Conformality Lost

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D. T. Son
M. Stephanov
D.B.K

arXiv:0905.4752

Motivation: QCD at LARGE N_c and N_f

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Colors

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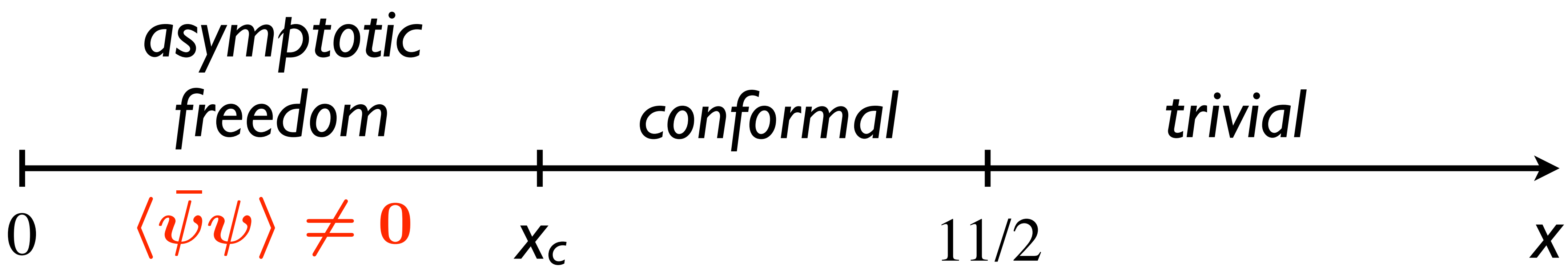
Colors *Flavors*

Define **$x = N_f/N_c$**, treat as a continuous variable



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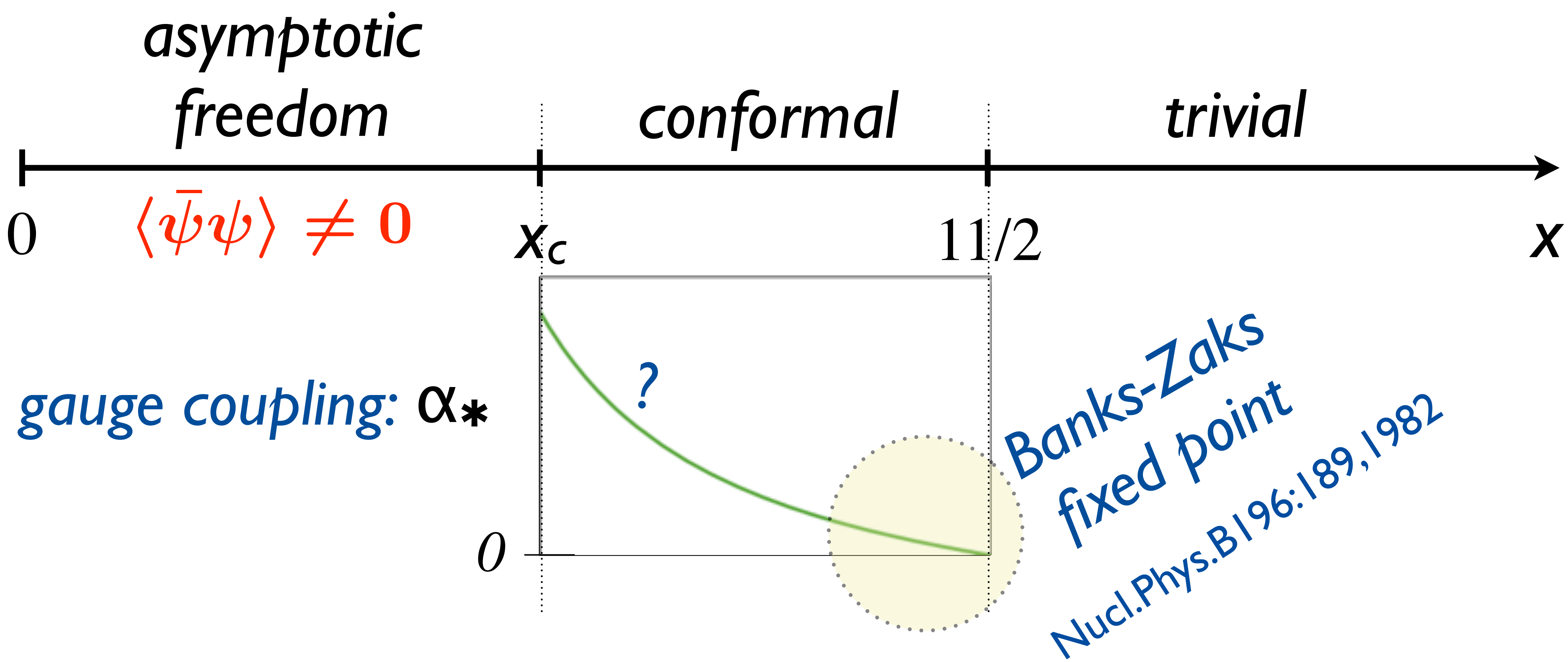

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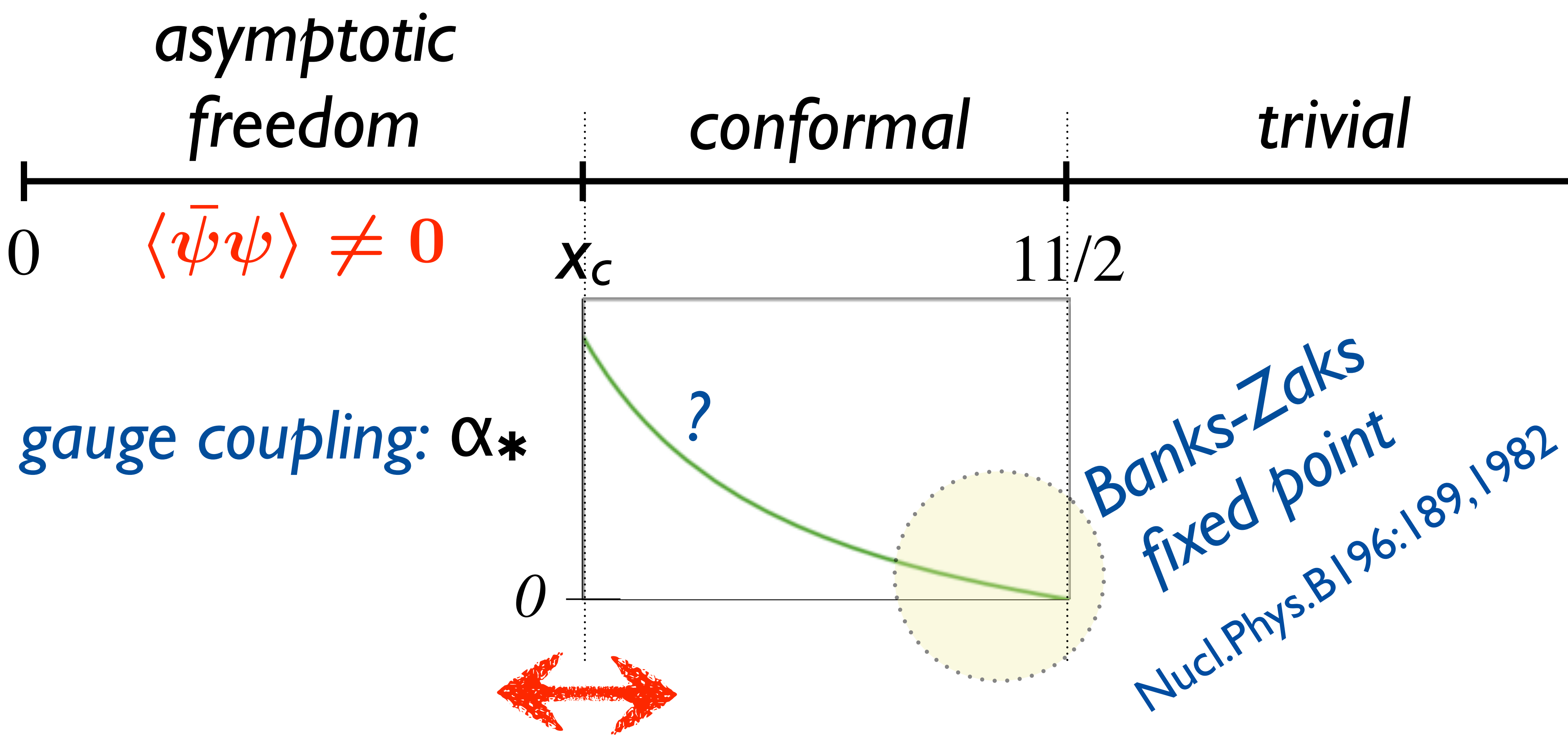

What is the nature of this transition?

How does the IR scale appear as conformality is lost?

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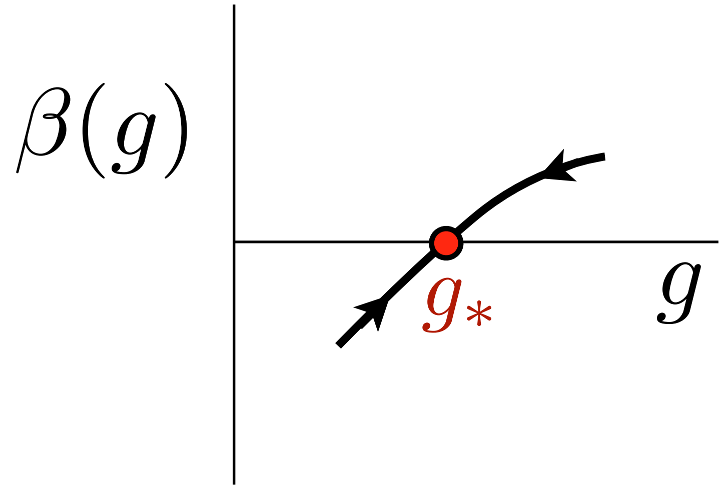
VI. QCD with many flavors? A partner theory QCD* with a nontrivial UV fixed point?

A theory with an infrared conformal fixed point at $g=g_*$ has a zero in the beta function:

$$\beta(g) = \mu\frac{\partial g}{\partial\mu} = \frac{\partial g}{\partial t}$$



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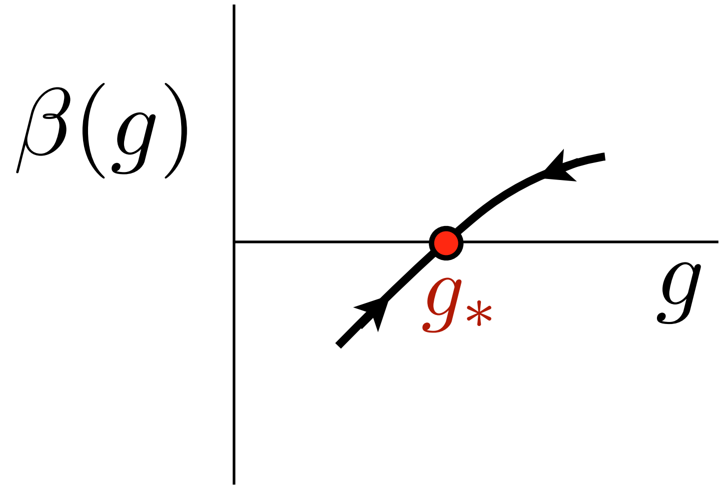

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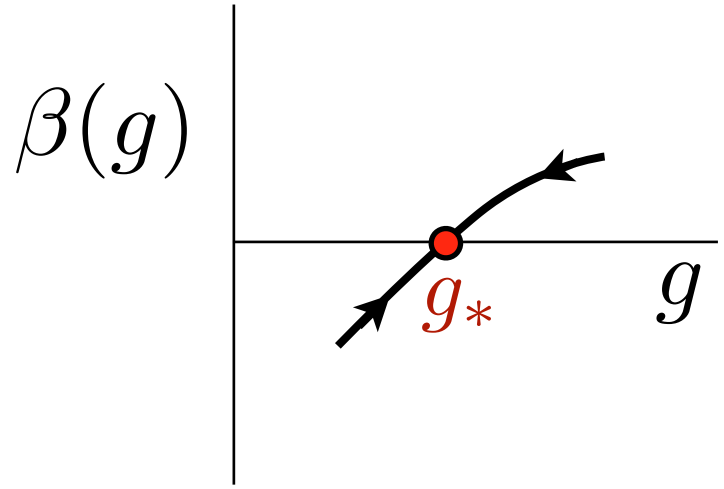

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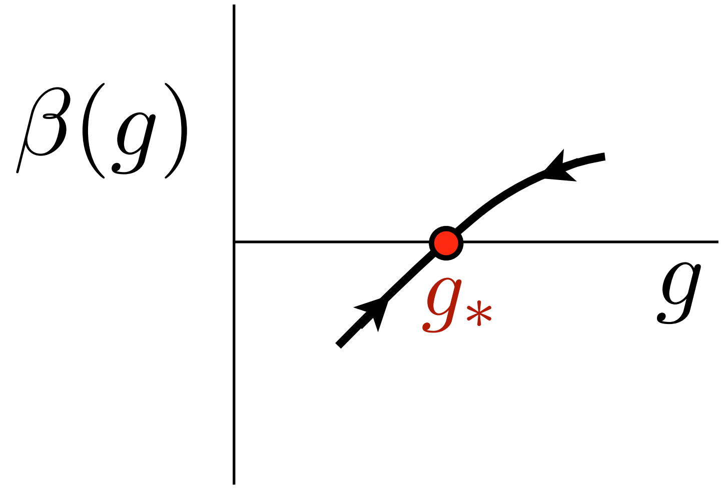

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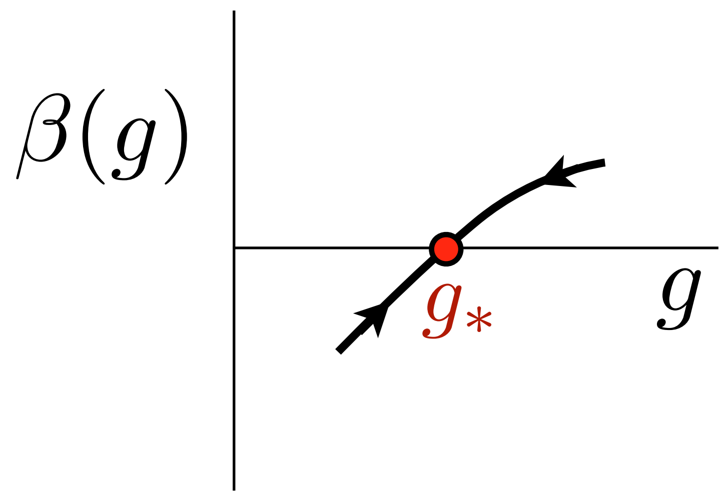

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How is conformality lost?

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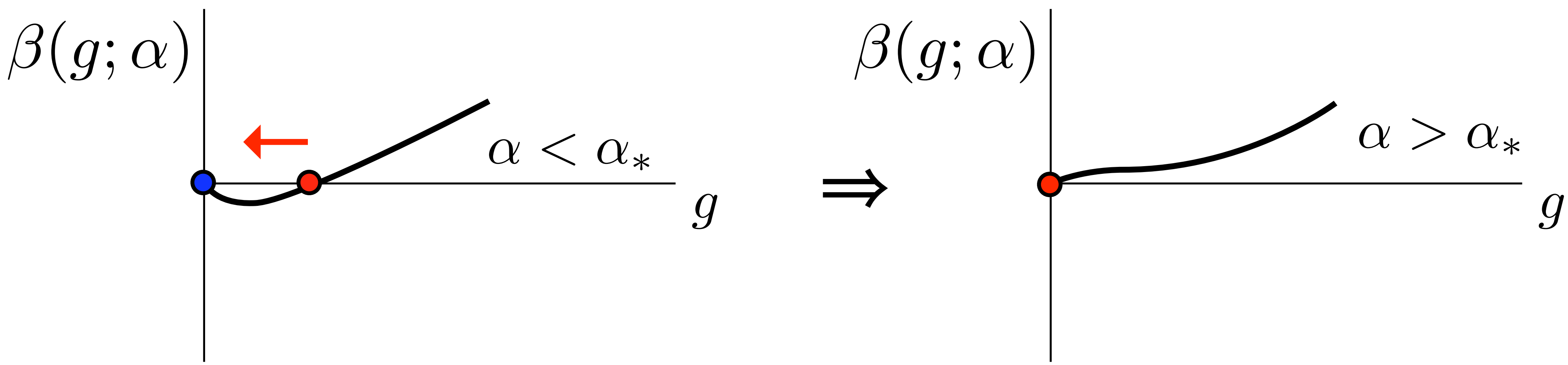

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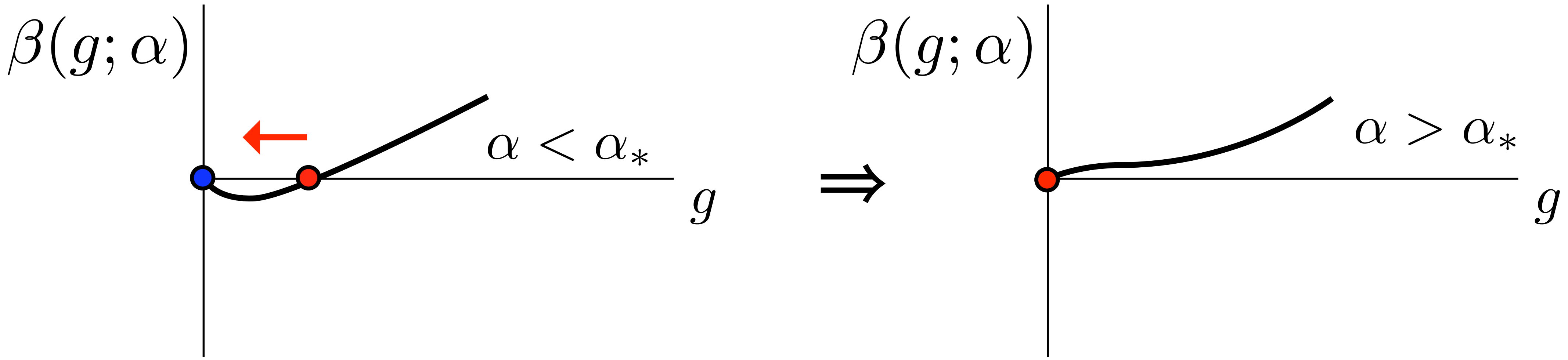

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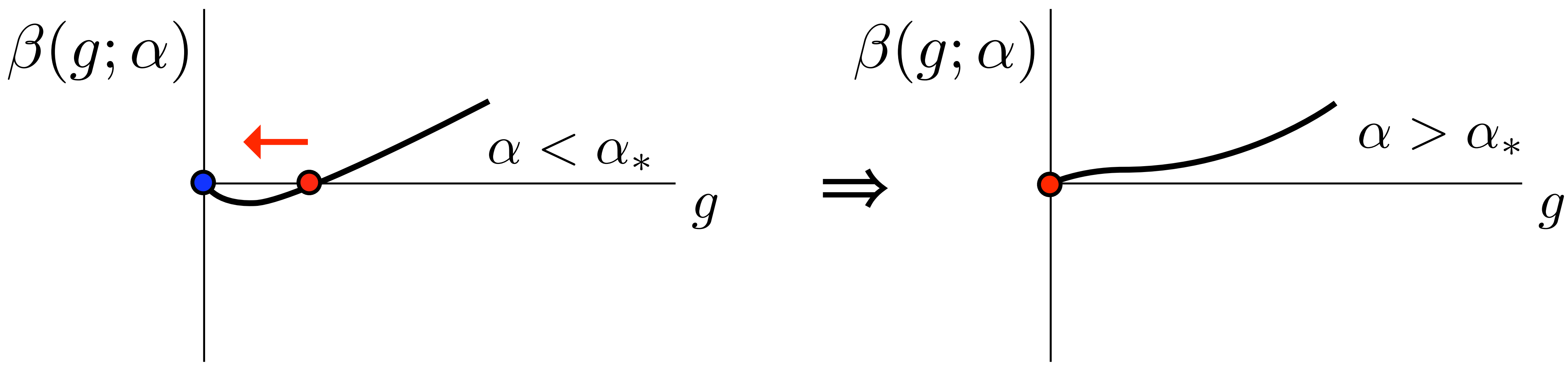

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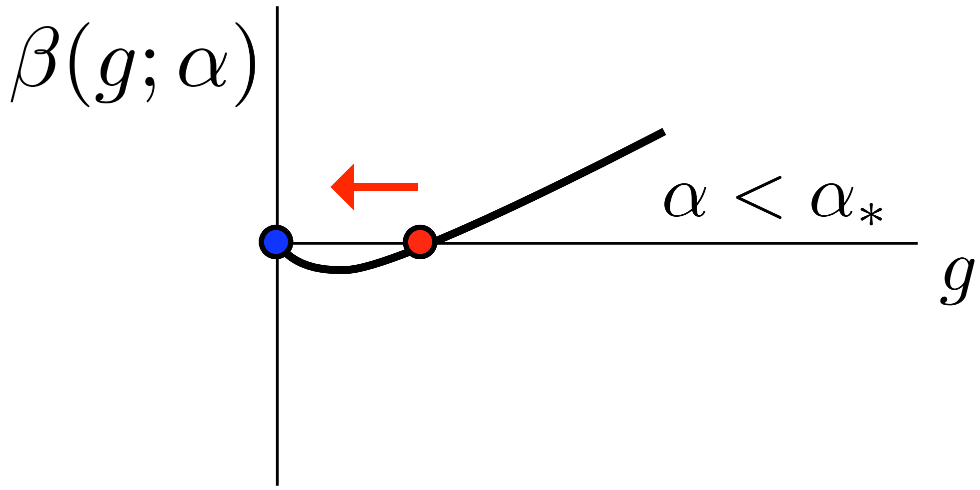

