archival observations.

The problems below are meant to be doable by hand, so please show your work. If you want to use your favorite programing language/spreadsheet as a calculator or to check your results, that is fine.

2. You measure 112 counts from a star you are observing, and you measure the background (in the same area and exposure time) to be 500 counts. Assuming Poisson statistics what are the errors on each of these count rates? What is the total error in the signal-to-noise ratio for this star (considering only background and source noise)?

3. Here we will be comparing the color of galaxies in the Virgo cluster to non-cluster galaxies.

a. Go the the VizieR Service (http://vizier.u-strasbg.fr/viz-bin/VizieR) and find the SDSS DR9 catalog (just search for SDSS). Search for galaxies within 20' of M87 with $rmag <$ 18, $z = 0.001..0.01$, and $cl = 3$ (to select galaxies). You can put in the search criteria next to the individual column names. Record the u-r colors.

b. Run the same search but for a position of RA=187.0, Dec=0.0 and radius of 1.3 degrees (you need a larger radius to get a similar sample size since the galaxy density in the field is smaller). Again, record the u-r colors.

c. Use the rank-sum test to assess whether the typical colors of the two samples are the same (See attached scan). There is one significant outlier in the Virgo galaxies sample. If you remove this are the colors significantly different?

d. Figure 1 of astro-ph/0309710 shows a color magnitude diagram for low-redshift SDSS galaxies. Do cluster galaxies tend to lie on the red sequence or in the blue cloud? What about non-cluster galaxies? (Qualitatively, no stats needed here.)

4. The following table lists K-band luminosities and X-ray luminosities for a set of elliptical galaxies. Here the X-ray emission comes from hot thermal gas in the galaxy halo. K-band luminosity is a good indicator of galaxy stellar mass, so we might expect these two quantities to be correlated.

a. Fit the data to a line using the least-squares method. The data are listed as the log of the luminosities, and you should fit this as $\log(L_X) = \mathrm{a}\,\log(L_K) + \mathrm{b}$.

b. Estimate the errors on the fit parameters. You can assume the variables are normally (Gaussian) distributed. How might you estimate the errors if you did not assume a Gaussian distribution (describe in words)?

c. Calculate the χ^2. Is this a good fit?

Table 1.

$\log(L_K/L_\odot)$	$\log(L_X)$(ergs cm^{-2} s^{-1})
11.50	40.70 ± 0.12
11.35	40.35 ± 0.15
11.20	40.10 ± 0.15
10.98	39.45 ± 0.13
11.75	41.05 ± 0.10
10.60	39.10 ± 0.14
11.30	40.50 ± 0.17
11.10	40.00 ± 0.12

Note. — K-band luminosities are in units of the K-band luminosity of the Sun which is assumed to be $K_\odot = 3.39$.

The Wilcoxon/Mann–Whitney rank-sum test. This is a test for differences of location between two populations. Independent random samples $x_1, x_2, \ldots, x_m$ and $y_1, y_2, \ldots, y_n$ are given. To test the null hypothesis H_0 that $\text{Prob}(x > y) = \text{Prob}(x < y)$ against the alternative *one-sided* hypothesis that $\text{Prob}(x > y) < \text{Prob}(x < y)$ (so that the *x*s tend to be smaller than the *y*s) we use as test statistic U, the number of pairs x_i, y_j for which $x_i > y_j$. If U is less than the appropriate lower quantile we reject H_0 and accept H_1.

Similarly if H_1 is the hypothesis that $\text{Prob}(x > y) > \text{Prob}(x < y)$ we use as test statistic U', the number of pairs for which $x_i < y_j$. U' and U have the same null distribution, so that $U'_{[P]} = U_{[P]}$. Note that (ignoring ties) $U + U' = mn$.

Equivalently, if the *x*s and *y*s are pooled and ranked in ascending order of magnitude from 1 to $m+n$, we define

$$U = R_X - \tfrac{1}{2}m(m+1) \quad U' = R_Y - \tfrac{1}{2}n(n+1)$$

where R_X and R_Y are the sums of the ranks of the *x*s and *y*s respectively.

