Dynamics of the Peccei-Quinn Scale

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Tom and Willy have been important figures in my career, from the very beginning until now. I met Willy when I was a graduate student, and he was collaborating with my advisor on questions in heavy quark physics. I met Tom about five years later in Tel Aviv, when we were both struggling to understand supersymmetry, and thinking about Witten's *inverted hierarchy*. Our research, and our careers, have intersected repeatedly through the years.

Axions have been one of the themes. Willy and I started thinking about the invisible axion one year when we were both at the IAS, along with Mark Srednicki, who was then at Princeton. This was new territory for me; we stumbled on it while trying to understand low energy supersymmetry. It was the most exciting thing which had happened to me in physics up to that time. Not much later, Willy started to bug me about the cosmology of the axion, and gradually we (and Abbott, Sikivie, Preskill, Wise and Wilczek) understood the problem, and that the axion could be the dark matter.

But the axion idea, as I'll review, while cute, is in many ways farfetched. Tom and I have attacked this question repeatedly. Is the axion cosmology as conventionally conceived? Do PQ symmetries make sense in field theory? In string theory?
Tom and Willy have been my friends for more years than I want to count; they've also been among my most important mentors. So I should begin with a disclaimer that the errors and misconceptions in this talk are entirely my own. I hope they'll find this at least mildly entertaining.

Outline

1. Axions: Their virtues, deficiencies. Axions to account for dark matter/dark matter to account for axions.
2. PQ Breaking In String Theory, Higher Dimensions (Without, with supersymmetry)
3. PQ Breaking in Field Theory Without Supersymmetry: Naturalness of $f_a = 10^{12}$
4. PQ Breaking in Field Theory With Supersymmetry:
   1. Intermediate Scale Breaking: Unlikely(?)
   2. Low Scale Breaking (Gauge Mediation): Points to scale of SUSY Breaking, f_a.

Setting: The Strong CP Problem and Proposed Solutions

1. $m_u = 0$ Simple. Could result as an accident of discrete flavor symmetries (Banks, Nir, Seiberg), or a result of “anomalous" discrete symmetries as in string theory.
2. CP exact microscopically, $\theta = 0$; spontaneous breaking leads only to small effective θ (Nelson, Barr). In critical string theories, CP is an exact (gauge) symmetry, spontaneously broken at generic points in typical moduli spaces. A plausible framework.
3. Axions: global, anomalous symmetry. Axion adjusts to yield $\theta \approx 0$. Puzzling why such a good symmetry. (Critical) String theory answers question of why one should have an extremely good (but approximate) global symmetry: Peccei-Quinn symmetries which hold to all orders of perturbation theory. $e^{\frac{-8\pi^2}{g^2}}$

Problems with each of these solutions:

1. $m_u = 0$. Lattice computations seem to rule out. (Current discussions with Tom, Seiberg, Kaplan, others; some skepticism.)
2. Spontaneous CP: need to consider fixing of moduli. Flux vacua as a model: 10^{500}, say, arising from turning on many different fluxes. But only half of fluxes, typically, invariant under CP: $10^{500} \to 10^{250}$. So only a tiny fraction of states. Not clear that these are otherwise singled out in an interesting way (e.g. cosmologically, anthropically).
3. Axions: If nature is something like a critical string theory, not clear, when moduli are fixed, why axions should survive to low energies (Banks, Gorbatov, M.D.).

So doubts about all three possibilities.

Focus on Axions. How plausible? Reasonable expectations for f_a? What they might suggest about other Beyond the Standard Model Physics.

1. String Theory (PQ spontaneously broken in higher dimensions and/or by stringy effects)
2. PQ breaking visible in four dimensional effective Field Theory

and within these categories:

1. No low energy supersymmetry
2. Supersymmetry, intermediate scale breaking (“gravity mediation")
3. Supersymmetry, low scale breaking (*gauge mediation*)

Two puzzles with axions

1. Astrophysics, cosmology seem to constrain the axion decay constant to a rather narrow range. If axion is to be dark matter, expect $f_a \sim 10^{12}$ GeV. Why this number?
2. PQ symmetry a global symmetry, presumably an accident [Glashow et al; Kamionkowski, March-Russell]. Needs to be a very good symmetry if to solve strong CP.