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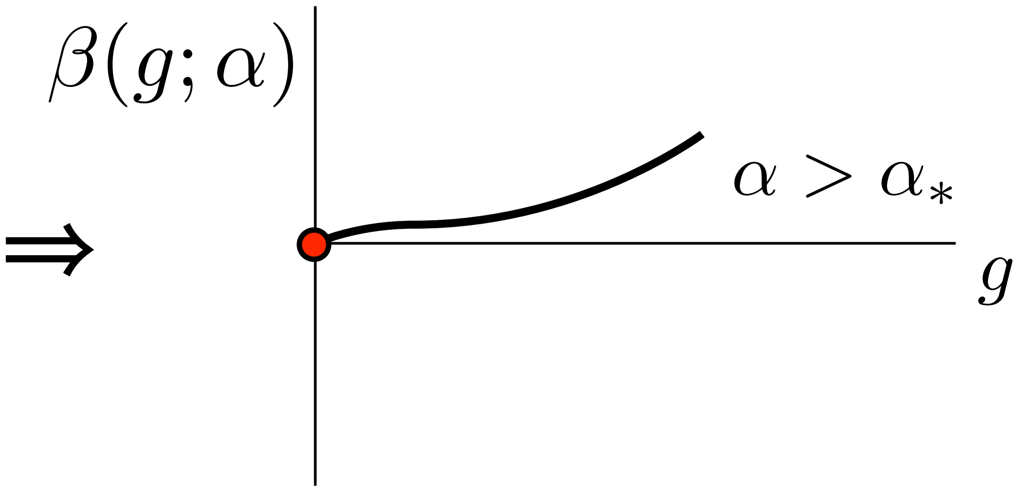

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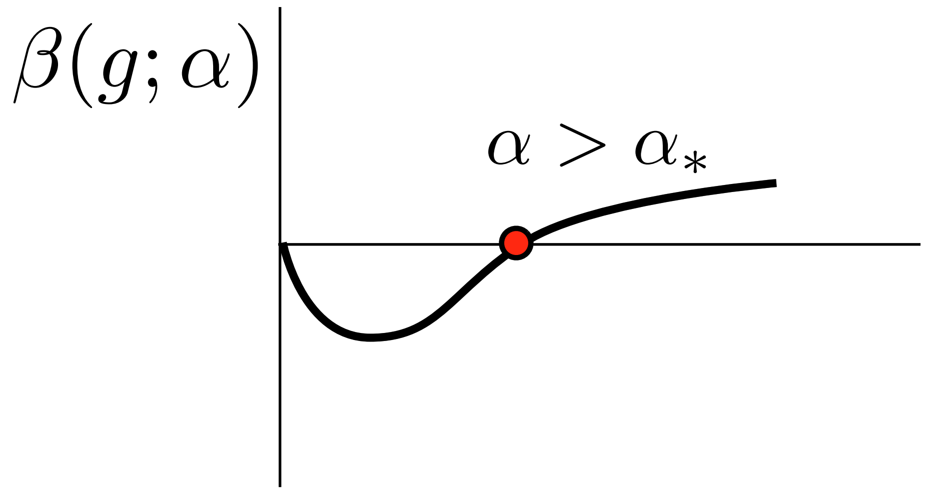

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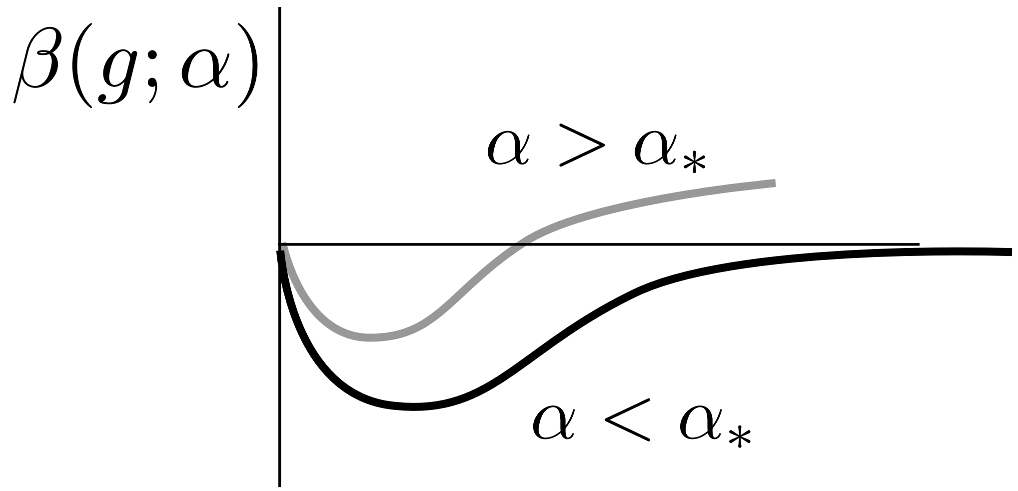

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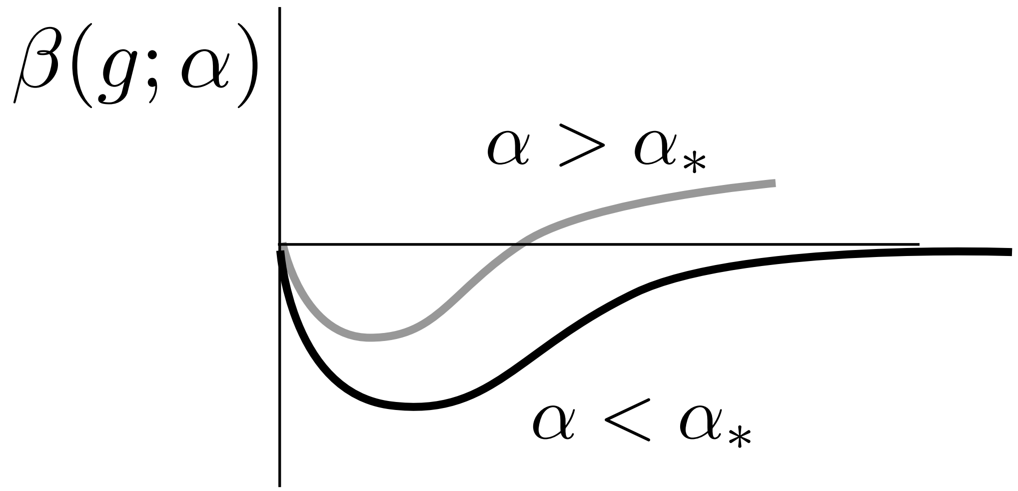

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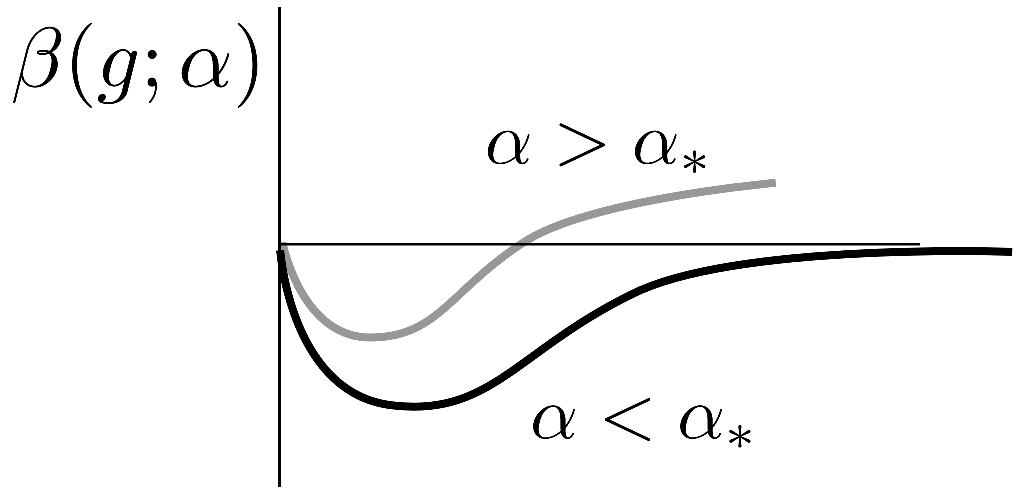

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➤ electric theory dual to a QED-like magnetic theory:

$$F_E \sim \frac{g^2 \ln(r\,\Lambda_{\rm UV})}{r^2} \qquad F_M \sim \frac{g_M^2}{r^2 \ln(r\,\Lambda_{\rm UV})} \qquad g_M \sim 1/g$$

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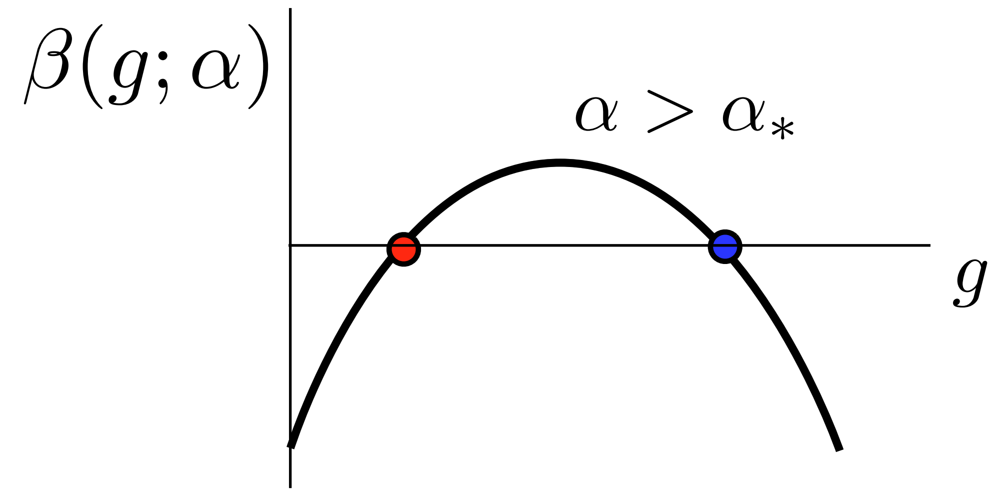

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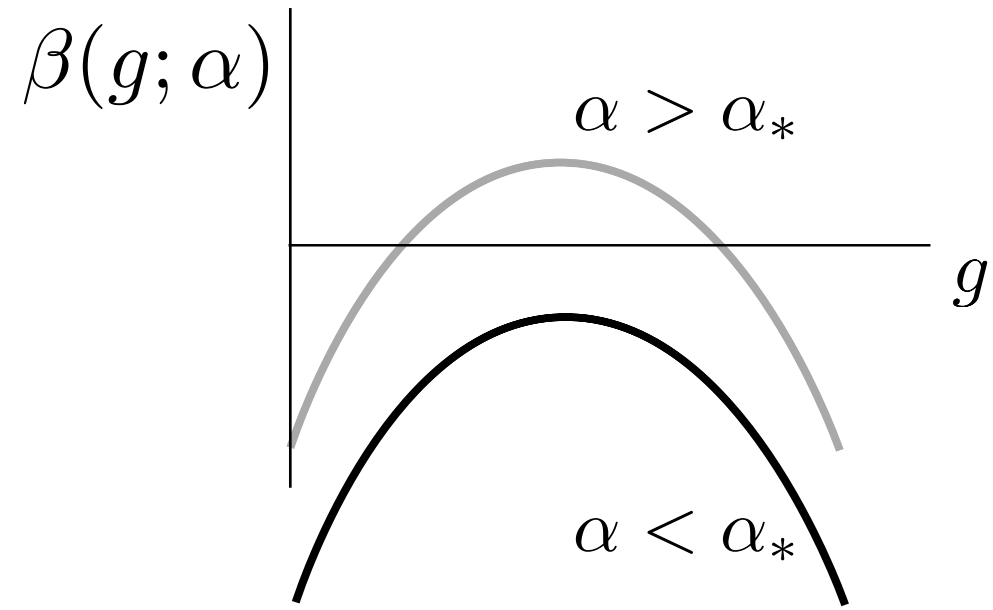

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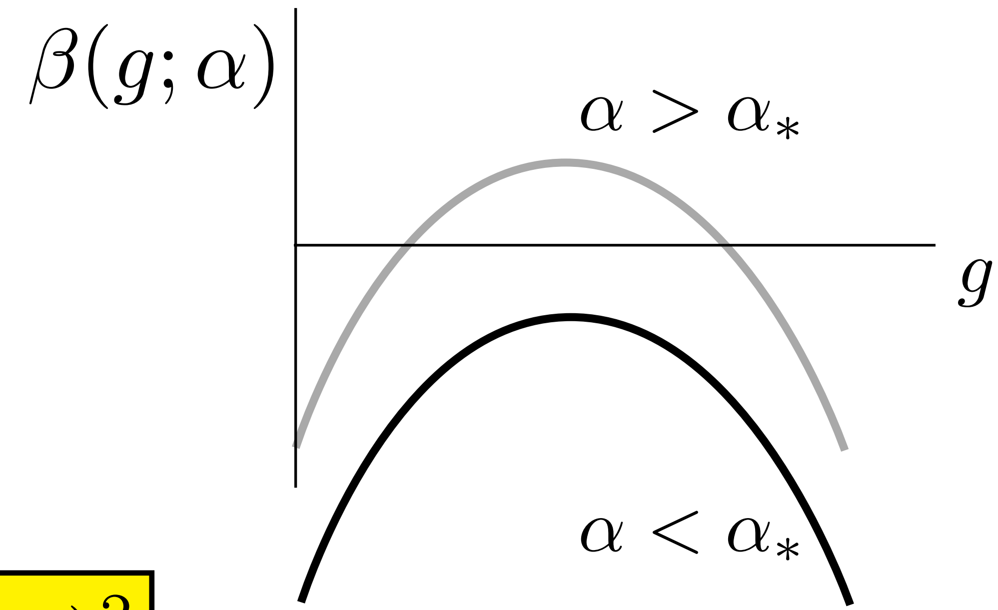

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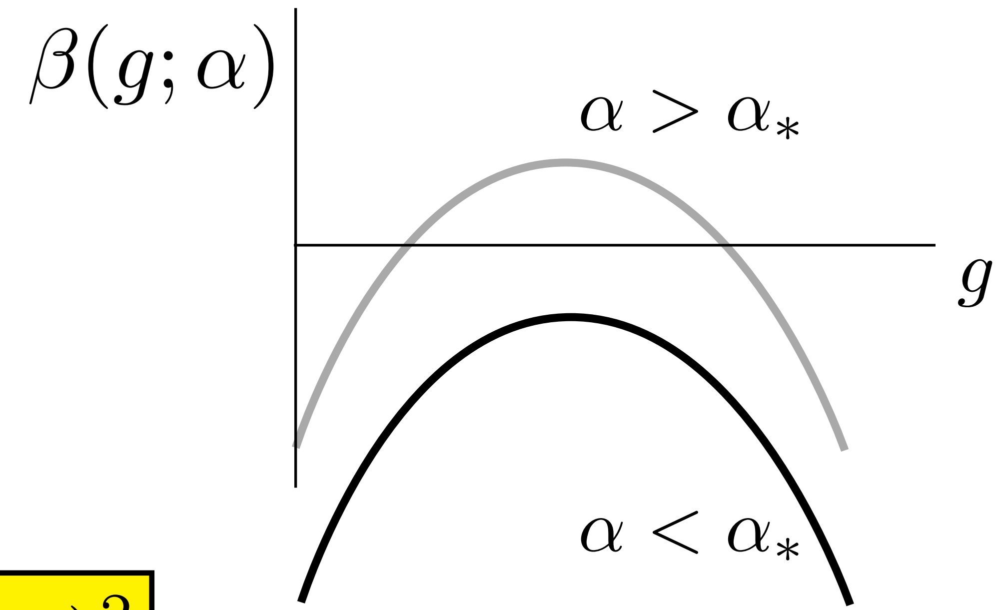

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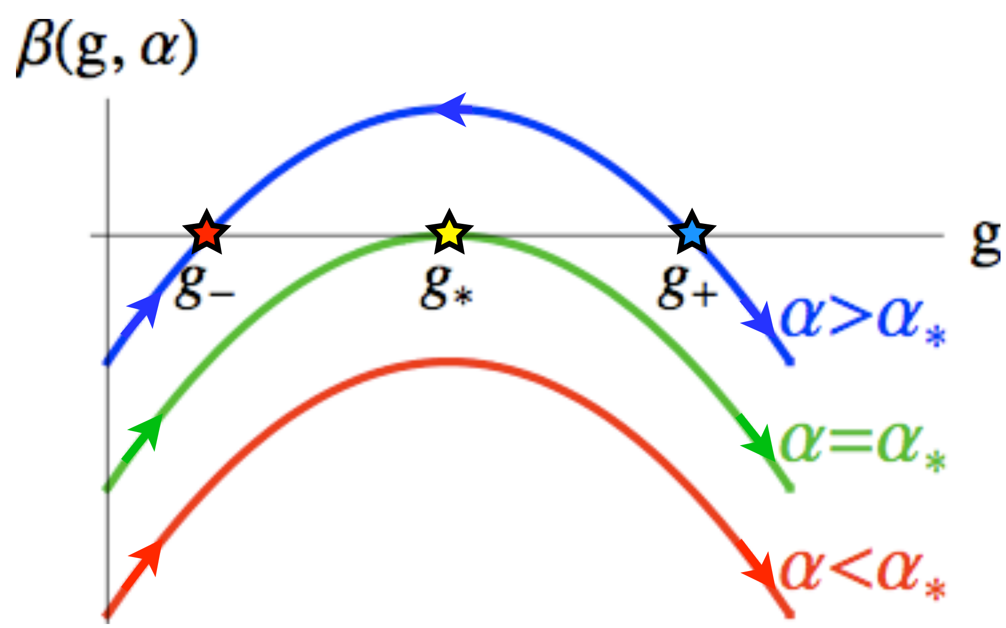

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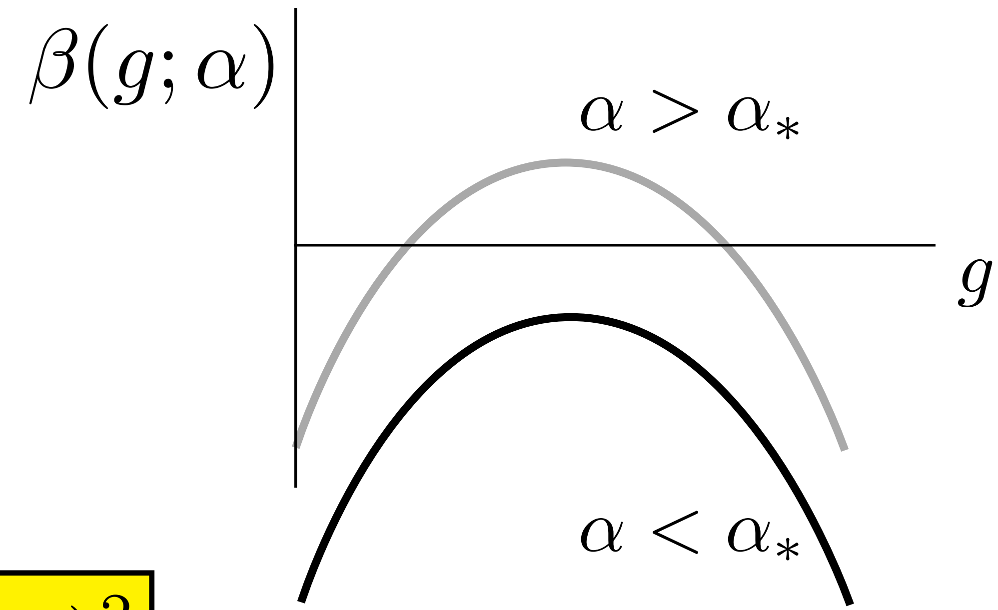