For the *two-sided* test with H_1: $\text{Prob}(x > y) \neq \text{Prob}(x < y)$ we use as test statistic U^*, the smaller of U and U'. The quantiles of U^* are related to those of U. For $P \leqslant \frac{1}{2}$:

$$U_{[P]} = U^*_{[2P]} \quad (\text{e.g. } U^*_{[.05]} = U_{[.025]})$$

The test was introduced by Wilcoxon, who used R_Y as test statistic.

Example 1. The performance of an electric switch can be measured by the number of times it can be operated before it fails. In order to decide whether the performance of switches of type A is significantly better than that of cheaper switches of type B, the performances of 6 switches of type A and 8 of type B were compared by repeatedly operating them and noting the order in which they failed. The failure order constitutes a performance ranking. The observed order is set out below.

Type	B	B	B	B	B	A	A	B	A	A	A	B	B	A
Failure order	1	2	3	4	5	6	7	8	9	10	11	12	13	14

A one-sided Mann–Whitney test can be used, with $m = 6$ (type A) and $n = 8$ (type B). The appropriate test statistic is U':

$$U' = R_B - \tfrac{1}{2}n(n+1) = 48 - 36 = 12$$

(R_B = sum of ranks of switches of type B). Now, for $m = 6$ and $n = 8$

$$U'_{[.05]} = U_{[.05]} = 11 \quad U'_{[.10]} = U_{[.10]} = 14$$

We conclude that the performance of switches of type A is significantly better at level 0.1 but not at 0.05.

Alternatively the one-sided Smirnov test (p. 69) or the two-sample runs test (p. 77) can be used.

Example 2. It is desired to test at 10% significance level whether treatment with a drug affects the mean reaction times. Eight individuals ($i = 1, 2, \ldots, 8$) are selected randomly; x and y are the reaction times before and after treatment.

i	1	2	3	4	5	6	7	8	
x_i	81	79	93	71	86	82	82	75	$R_+ = 30$
y_i	72	76	78	73	75	86	77	74	$R_- = 6$
$x_i - y_i$	+9	+3	+15	−2	+11	−4	+5	+1	$T^* = \min(R_+, R_-)$
R_i	6	3	8	2	7	4	5	1	$= 6$

The X- and Y-distributions will be identical and the d-distribution symmetric if the drug is ineffective; the Wilcoxon signed-rank test may therefore be used. We require a two-sided test, and use the test statistic T^* which has the value 6. Since, for $n = 8$, $T^*_{[.10]} = T_{[.05]} = 6$ we conclude that the change in mean reaction times is not significant at level 0.1.

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n	.90
1	.900
2	.684
3	.565
4	.493
5	.447
6	.410
7	.381
8	.358
9	.339
10	.323
11	.308
12	.296
13	.285
14	.275
15	.266
16	.258
17	.250
18	.244
19	.237
20	.232
21	.226
22	.221
23	.216
24	.212
25	.208
26	.204
27	.200
28	.197
29	.193
30	.190

Source: L.

Kolmogorov tests a
function for a given
probability distribu
test the null hypot
distribution functio
use the test statistic