Requirements of the Global Symmetry

$$V_{qcd} \approx -m_\pi^2 f_\pi^2 \cos(\frac{a}{f_a}).$$

Natural value of axion potential:

$$V_a = Q\, f_a^4 \cos(\frac{a}{f_a} - \theta_0)$$

so if axion to solve strong CP problem, need *quality factor*

$$Q < 10^{-62}.$$

In terms of Planck suppressed ops, $(f_a/M_p)^n$, $n > 10$, for $f_a = 10^{12}$; $n > 16$ for $f_a = 10^{15}$ Why should there be a PQ symmetry at all, and why such a good symmetry?

Accounting for A Very Good Global Symmetry

1. Critical String Theory: PQ symmetry accidental consequence of two form gauge invariance of the theory, or of discrete (presumably gauge) symmetries of string theory (e.g. subgroup of $SL(2, Z)$). A good symmetry of perturbation theory. So breaking could be exponentially small in some parameter, $\epsilon = e^{-8\pi^2/g^2}$.
2. If real world almost a critical string theory, need to account for fixing of moduli. KKLT scenario: fluxes lead to a small number, but all moduli fixed *at scales well above susy breaking scale*. No light axions. Perhaps axions in a small subset of states, but why are these special?
3. If PQ breaking at low energies, discrete symmetries might explain (e.g. Z_N, with N quite large). Doesn't sound particularly generic (Z_{12}, Z_{24}), unless correlated with other phenomena.

Axion as Dark Matter

Axions have long been considered a plausible dark matter candidate. Produced coherently in the early universe, by misalignment (I'll assume that the reheat temperature after inflation is below f_a). The energy density of axions is proportional to the square of the misalignment angle, θ_0.

$$\Omega_a h^2 \approx 0.15\theta_0^2 \left(\frac{f_a}{10^{12}}\right)^{7/6}.$$

This gives an upper bound on f_a, if $\theta_0 \sim 1$, of order 10^{12}. If the bound is saturated, the dark matter is accounted for. There is a lower bound coming from astrophysics (esp. supernova 1987a, globular clusters) of about 10^9 GeV.

Relaxing the Bound

Three possibilities:

1. Late decays of particles (inevitable if intermediate scale susy breaking; pseudomoduli) can allow f_a up to $10^{14} - 10^{15}$ (Dine,Fischler; Turner; Banks, Dine).
2. Small θ_0 relaxes the bound. Note that if the PQ transition occurs after inflation, different regions have different θ_0. So we could just be (un)lucky (Pi, Dine/Fischler).
3. As above, but perhaps anthropic considerations select for small θ_0 (Linde; Aguirre, Reiss, Tegmark, Wilczek). No definitive conclusion, but plausible that hospitable universes lie in a narrow range of Ω_a. Note the assumption of inflation means that, *if there is some peaking in the distribution*, some selection is inevitable.

Having introduced in an un-contentious way, anthropic selection (**thanks to Andrei, Anthony; apologies to Tom**) for θ_0, it is tempting to consider anthropic selection for

1. The existence of axions
2. Other parameters, such as f_a.

The first point requires that, in some theoretical framework, axions be a particularly “generic" type of dark matter. In the second, in an underlying landscape, one might expect that f_a varies. This might be interesting if requiring an axion to be the dark matter simultaneously explains the smallness of the observed θ.