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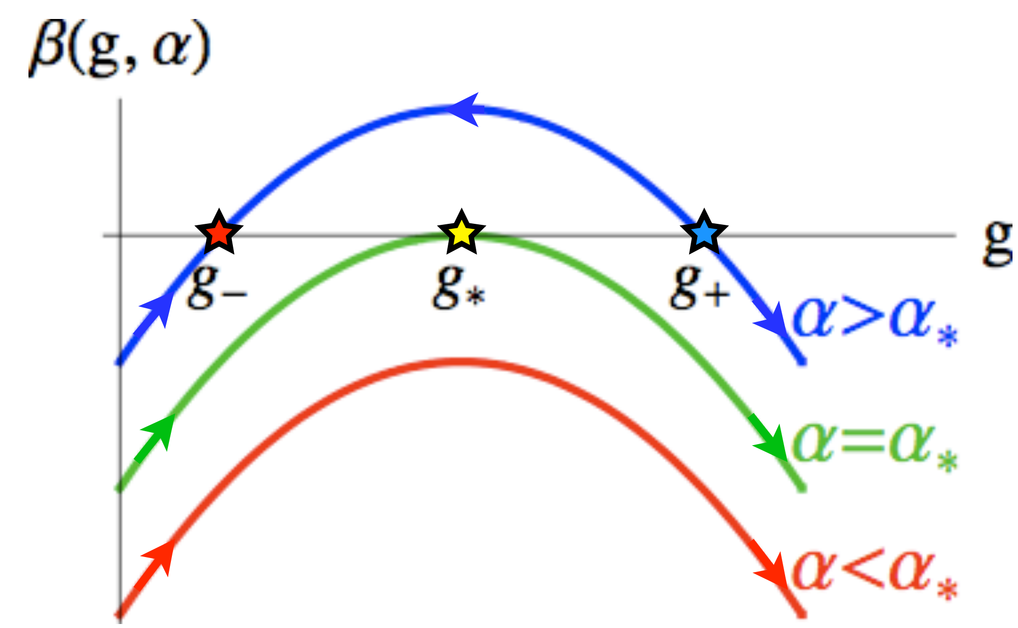

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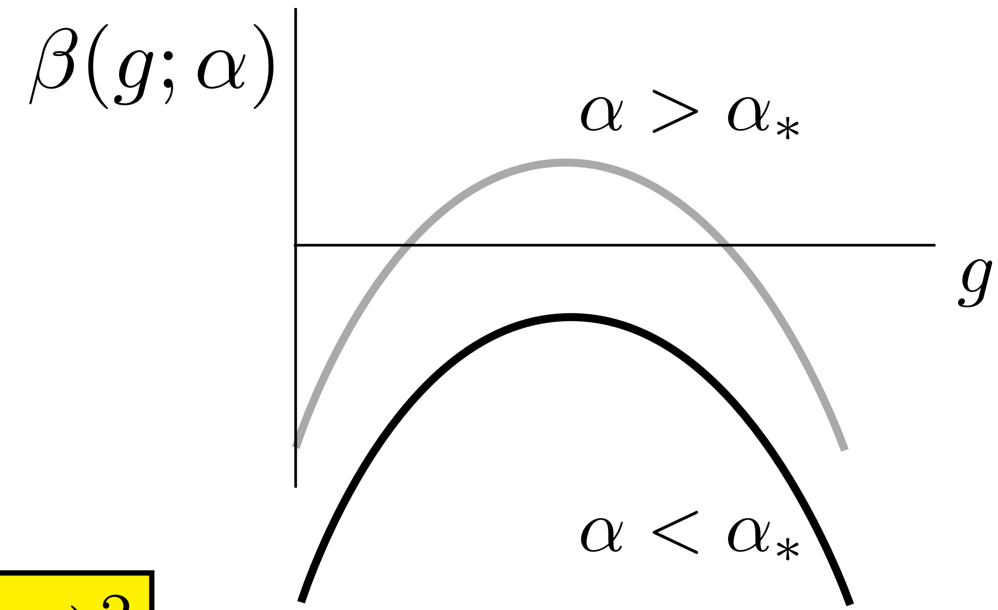

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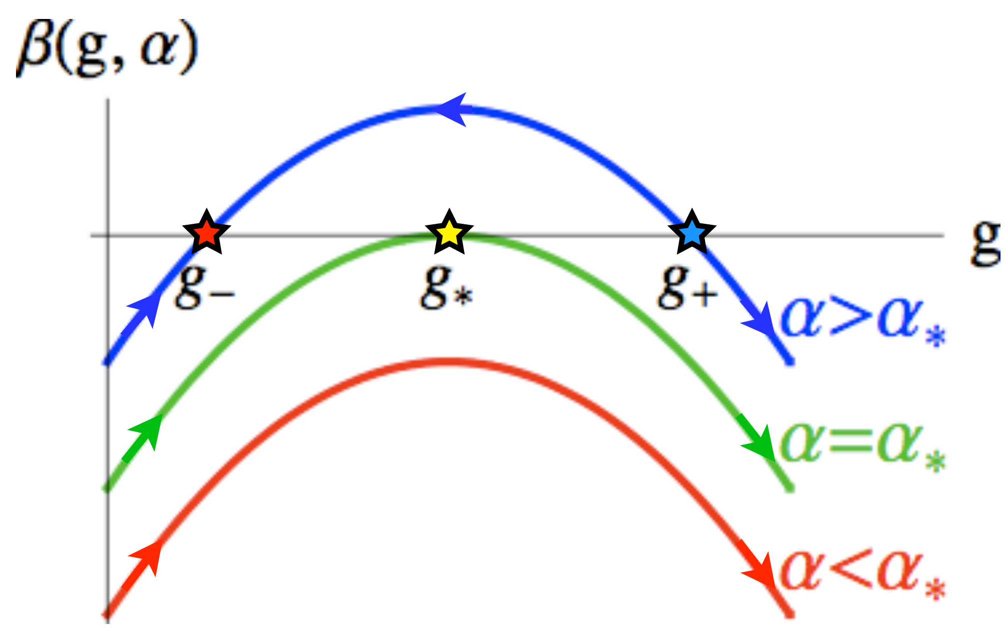

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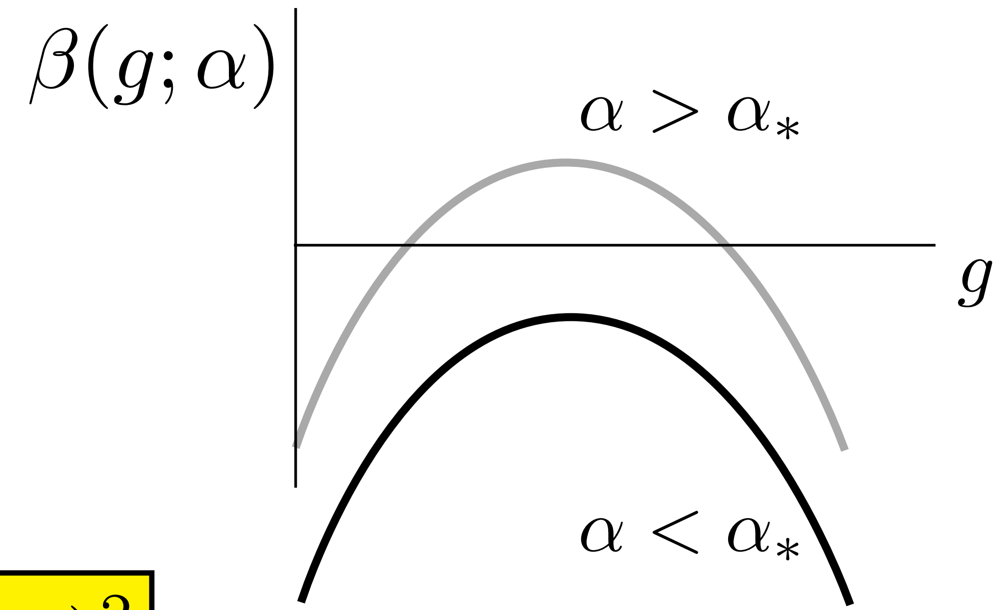

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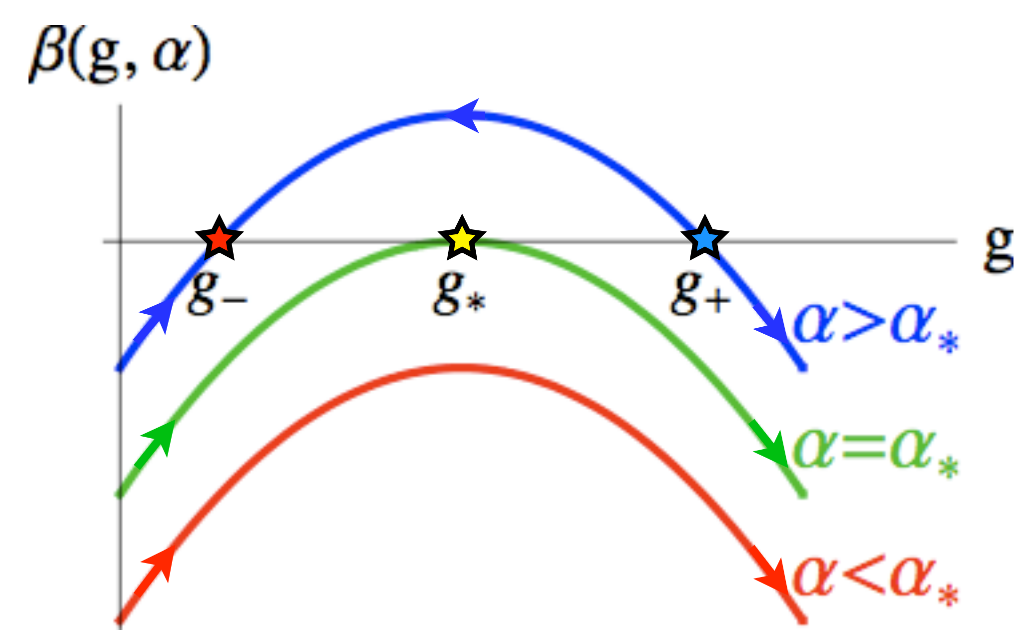

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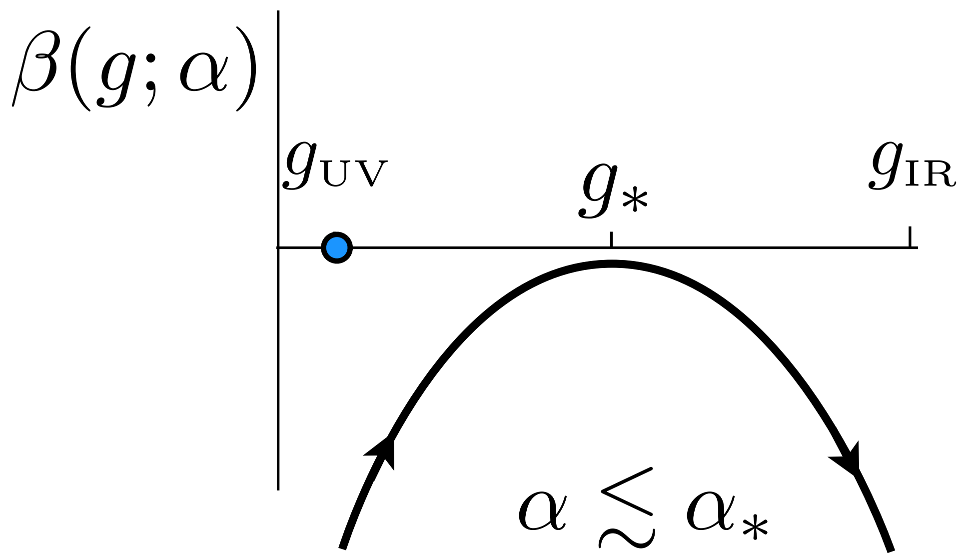

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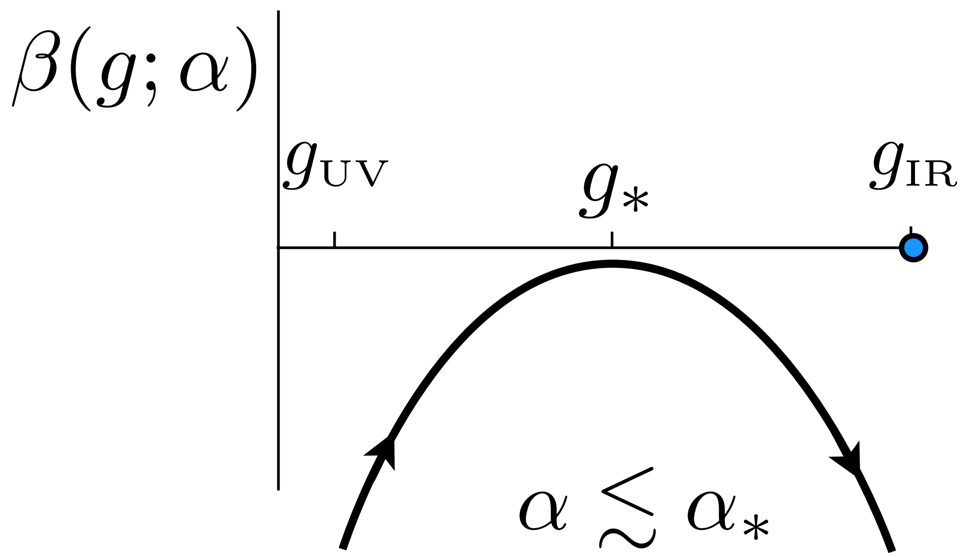

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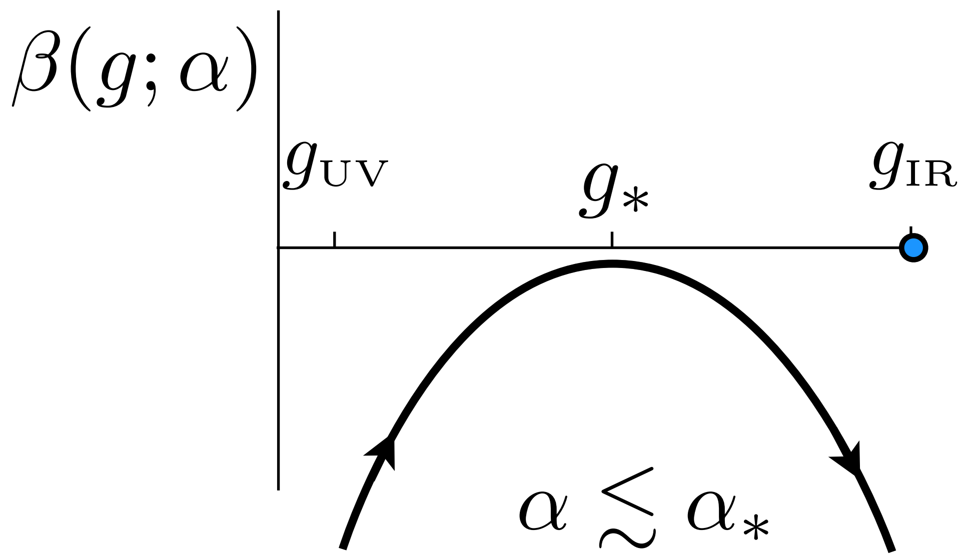

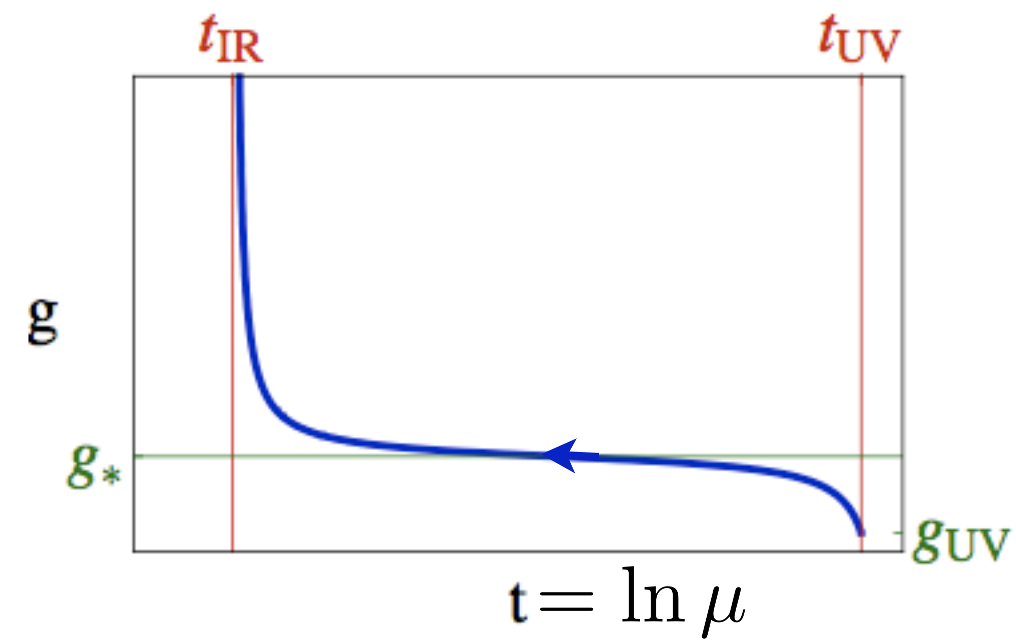

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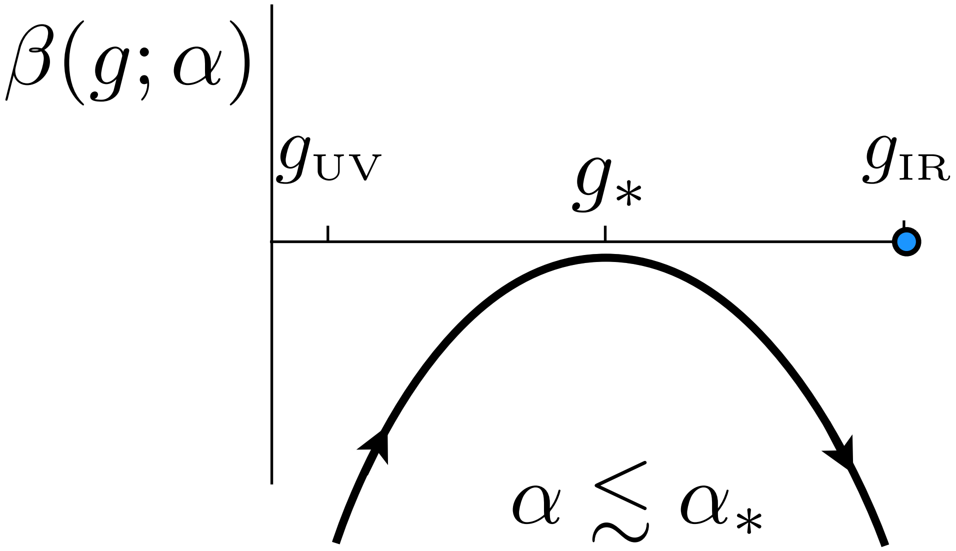

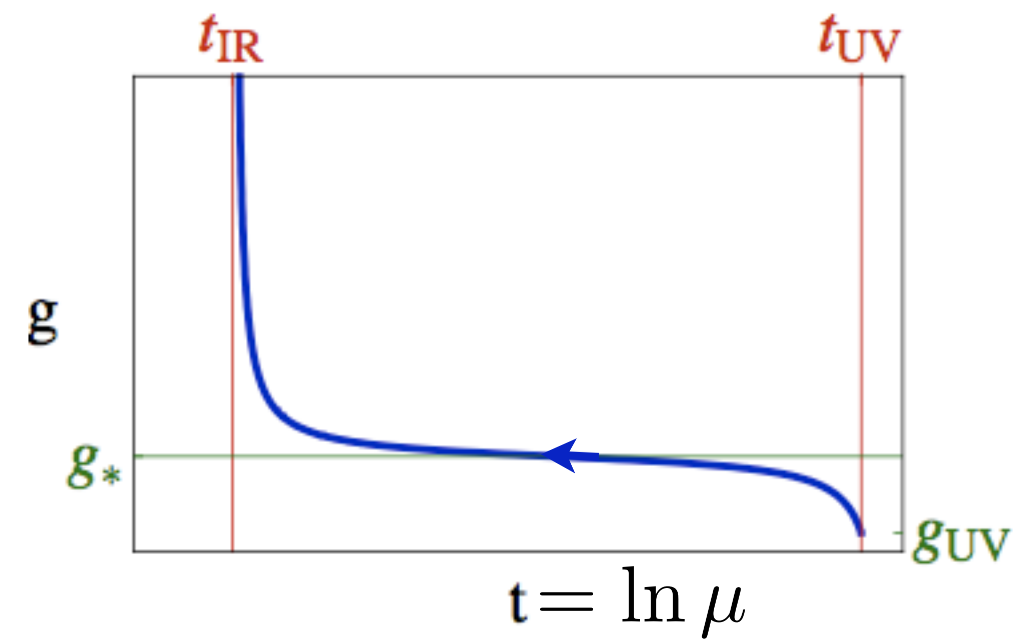

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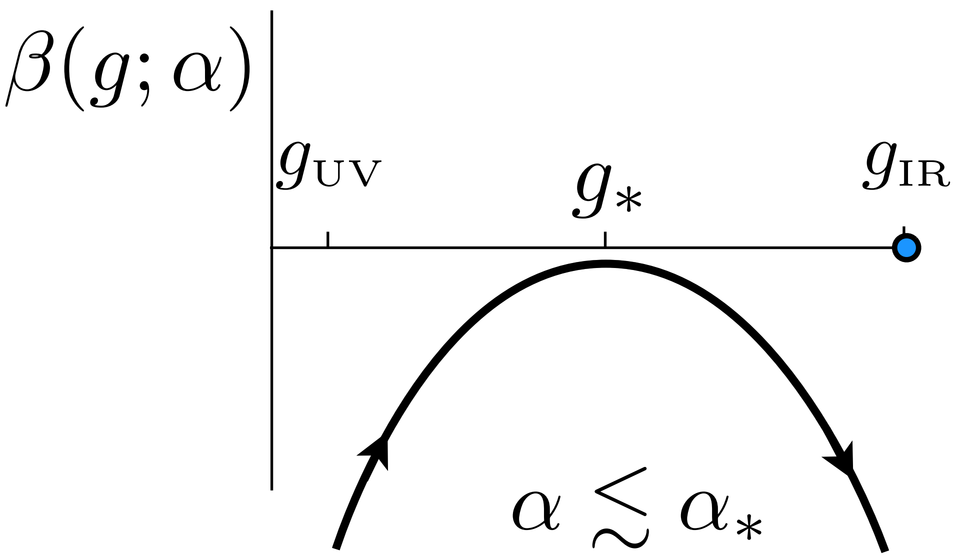