and reject H_0 in fav
If the alternative

To test H_0 again

The table gives u
large n, quantiles a
example, for $n =$ 1

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m	*n*	*P* = .001	.005	.01	.025	.05	.10
2	2	0	0	0	0	0	0
	3	0	0	0	0	0	1
	4	0	0	0	0	0	1
	5	0	0	0	0	1	2
	6	0	0	0	0	1	2
	7	0	0	0	0	1	2
	8	0	0	0	1	2	3
	9	0	0	0	1	2	3
	10	0	0	0	1	2	4
	11	0	0	0	1	2	4
	12	0	0	0	2	3	5
	13	0	0	1	2	3	5
	14	0	0	1	2	4	6
	15	0	0	1	2	4	6
	16	0	0	1	2	4	6
	17	0	0	1	3	4	7
	18	0	0	1	3	5	7
	19	0	1	2	3	5	8
	20	0	1	2	3	5	8
3	3	0	0	0	0	1	2
	4	0	0	0	0	1	2
	5	0	0	0	1	2	3
	6	0	0	0	2	3	4
	7	0	0	1	2	3	5
	8	0	0	1	3	4	6
	9	0	1	2	3	5	6
	10	0	1	2	4	5	7
	11	0	1	2	4	6	8
	12	0	2	3	5	6	9
	13	0	2	3	5	7	10
	14	0	2	3	6	8	11
	15	0	3	4	6	8	11
	16	0	3	4	7	9	12
	17	1	3	5	7	10	13
	18	1	3	5	8	10	14
	19	1	4	5	8	11	15
	20	1	4	6	9	12	16
4	4	0	0	0	1	2	4
	5	0	0	1	2	3	5
	6	0	1	2	3	4	6
	7	0	1	2	4	5	7
	8	0	2	3	5	6	8
	9	0	2	4	5	7	10
	10	1	3	4	6	8	11
	11	1	3	5	7	9	12
	12	1	4	6	8	10	13
	13	2	4	6	9	11	14
	14	2	5	7	10	12	16
	15	2	6	8	11	13	17
	16	3	6	8	12	15	18
	17	3	7	9	12	16	19
	18	4	7	10	13	17	21
	19	4	8	10	14	18	22
	20	4	9	11	15	19	23
5	5	0	1	2	3	5	6
	6	0	2	3	4	6	8
5	7	0	2	4	6	7	9
	8	1	3	5	7	9	11
	9	2	4	6	8	10	13
	10	2	5	7	9	12	14
	11	3	6	8	10	13	16
	12	3	7	9	12	14	18
	13	4	8	10	13	16	19
	14	4	8	11	14	17	21
	15	5	9	12	15	19	23
	16	6	10	13	16	20	24
	17	6	11	14	18	21	26
	18	7	12	15	19	23	28
	19	8	13	16	20	24	29
	20	8	14	17	21	26	31
6	6	0	3	4	6	8	10
	7	1	4	5	7	9	12
	8	2	5	7	9	11	14
	9	3	6	8	11	13	16
	10	4	7	9	12	15	18
	11	5	8	10	14	17	20
	12	5	10	12	15	18	22
	13	6	11	13	17	20	24
	14	7	12	14	18	22	26
	15	8	13	16	20	24	28
	16	9	14	17	22	26	30
	17	10	16	19	23	27	32
	18	11	17	20	25	29	35
	19	12	18	21	26	31	37
	20	13	19	23	28	33	39
7	7	2	5	7	9	12	14
	8	3	7	8	11	14	17
	9	4	8	10	13	16	19
	10	6	10	12	15	18	22
	11	7	11	13	17	20	24
	12	8	13	15	19	22	27
	13	9	14	17	21	25	29
	14	10	16	18	23	27	32
	15	11	17	20	25	29	34
	16	12	19	22	27	31	37
	17	14	20	24	29	34	39
	18	15	22	25	31	36	42
	19	16	23	27	33	38	44
	20	17	25	29	35	40	47
8	8	5	8	10	14	16	20
	9	6	10	12	16	19	23
	10	7	12	14	18	21	25
	11	9	14	16	20	24	28
	12	10	16	18	23	27	31
	13	12	18	21	25	29	34
	14	13	19	23	27	32	37
	15	15	21	25	30	34	40
	16	16	23	27	32	37	43
	17	18	25	29	35	40	46
	18	19	27	31	37	42	49
	19	21	29	33	39	45	52
	20	22	31	35	42	48	55

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Source: For
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Statistical Tables for the Social, Biological, and Physical Sciences
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