Axion as Accidental Dark Matter

In order to constitute the dark matter, how light does axion have to be? Our basic requirement is that the axion not dominate the energy density for temperatures above about 1 eV. If the Peccei-Quinn symmetry is violated by some higher dimension operator, scaled by M_p, such as

$$\delta V = \frac{he^{i\alpha}}{M_p^n}\phi^{n+4} = hf_a^4\left(\frac{f_a}{M_p}\right)^n \cos(a/f_a + \alpha)$$

Then the axion mass is:

$$m_a^2 = f_a^2\left(\frac{f_a}{M_p}\right)^n$$

On the other hand, the initial axion energy density is of order $f_a^2/M_p^2 = 10^{-12}\left(\frac{f_a}{10^{12}}\right)^2$. So we require:

$$10^{27}\left(\frac{f_a}{M_p}\right)^{\frac{n+10}{4}} < 1$$

For $f_a = 10^{12}$, this indeed requires $n > 8$, but the requirement of small enough θ means $n > 10$.
It is hard to assess the relative likelihood of these two cases; e.g. if due to discrete symmetry, a large discrete symmetry in each case, but one might worry that a larger symmetry is exponentially less likely.

Interestingly, for larger f_a there is a crossover; the requirement of dark matter insures small enough θ for $f_a \sim 10^{14}$. However, the required n's are huge, more than 20!
In string theory, within our present, limited understanding, the problem looks different.

The θ Problem in String Theory

Issues:

1. Mechanism which fixes moduli must leave one Peccei-Quinn symmetry nearly exact. Not clear that this is generic.
   Simplest version of KKLT scenario: all moduli fixed at high scales. Lightest is Kahler modulus: $W = W_0 + e^{-\rho}$, $W_0 > m_{3/2}$. W_0 breaks the PQ symmetry. Multiple Kahler moduli? Perhaps some don't appear in superpotential, or appear suppressed by $e^{-n\rho}$. (If, e.g., $e^{-\rho} = m_{3/2}^2/M_p^2$, need $n \geq 3$). Not clear whether such a phenomenon is generic; what might select for it. Perhaps dark matter?.
2. Moduli problem: saxion mass must be 30 TeV or so if to decay before nucleosynthesis (Banks, Kaplan, Nelson).

String Theory: A picture without supersymmetry

E.g. suppose that there is a small parameter, $e^{\frac{-8\pi^2}{g^2}} = \epsilon$.
First, suppose no susy. If ϵ is the strength of PQ breaking, and if $f_a = 10^{15}$, need

$$\epsilon = 10^{-74}$$

to account for θ.
This is *weaker* than the condition to account for dark matter!

String Theory With Supersymmetry

If supersymmetry is broken at low energies, there is another small parameter. Take $\epsilon = m_{3/2}^2/M_p^2$. Need ϵ^3 suppression to account for dark matter. Again, this is enough to explain θ_{qcd}. Alternatively, there might be some other small quantity, $\epsilon' \ll \epsilon$. But if SUSY broken at intermediate scale, other dark matter candidates, perhaps more generic.
Gauge mediation (below).

Field Theory Models Without Supersymmetry

Perhaps axion a generic dark matter candidate.
$f_a \ll M_p$ fine tuned without SUSY?
Can now pose the following question. As Aguirre et al argue, initial value of θ_0 might be selected so as to account for a large f_a. In a non-supersymmetric theory, we would expect small f_a very fine tuned; $\theta_0 < 10^{-3}$ far less so (e.g. if f_a selected from a landscape of possibilities, small θ_0 would seem far more probable).

Dynamical Breaking of PQ Symmetry

Different if f_a dynamical. E.g. $SU(N)$ theory,
$Q = (N, 5)$ $\overline{Q} = (\overline{N}, \overline{5})$, $q = (N, 1)$, $\overline{q} = (\overline{N}, 1)$.
Then

$$\langle \bar{Q} Q \rangle \approx \Lambda^3 \quad \langle \bar{q} q \rangle \approx \Lambda^3$$

break a PQ symmetry with QCD anomaly; $f_a \approx \Lambda$.
$\Lambda = M_p e^{\frac{-8\pi^2}{b_0 g^2}}$; if g^2 uniformly distributed, small f_a is favored over small θ_0 (can quantify).
A "naturalness" argument that axions might be observable in cavity experiments (Sikivie, Van Bibber, Rosenberg...).