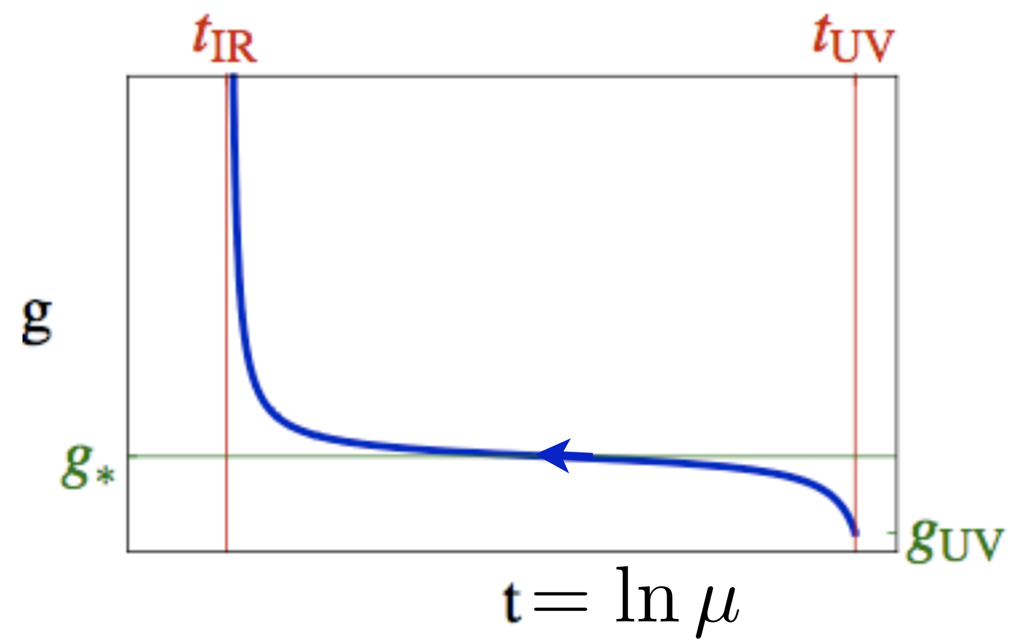

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(Not like 2nd order phase transition: $\Lambda_{\rm IR} \simeq \Lambda_{\rm UV}\sqrt{|\alpha_* - \alpha|}$)

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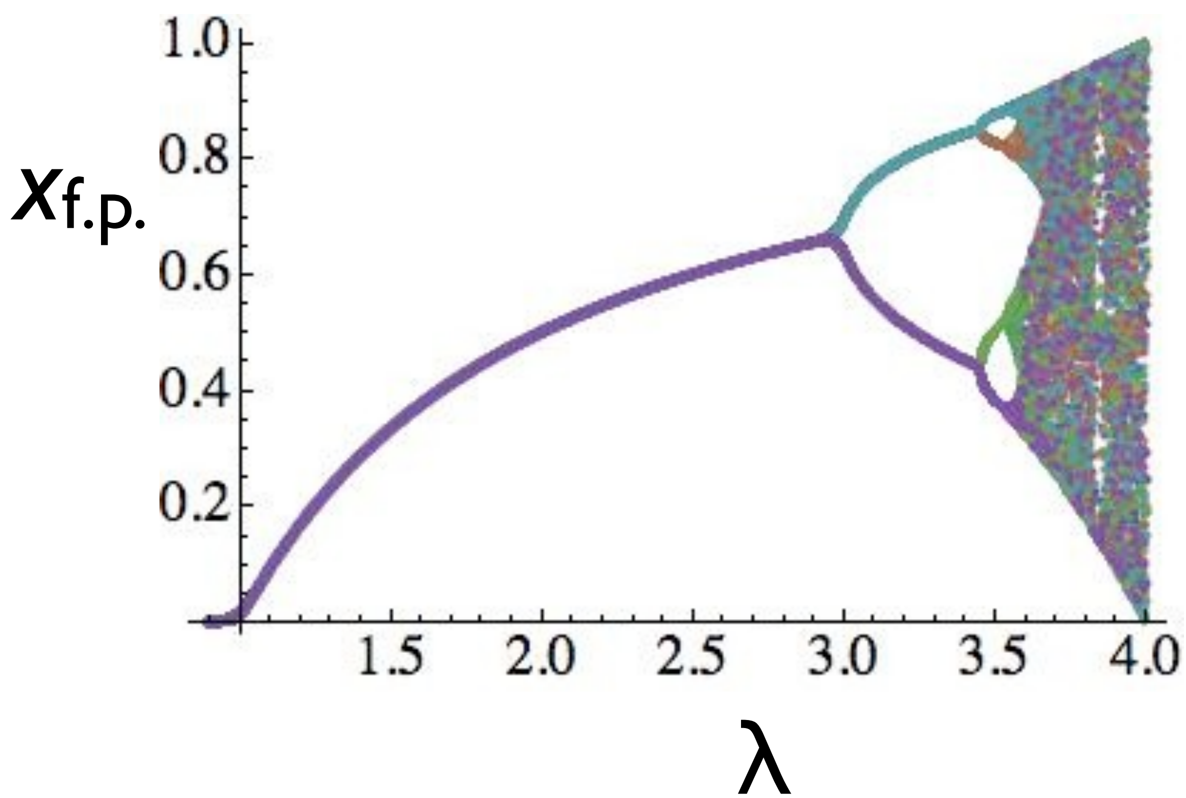

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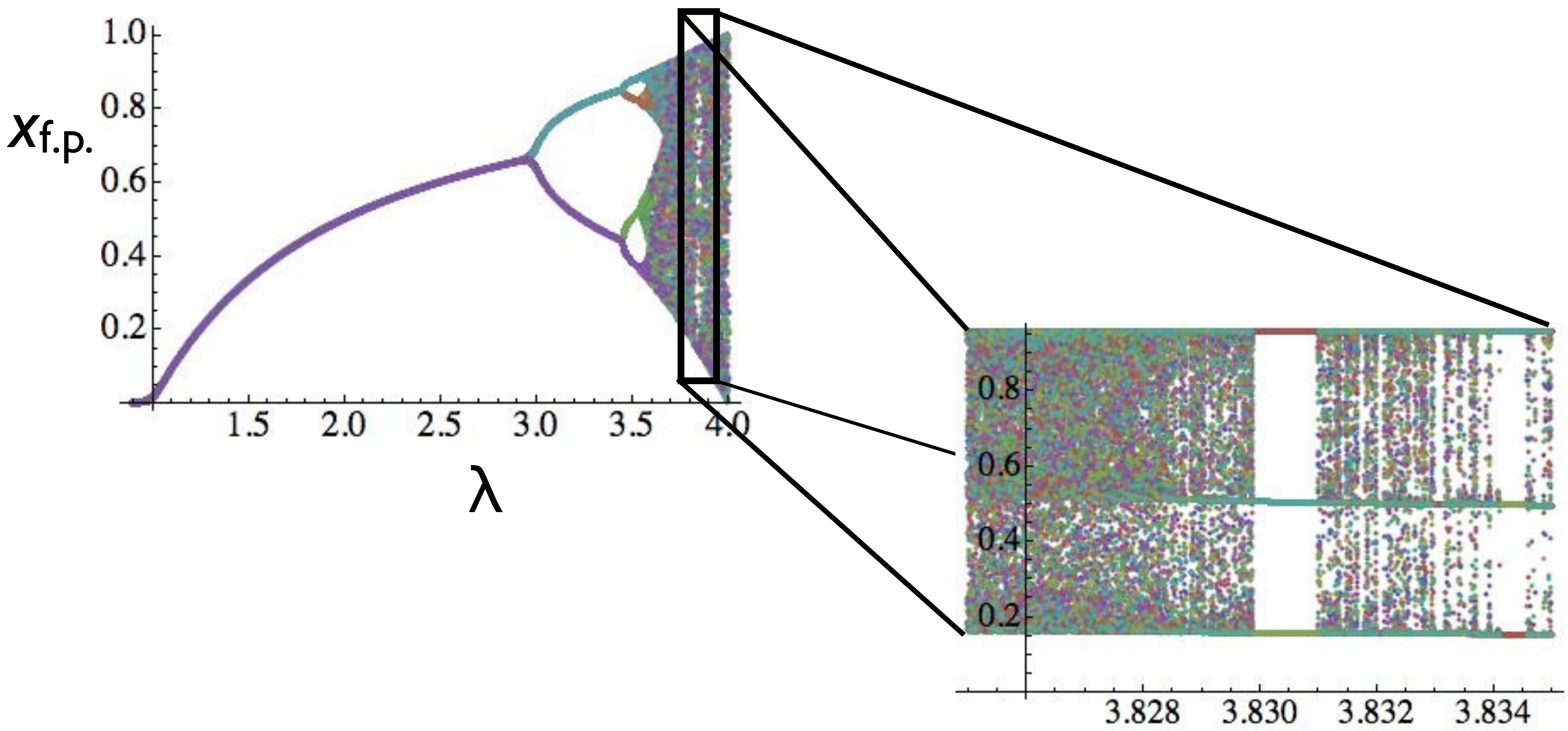

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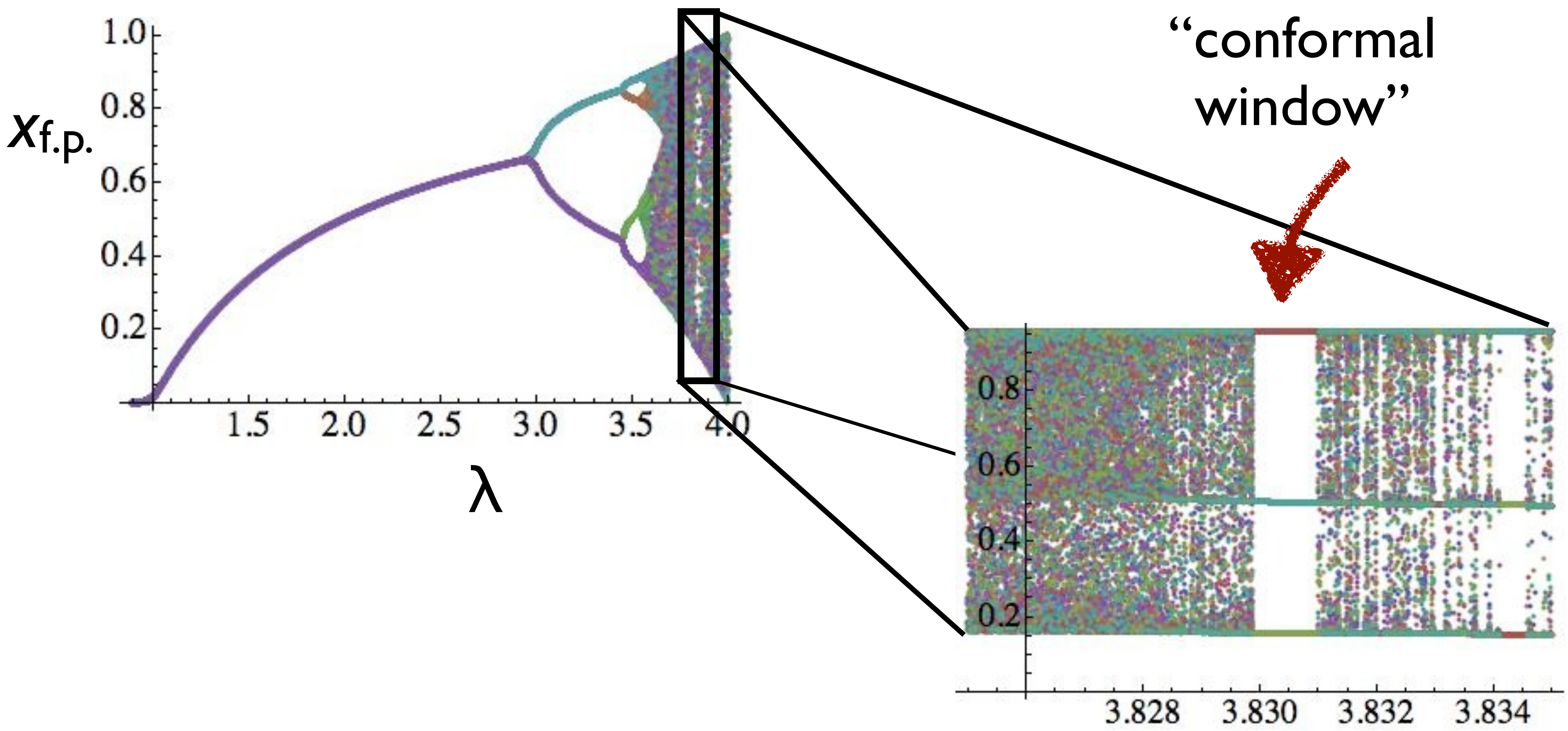

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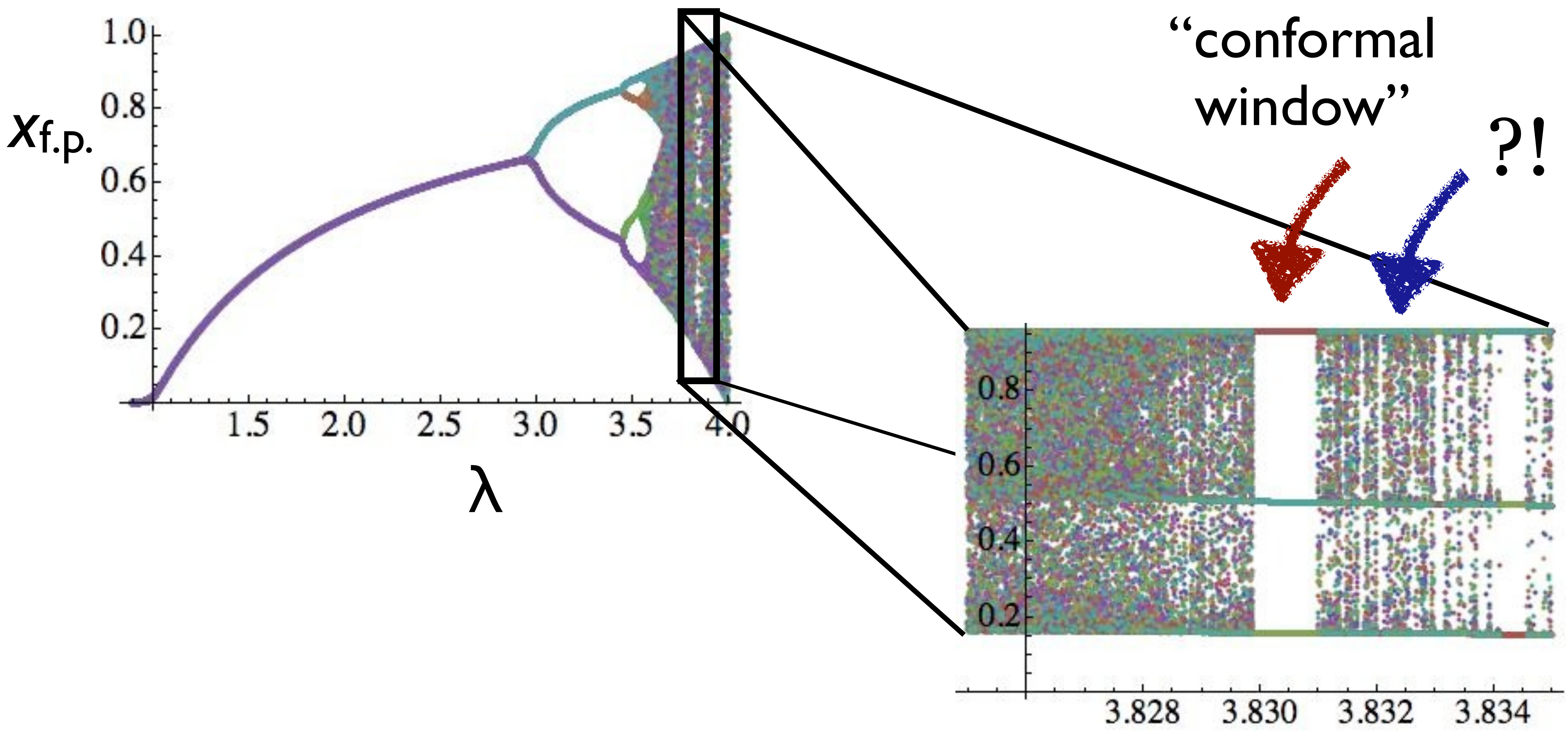

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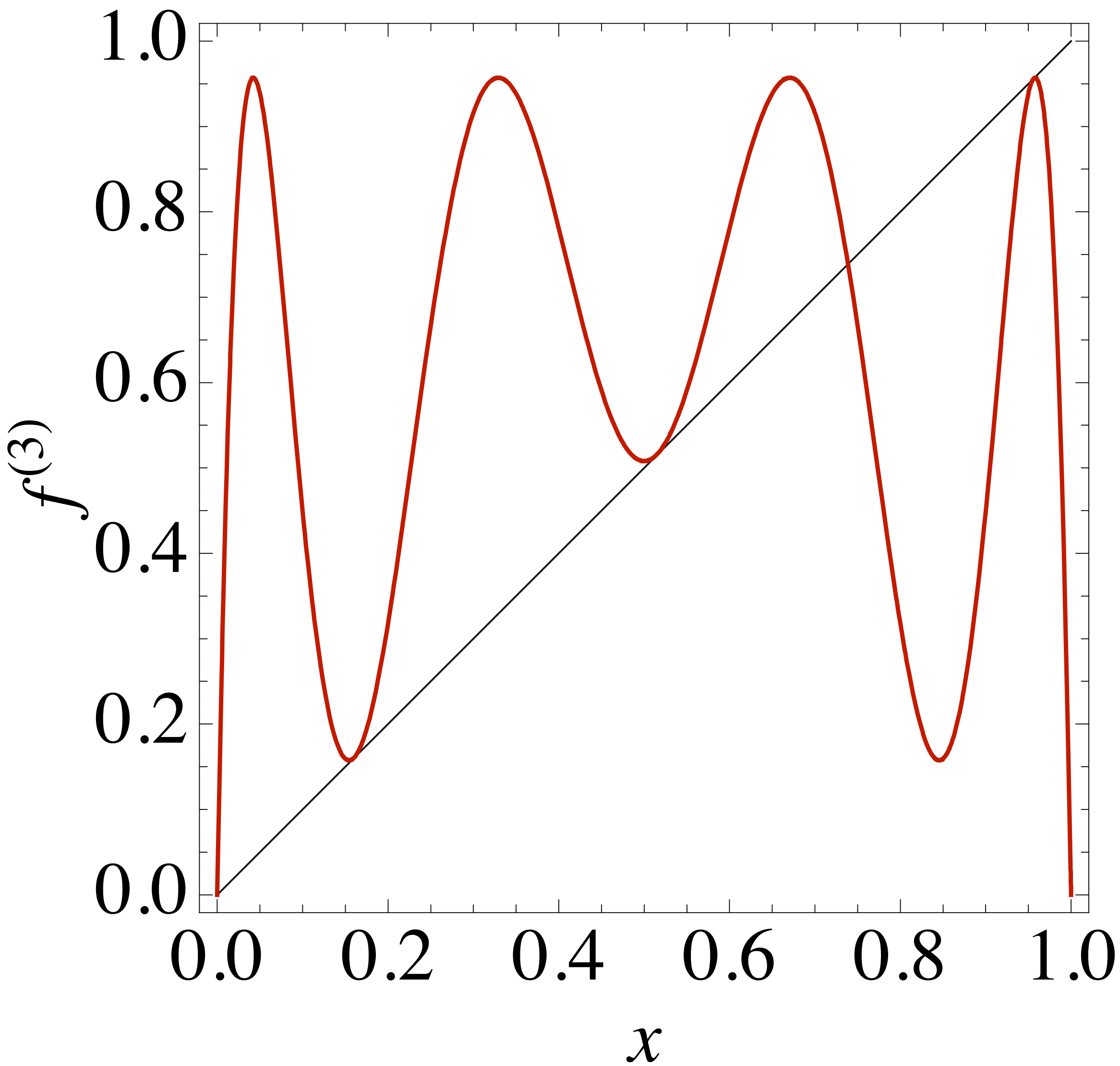

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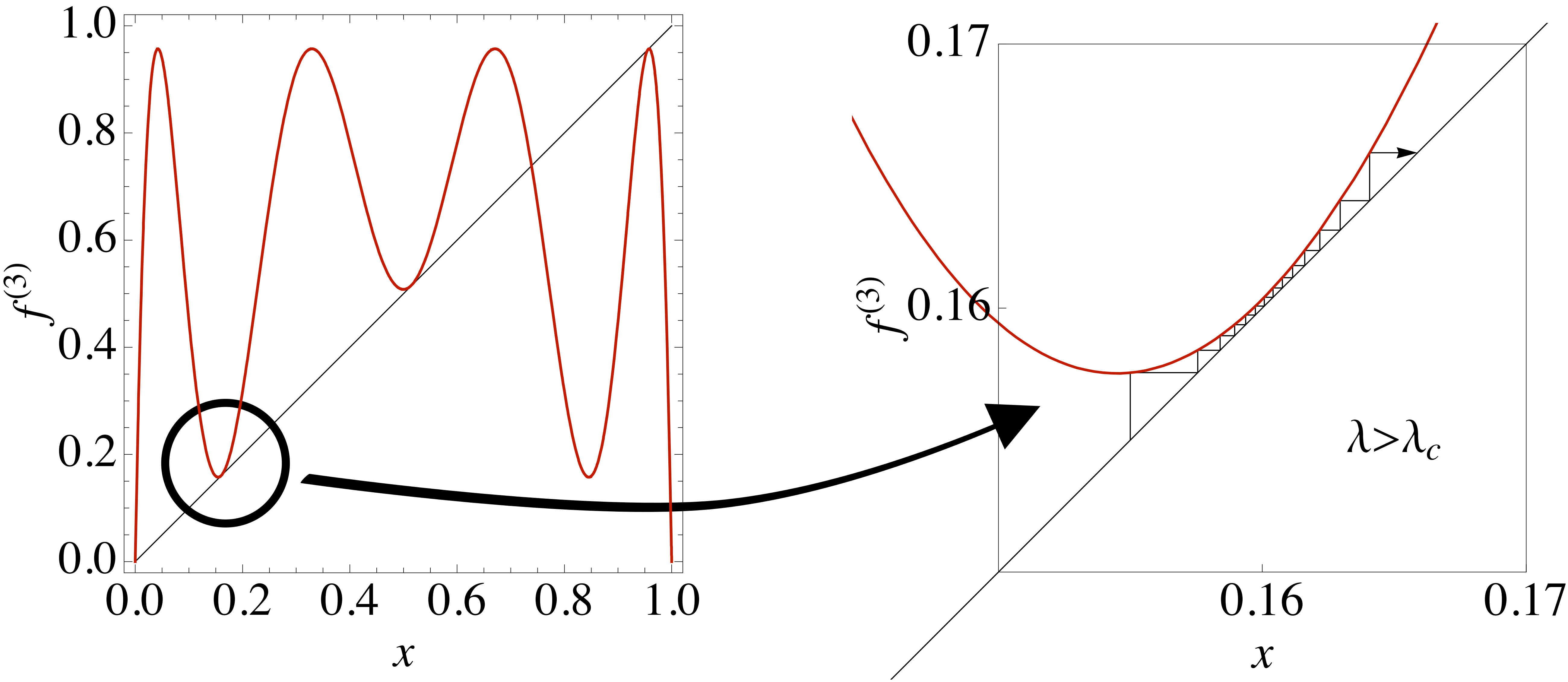

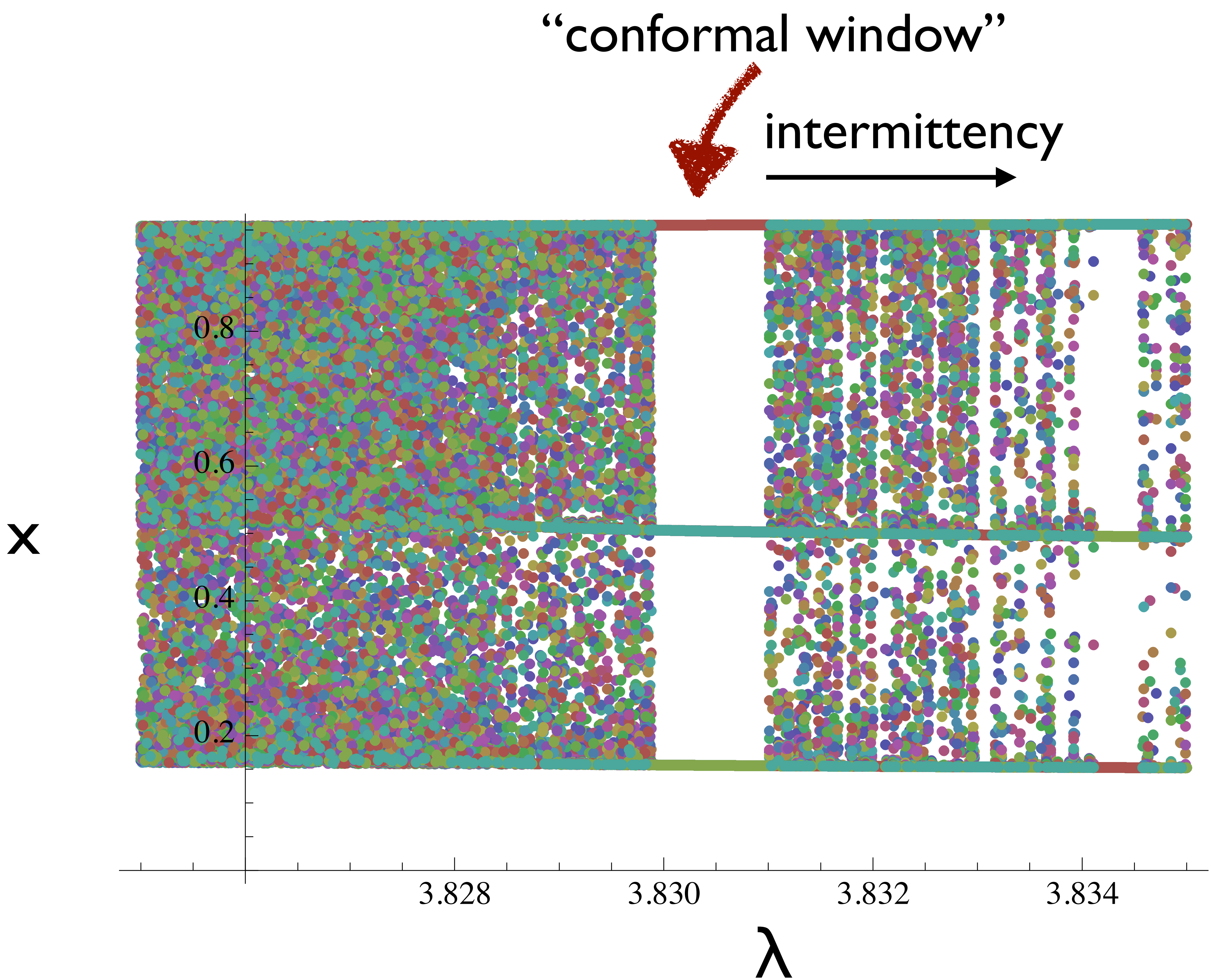
"conformal window"
intermittency
x
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0.6
0.4
0.2
3.828
3.830
3.832
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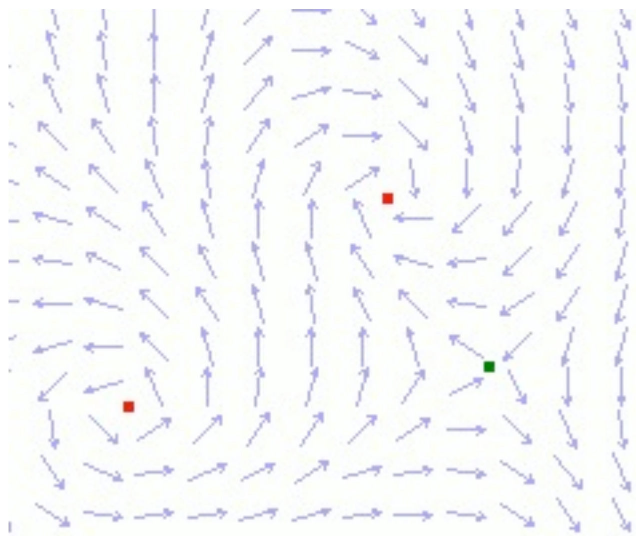
Vortices in XY model

box size R, vortex core size a:



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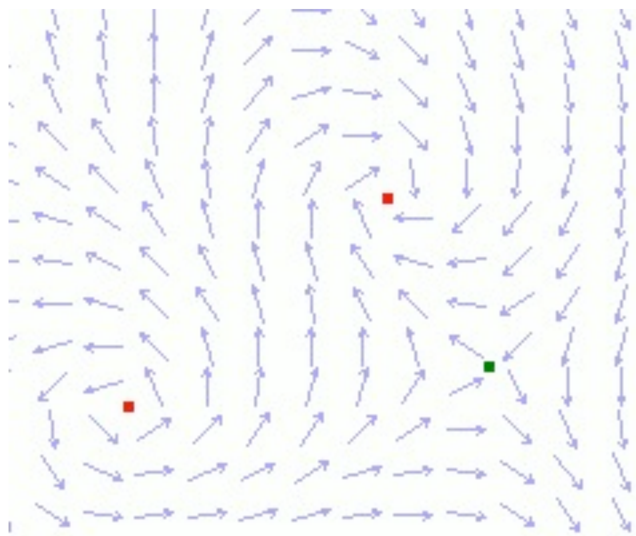

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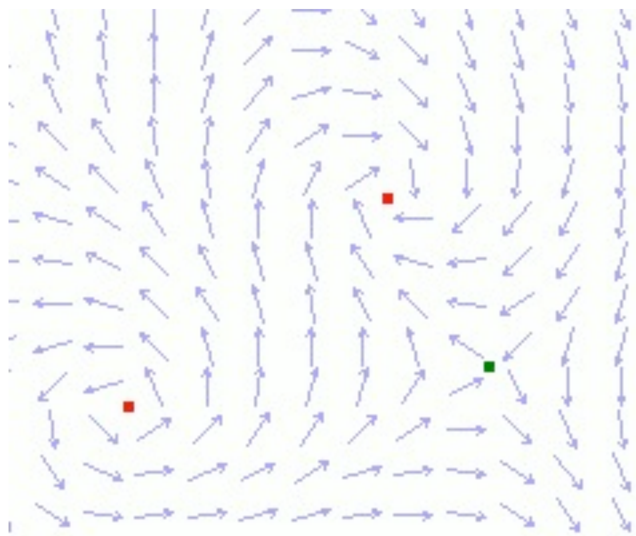

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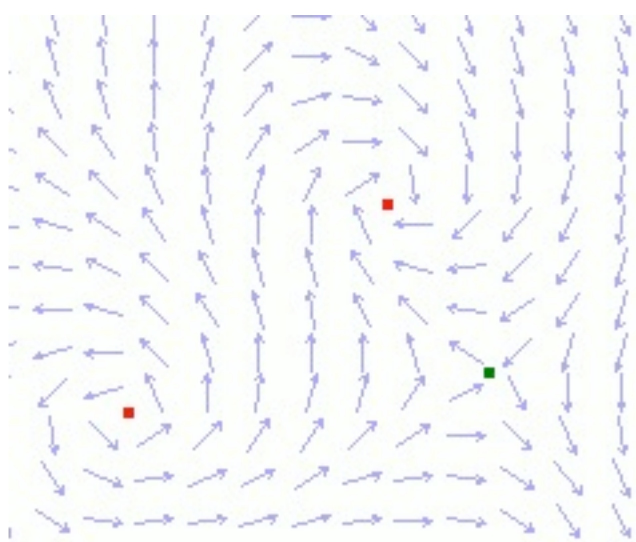

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The XY model is equivalent to the Sine-Gordon model

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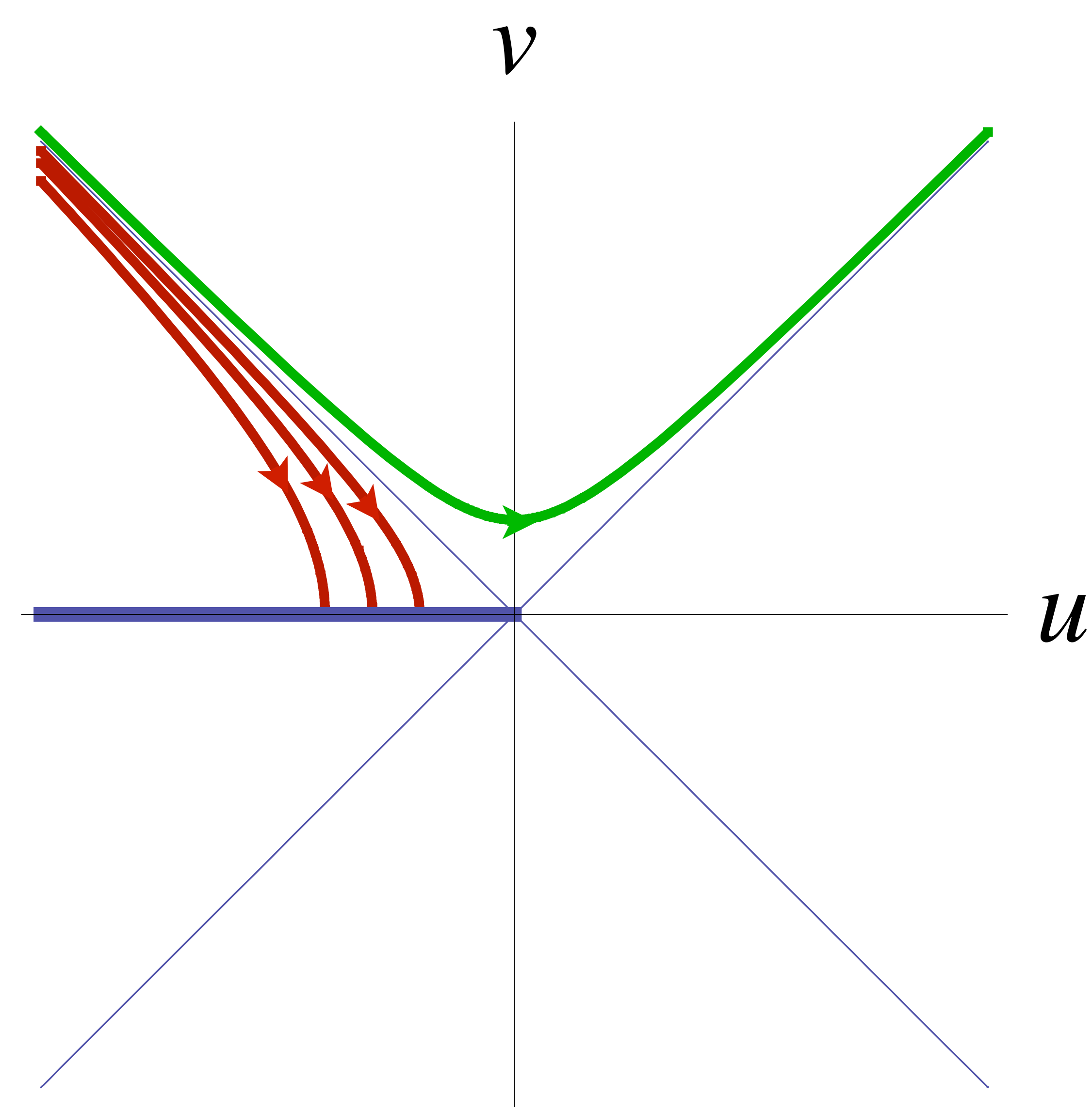

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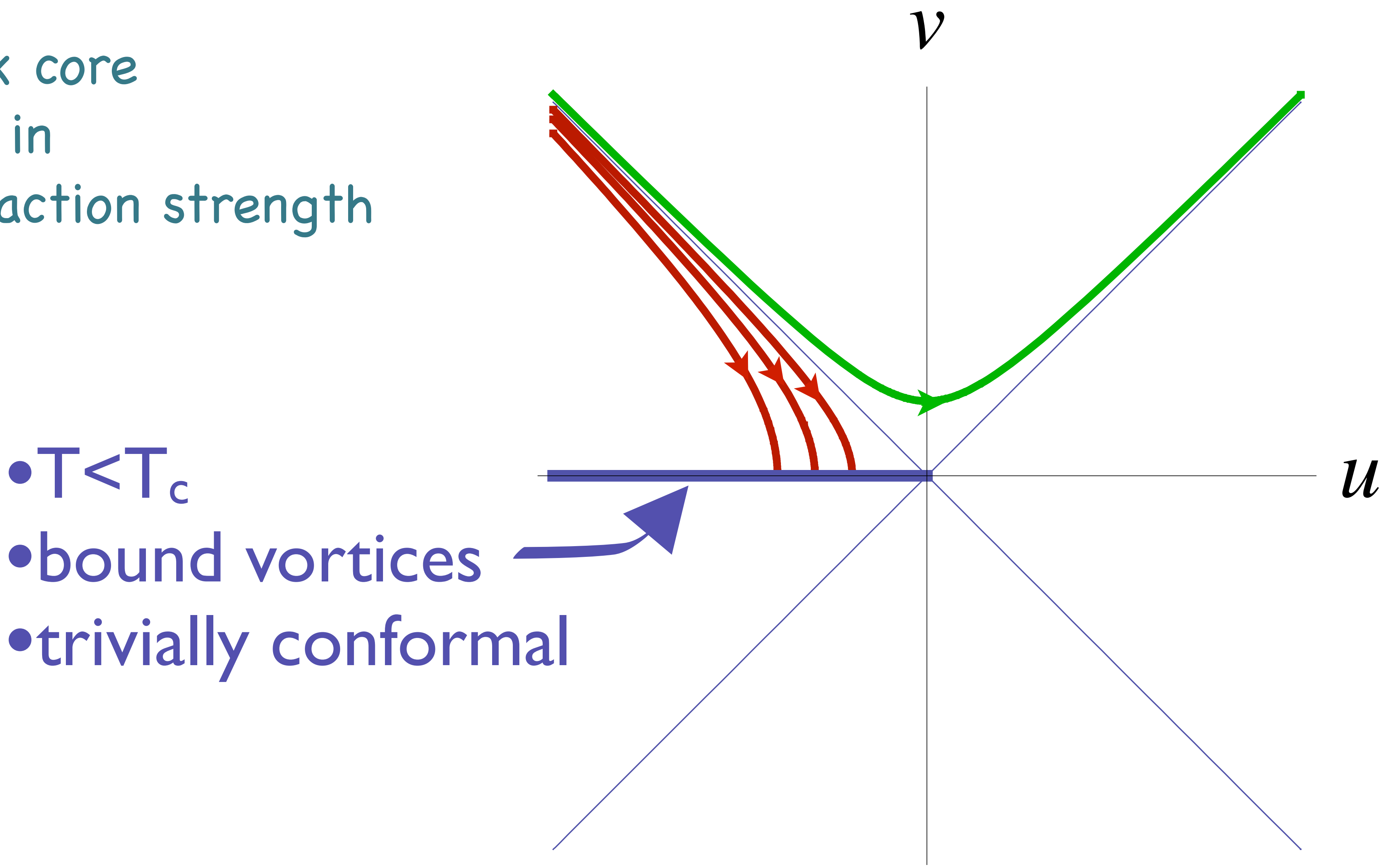