In this dynamical context, a smaller discrete symmetry might account for the quality of the PQ symmetry. If $f_a = 10^{12}$, ops like

$$\delta\mathcal{L} = \frac{(\bar{q}q)^n}{M_p^{3n-2}}$$

explicitly break the PQ symmetry. Need $n > 3$. Dark matter requires only $n = 3$. Perhaps (un) lucky.

Intermediate scale SUSY breaking

Such a structure would seem natural in string theory, but one might expect a large f_a.
Cosmological issues with moduli (Banks, Kaplan, Nelson). But for our considerations today, biggest problem is that there are other, likely more generic, dark matter candidates.

Low Scale Supersymmetry Breaking (Gauge Mediation)

Gauge mediation: no single compelling dark matter candidate.
Natural to examine the axion.
Calling f the underlying scale of supersymmetry breaking, roughly $10^5\mathrm{GeV} < f < 10^9\mathrm{GeV}$.
Suppose that the saxion couples to the messengers/susy breaking sector through Planck or Gut suppressed operators. Even in the latter case, and for $f = 10^9$, $m_s \sim 1GeV$. Its lifetime is of order

$$\Gamma \approx \frac{m_s^3}{M^2} \approx 10^{-32}$$

long after nucleosynthesis. Lower f_a, f: too long.
Perhaps there is a solution, but seems even more contrived (tuned) than the heavy modulus in the intermediate scale case.
Suggests that $\mathcal{A}$ should couple directly to Messengers.

A simple model with unbroken supersymmetry:

χ neutral under PQ, $S_\pm$ charges ± 1.

$$W = \chi(S_+ S_- - \mu^2) + S_+ q\bar{q} + S_- \ell\bar{\ell}$$

Write $S_\pm = \mu exp(\pm \mathcal{A}/\mu)$. $\mathcal{A}$ is a modulus; it's real part determines f_a. The imaginary part is the axion.

Adding supersymmetry breaking

It is not hard to write models in which susy is broken and the value of f_a is fixed.
Quite generally, a light scalar (pseudomodulus) determines f_a.
This scalar is not necessarily the saxion; indeed, the saxion, axino are not necessarily especially light, nor are they necessarily mass eigenstates (i.e. identifiable as special particle types).
But the light modulus is an issue for *cosmology*.
If axion is dark matter ($f_a > 10^{12}$), requires:

1. $f_a < 10^{13}$ GeV
2. $m_{3/2} > 1$ MeV

$$m_P^2 = (\text{loop factor})\frac{F^2}{f_a^2}. \tag{1}$$

Require decay before nucleosynthesis:

$$\Gamma \approx \frac{m_P^3}{f_a^2}\left(\frac{\alpha_i}{4\pi}\right)^2 \tag{2}$$

$$= 10^{-24}\ \text{GeV}\left[\left(\frac{F}{10^{16}\ \text{GeV}}\right)^3\left(\frac{10^{12}}{f_a}\right)^5\right]$$

Conclusions

A few lessons (some surprising, at least for me):

1. Spontaneous CP not a likely solution of the strong CP problem, so if lattice results confirmed, left with axion.
2. Existence of a PQ symmetry, good enough to solve strong CP, *might* be correlated with the problem of dark matter
3. In non-supersymmetric theories, low f_a is natural if PQ breaking dynamical
4. In string theory, existence of axions likely also correlated with existence of a very small parameter (and, with dark matter)
5. If we discover evidence for gauge mediation, and if we are convinced that $m_u \neq 0$, then strong CP accounts for a small value of f_a, points to high scales of susy breaking.

THE END