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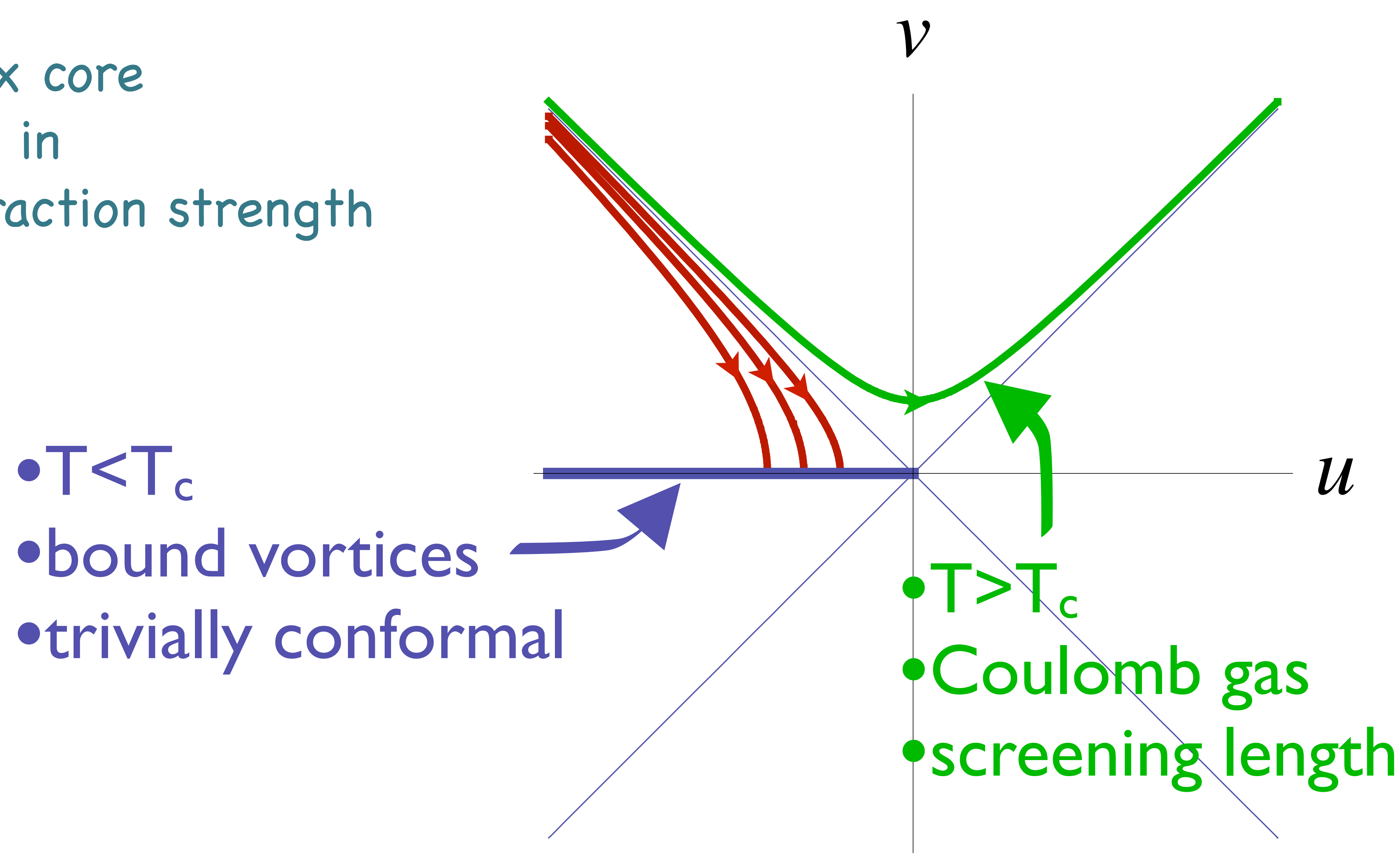

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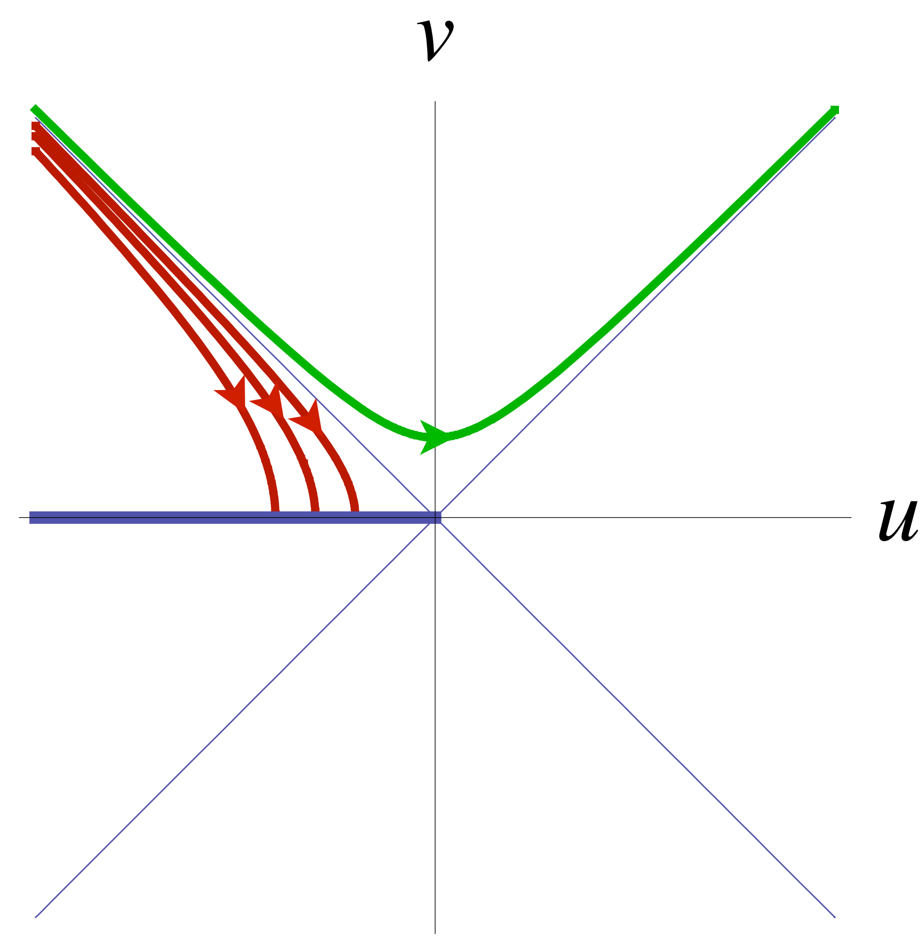

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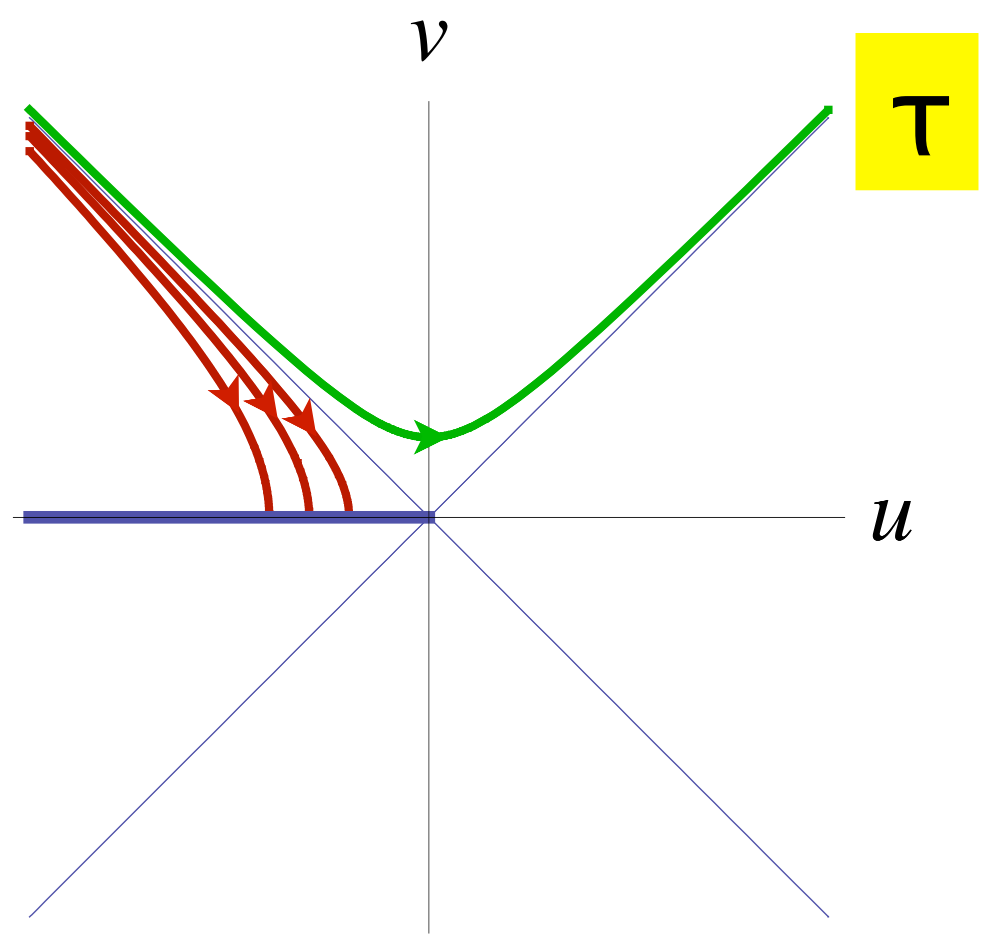

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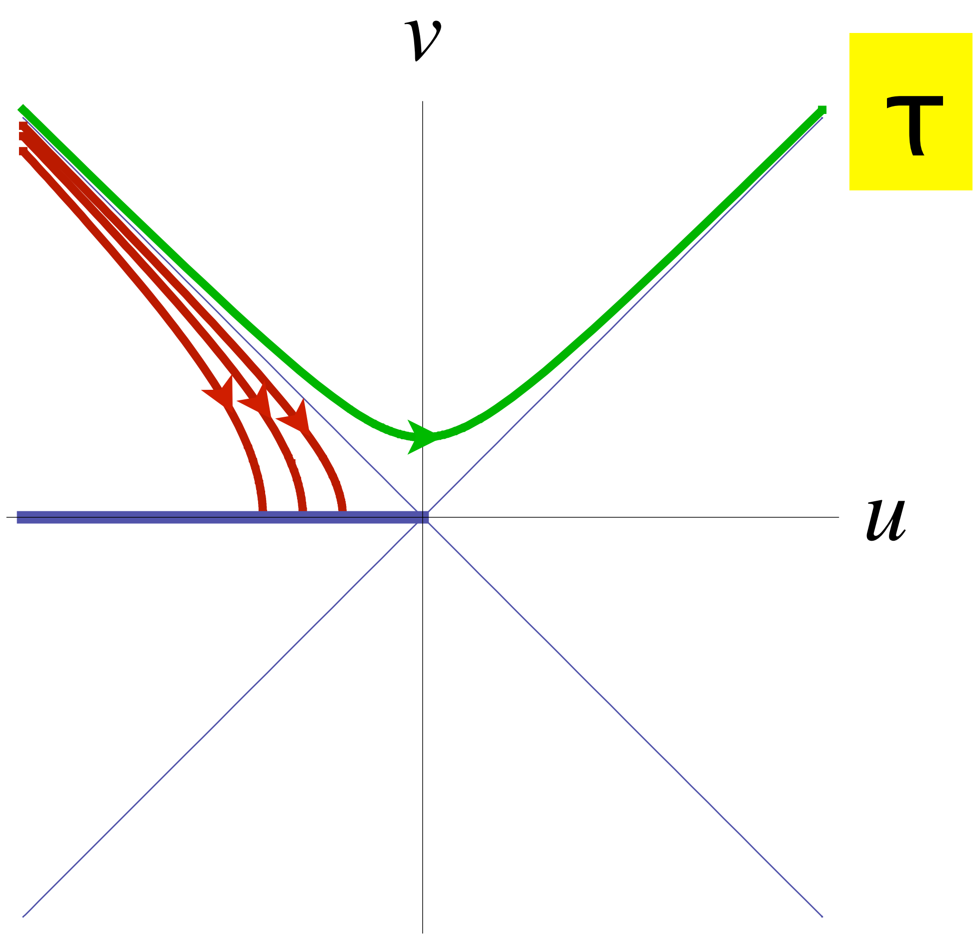

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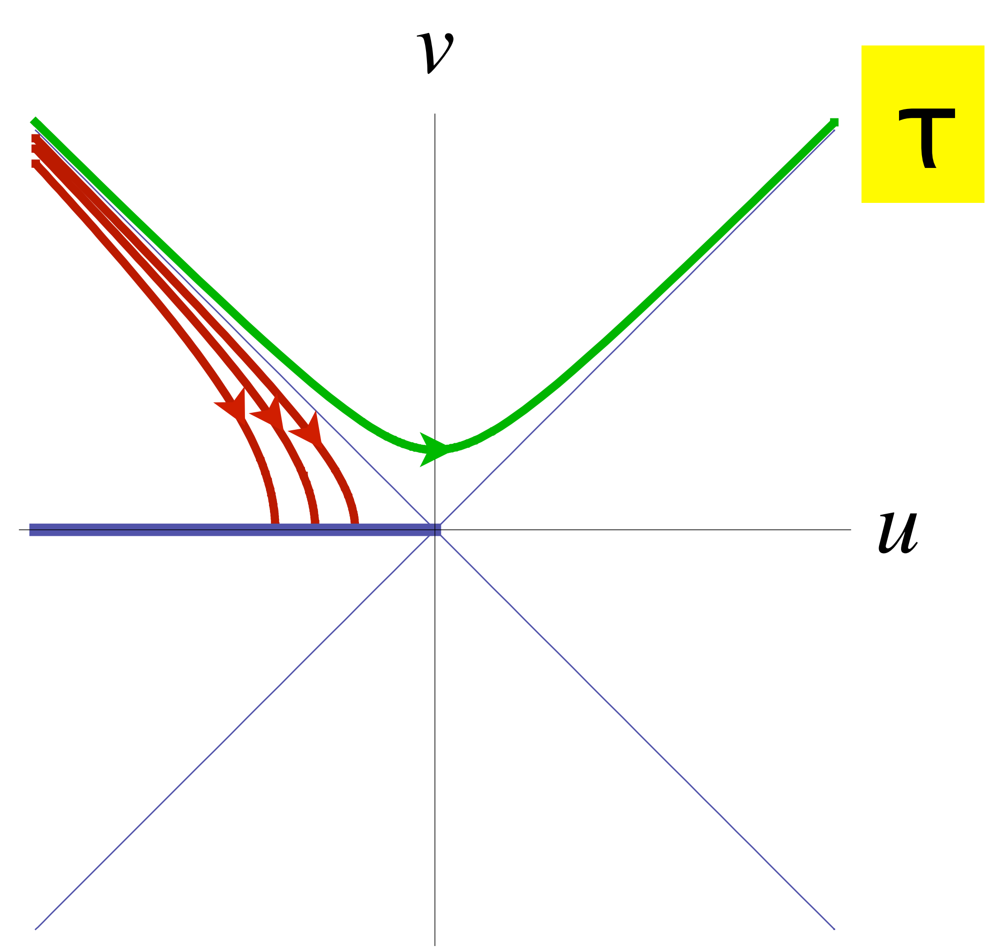

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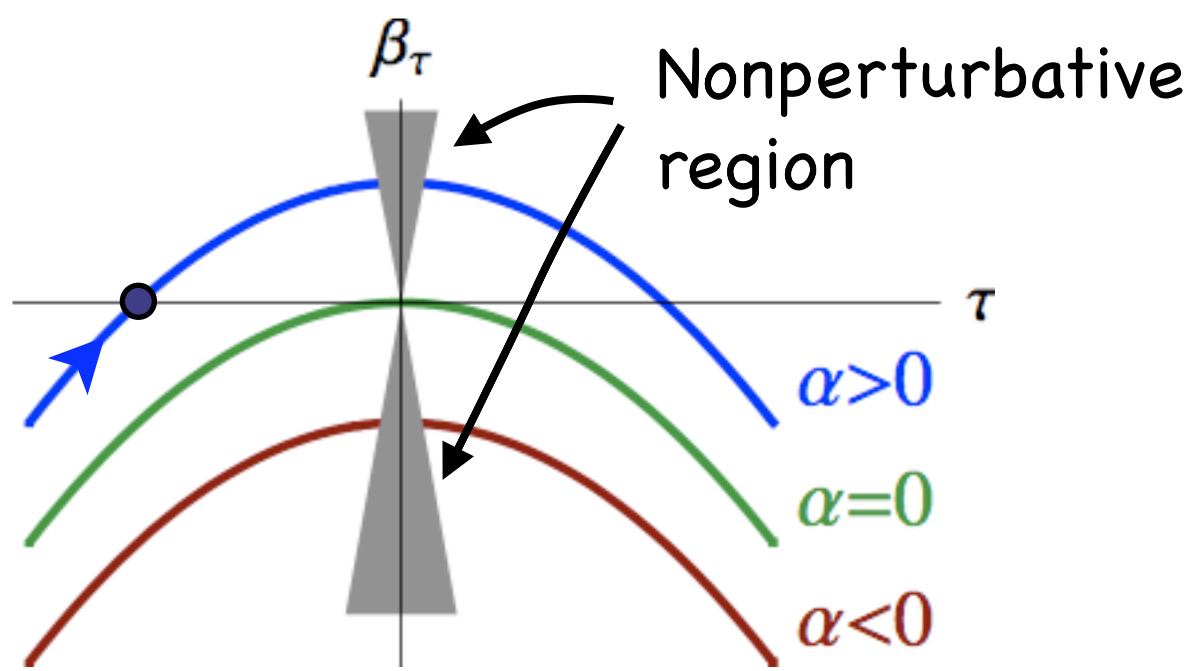
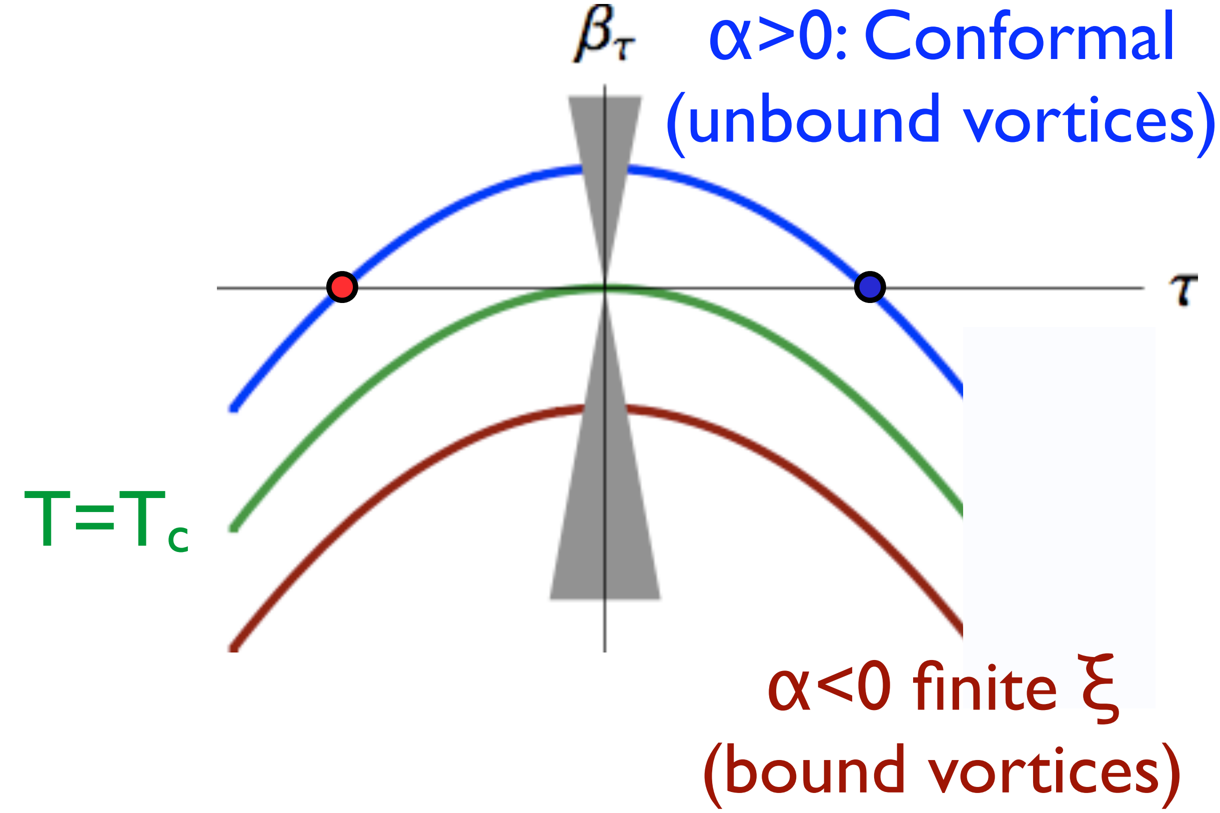


Correlation length in BKT transition:



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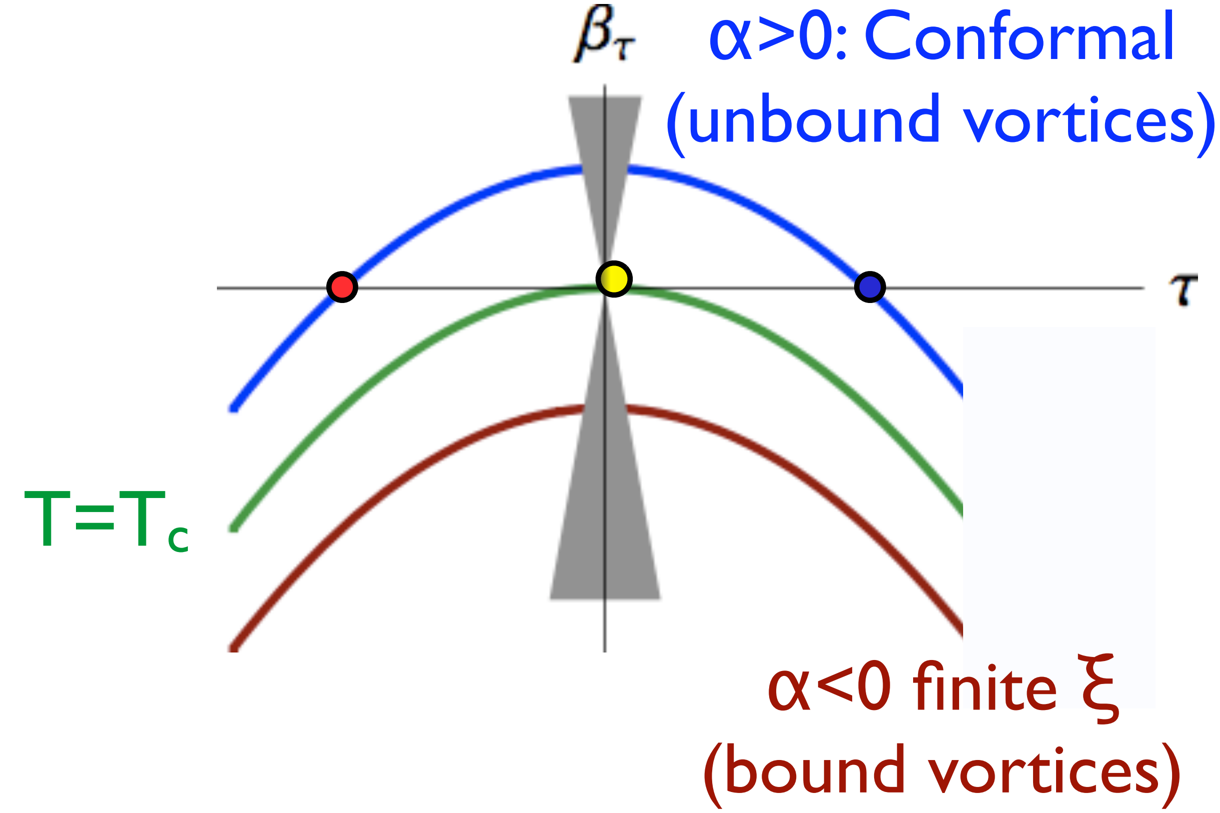

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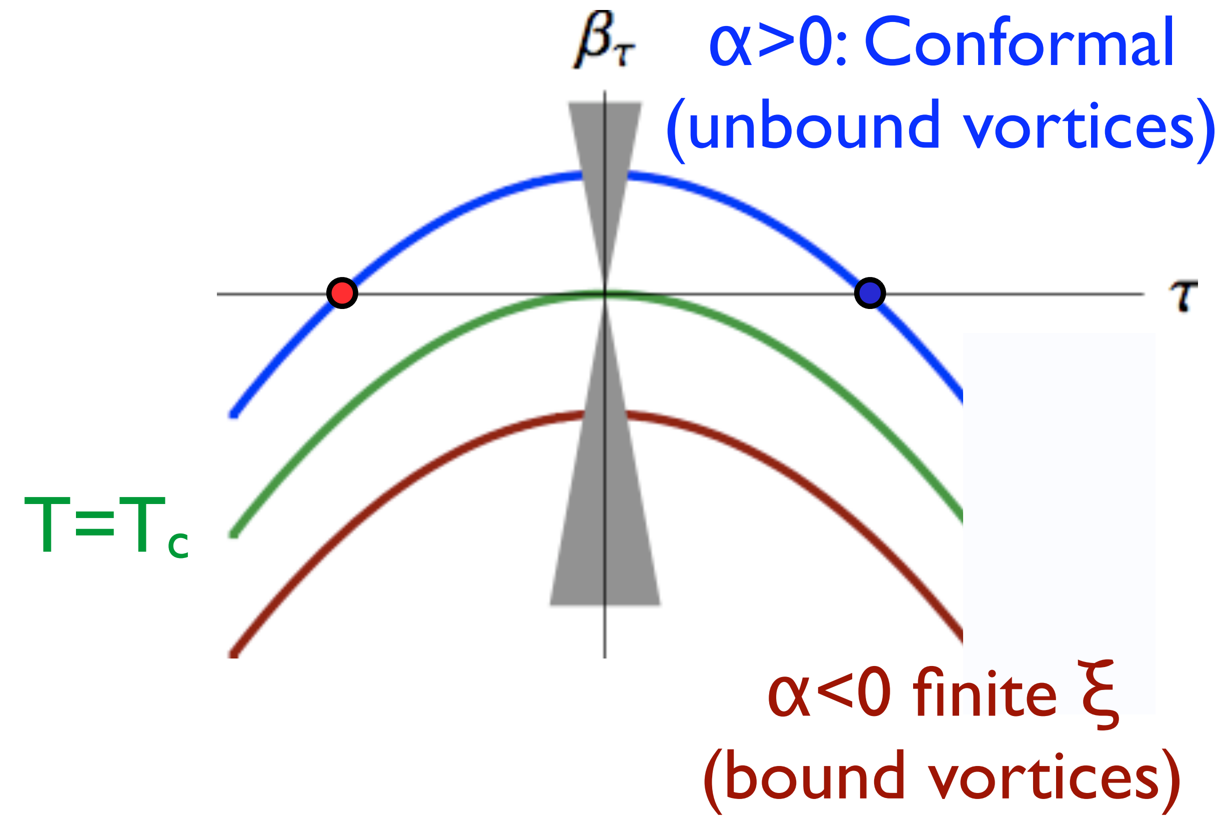

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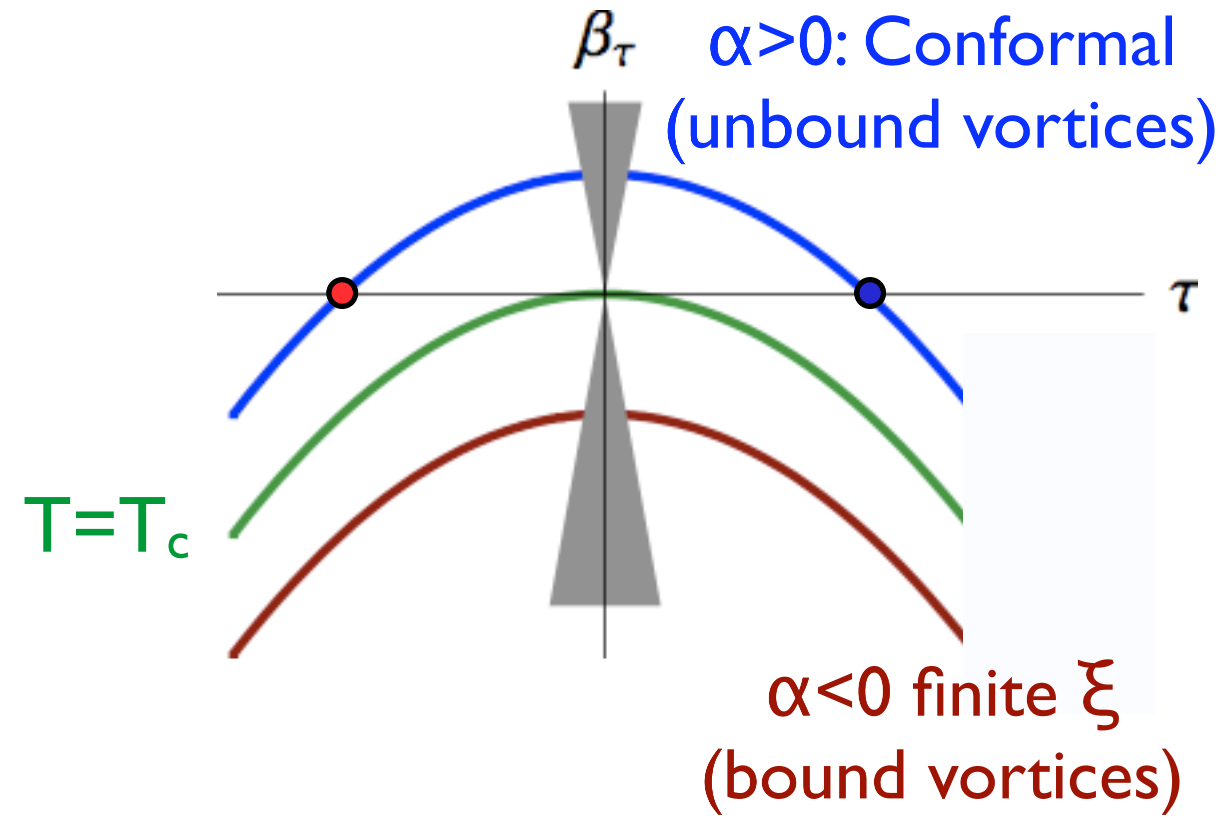

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...giving rise to an IR scale (like Λ_{QCD}) which sets the scale for the finite correlation length for α<0:

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Next: other examples:

- QM with $1/r^2$ potential
- AdS/CFT
- Defect Yang-Mills
- QCD with many flavors

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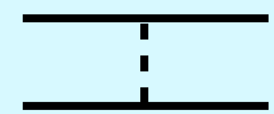
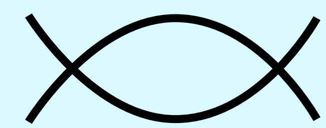
Find g runs:  + 

$$\beta(g;\alpha) = \mu\frac{\partial g}{\partial\mu} = \left(\alpha + \frac{\epsilon^2}{4}\right) - (g-\epsilon)^2$$

Same as toy model! $\alpha_* = -\varepsilon^2/4$, $g_* = \varepsilon$

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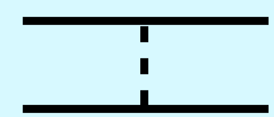
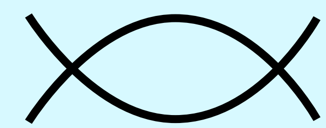
Find g runs:  + 

$$\beta(g;\alpha) = \mu\frac{\partial g}{\partial\mu} = \left(\alpha + \frac{\epsilon^2}{4}\right) - (g-\epsilon)^2$$

Same as toy model! $\alpha_* = -\varepsilon^2/4$, $g_* = \varepsilon$

$\alpha > \alpha_*$: conformal

$\alpha = \alpha_*$: critical

$\alpha < \alpha_*$: g blows up in IR

RG treatment of $1/r^2$ potential: *I. Perturbative*

$\alpha_* \equiv -(d-2)^2/4$ so work in $d=2+\varepsilon$

δ-function

$$S = \int dt\, d^d\mathbf{x} \left(i\psi^\dagger \partial_t \psi - \frac{|\nabla\psi|^2}{2m} + \frac{g\pi}{2}\psi^\dagger\psi^\dagger\psi\psi \right)$$
$$- \int dt\, d^d\mathbf{x}\, d^d\mathbf{y}\, \psi^\dagger(t,\mathbf{x})\psi^\dagger(t,\mathbf{y}) \frac{\alpha}{|\mathbf{x}-\mathbf{y}|^2}\psi(t,\mathbf{y})\psi(t,\mathbf{x})$$

propagator: $\dfrac{i}{\omega - \mathbf{p}^2/2m}$

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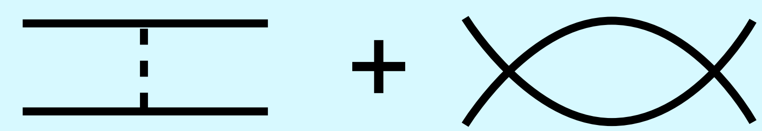

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$$B \sim \left(\frac{\Lambda_{\rm IR}^2}{m}\right) \sim \left(\frac{\Lambda_{\rm UV}^2}{m}\right) e^{-2\pi/\sqrt{\alpha_* - \alpha}}$$

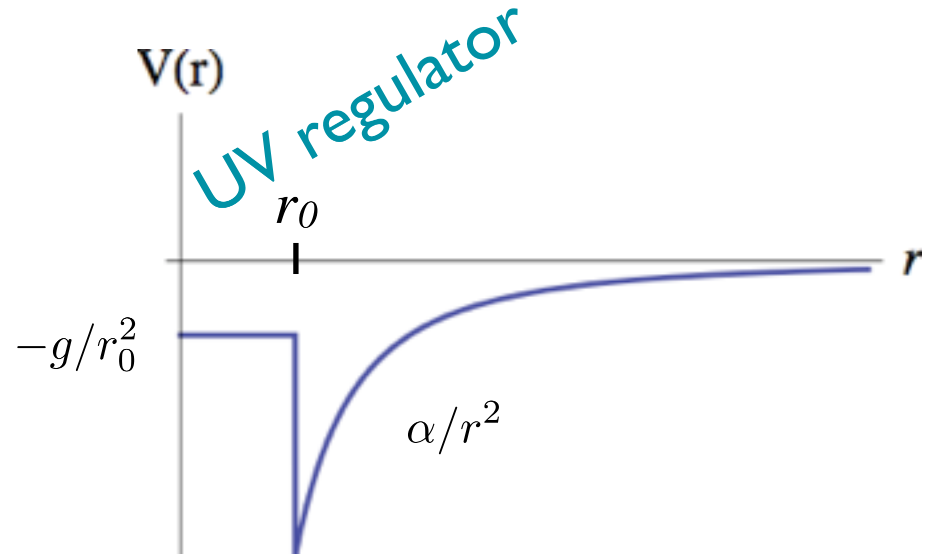
BKT scaling

bound state energy

RG treatment of $1/r^2$ potential: *II. Non-perturbative*

regulate with square well:

$$V(r) = \begin{cases} \alpha/r^2 & r > r_0 \\ -g/r_0^2 & r < r_0 \end{cases}$$



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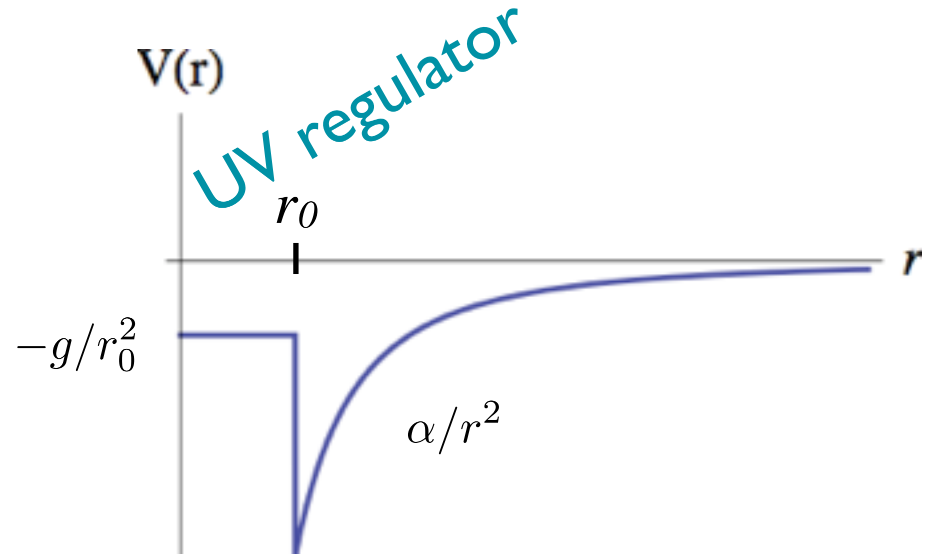

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Solve for c_+/c_- (a physical dimensionful quantity) and require invariance: $d(c_+/c_-)/dr_0 = 0$:

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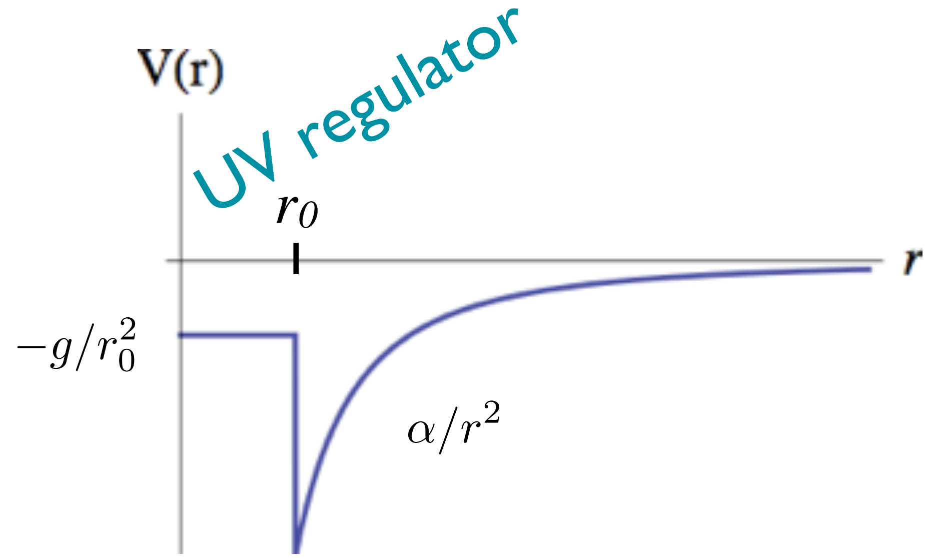

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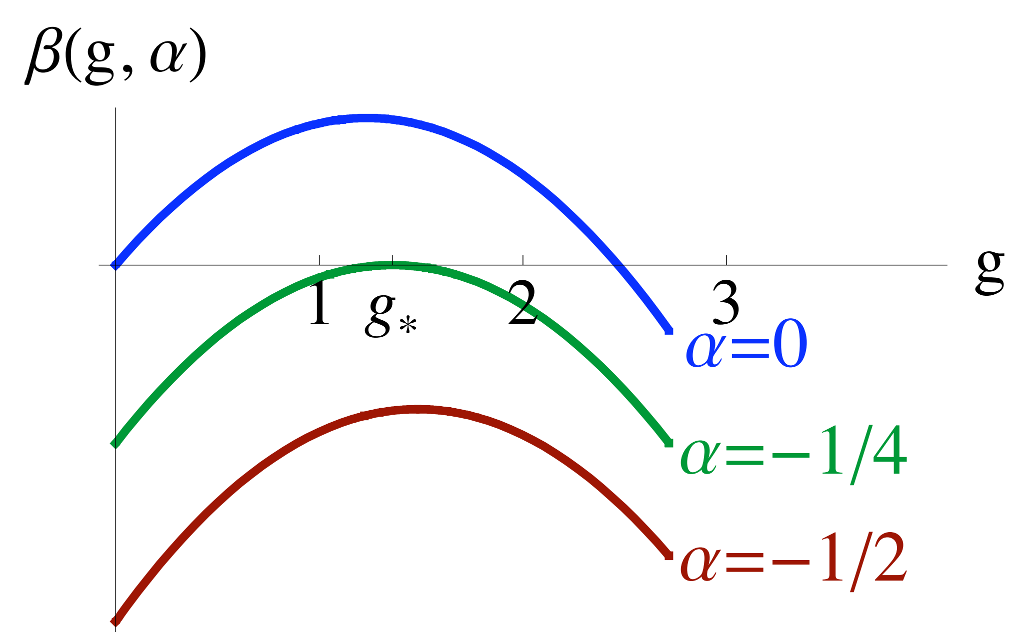
Find <u>exact</u> β-function for g. Eg, for d=3:

$$\beta = \frac{2\sqrt{g}\left(\alpha + \sqrt{g}\cot\sqrt{g} - g\cot^2\sqrt{g}\right)}{-\cot\sqrt{g} + \sqrt{g}\csc^2\sqrt{g}}$$

$\alpha_* = -1/4$, $g_* \approx 1.36$



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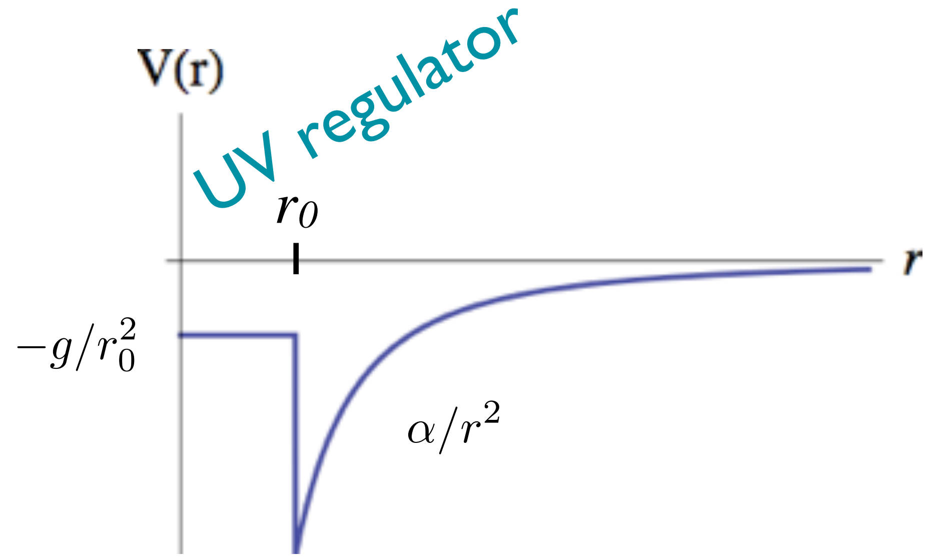

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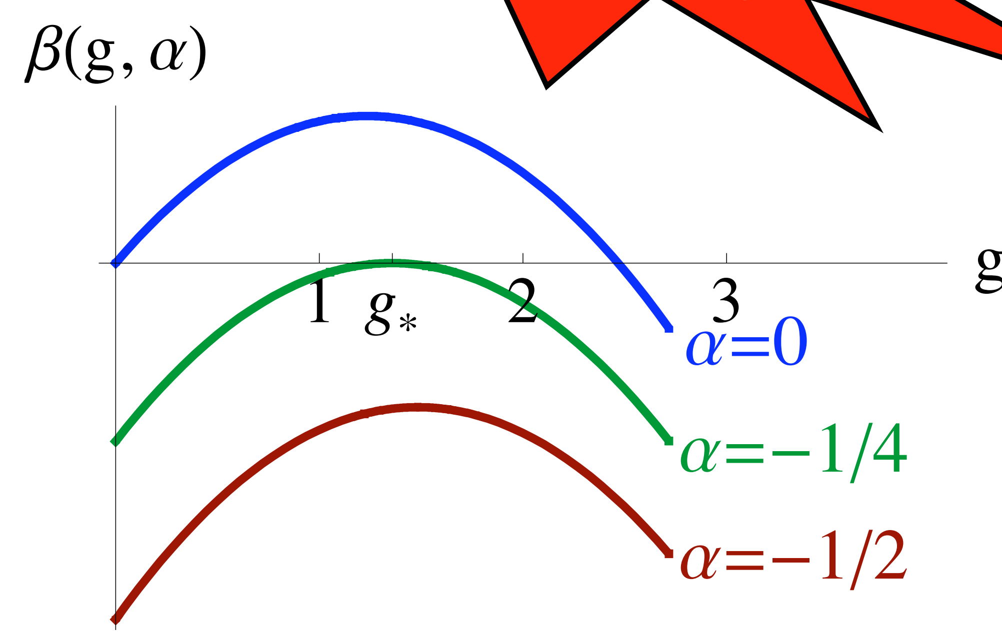


Even better: define

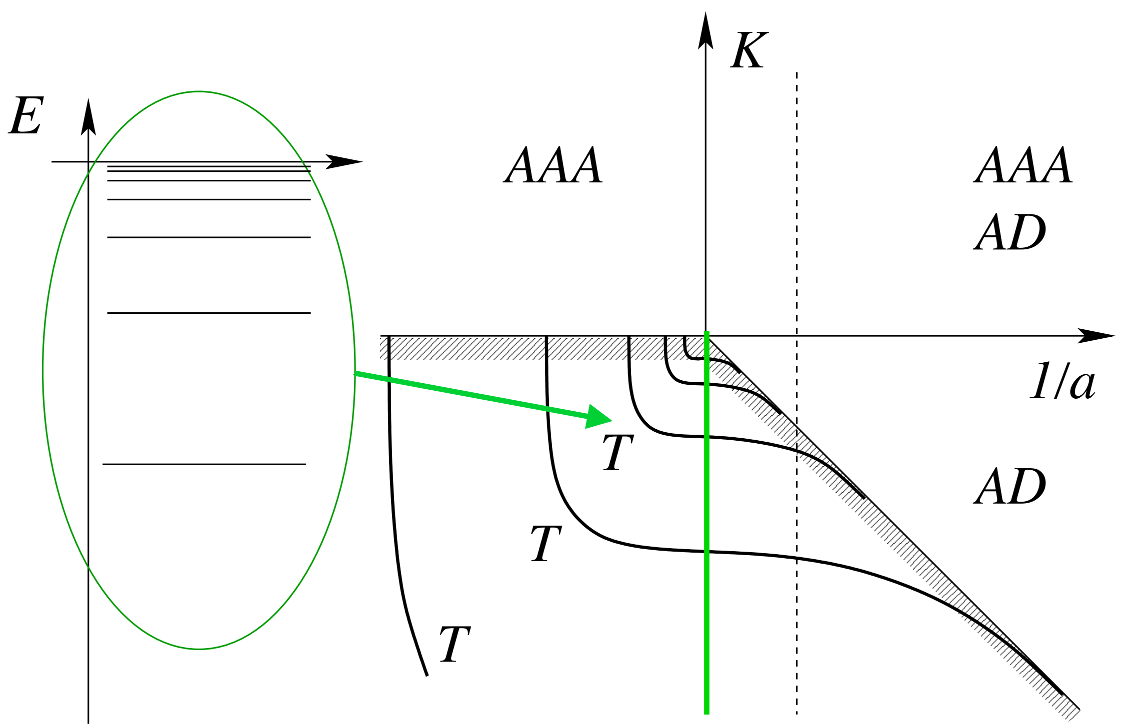
$$\gamma = \left(\frac{\sqrt{g}\, J_{d/2}(\sqrt{g})}{J_{d/2-1}(\sqrt{g})} \right)$$

Condition $d(c_+/c_-)/dr_0$ yields exact β-function in d-dimensions:

$$\beta_\gamma = \frac{\partial\gamma}{\partial t} = (\alpha - \alpha_*) - (\gamma - \gamma_*)^2 \ , \qquad \gamma_* = \frac{d-2}{2}$$

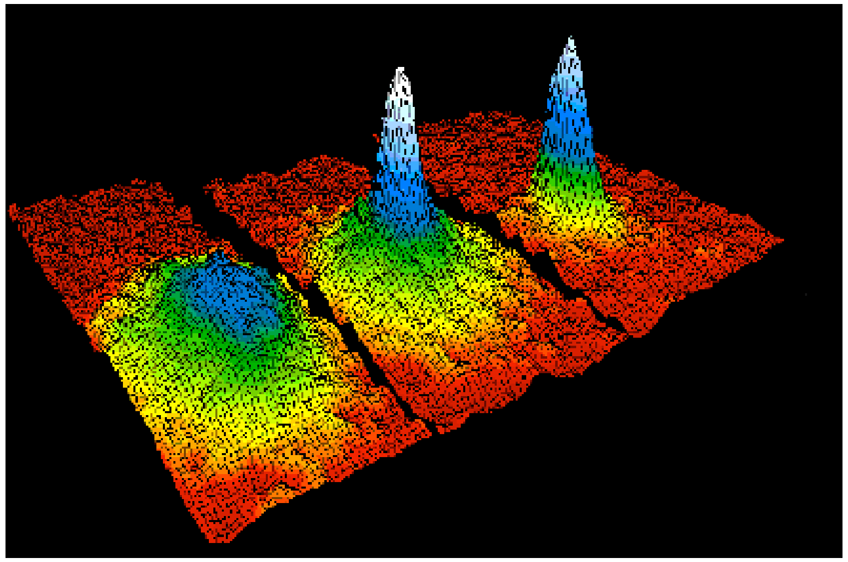
- Toy model is exact!
- γ is a periodic function of g, γ=±∞ equivalent
- Limit cycle behavior for $\alpha<\alpha_*$: explains "Efimov states" for trapped atoms at Feschbach resonance

(V. Efimov, Phys. Lett. **33B** (1970) 563)



a = atom-atom scattering length

A=atom
D=dimer
T=trimer



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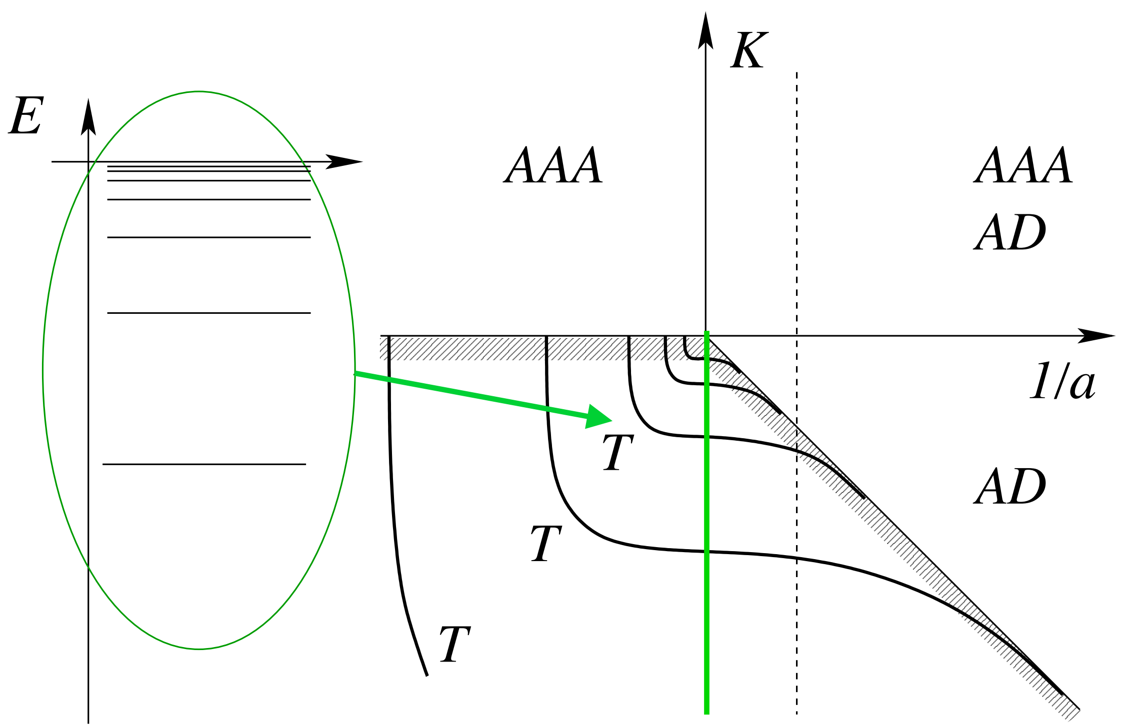

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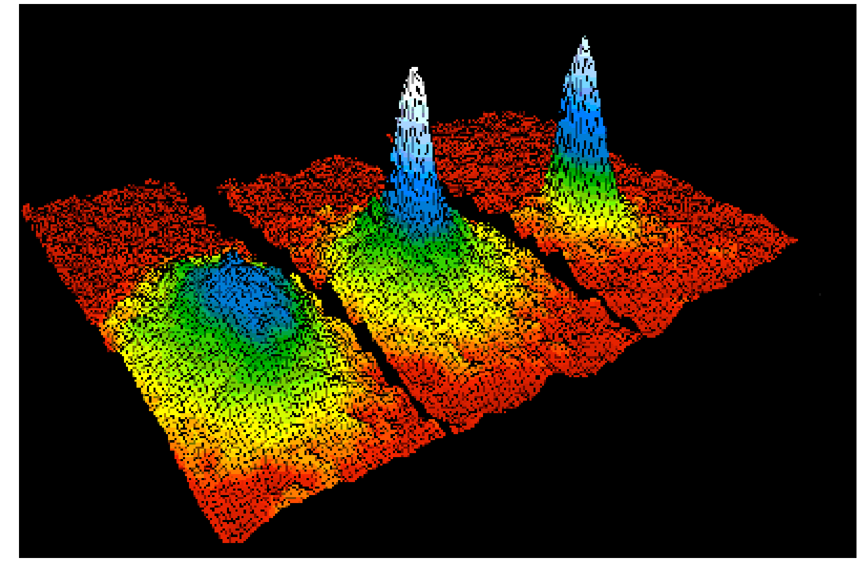

$$E_{n+1}/E_n = 1/515.03 \ , \quad n \to \infty$$

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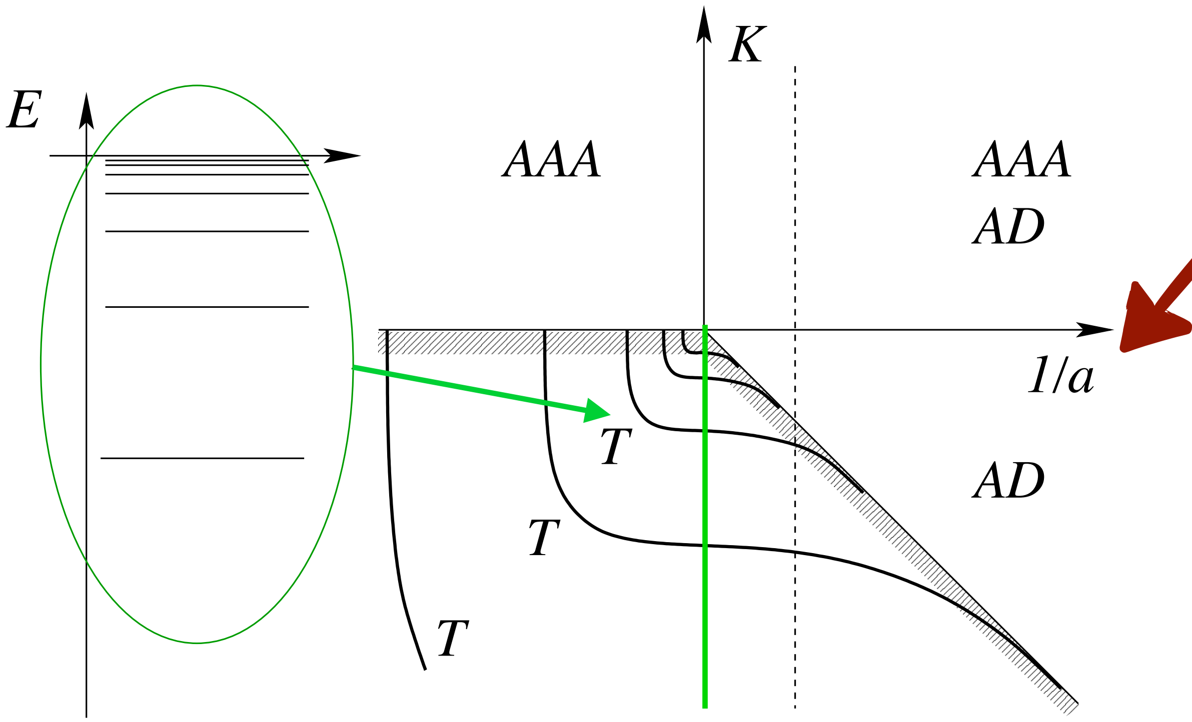

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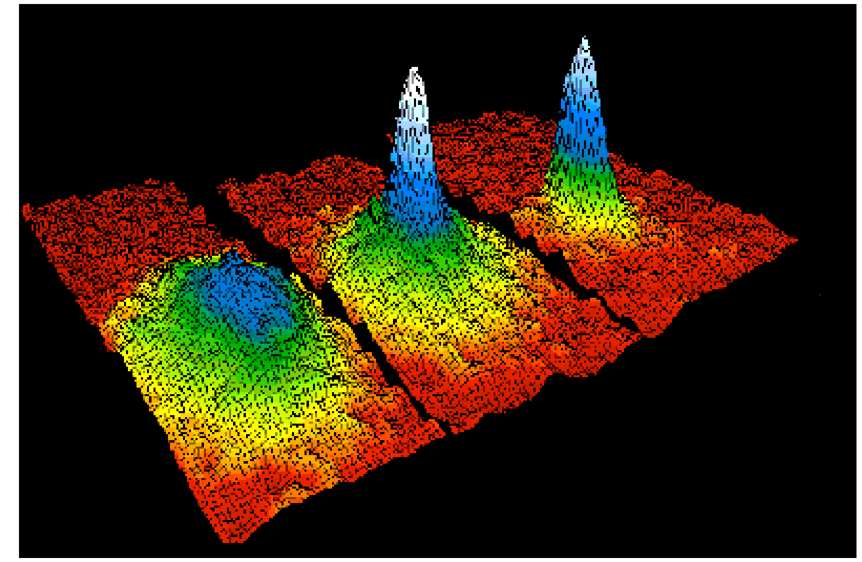

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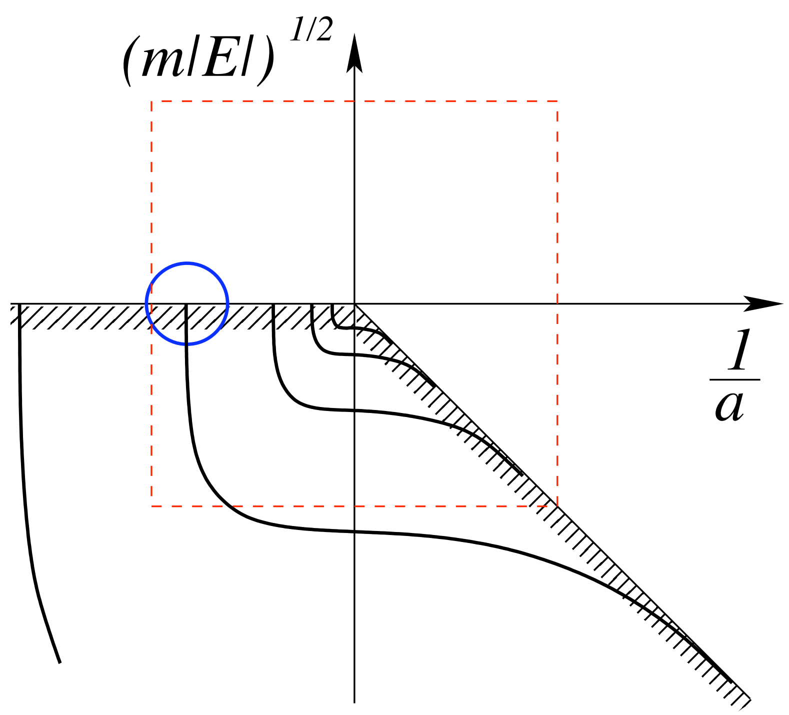
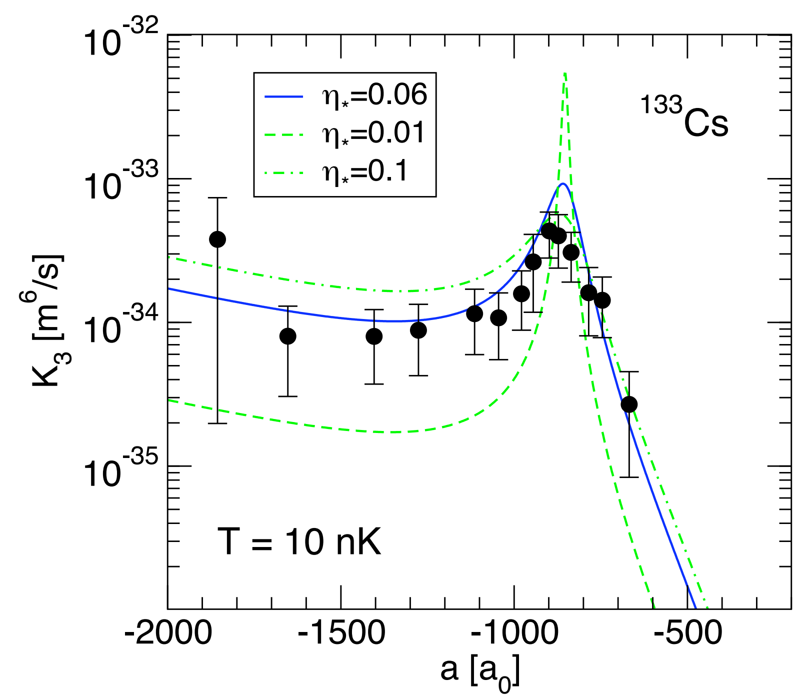


Experimental evidence for Efimov states in ^{133}Cs

(Kraemer et al. (Innsbruck), Nature **440** (2006) 315)



$a < 0: \qquad a'_* = -875 a_0$

Conformal phases: measure correlations, not β-functions!

Look at operator scaling dimensions:

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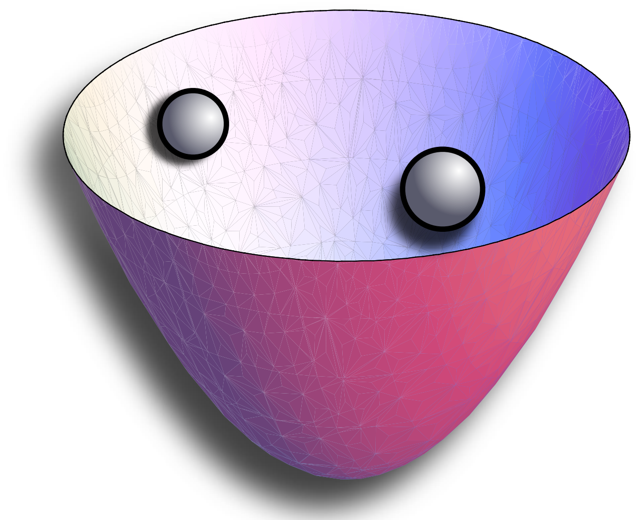

From Nishida & Son, 2007:

- Replace $V(r_1-r_2) \rightarrow V(r_1-r_2) + \frac{1}{2}\,\omega^2|r_1{}^2+r_2{}^2|$
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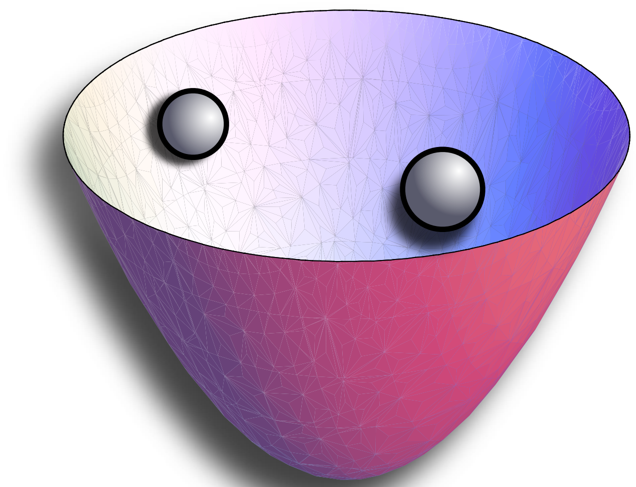

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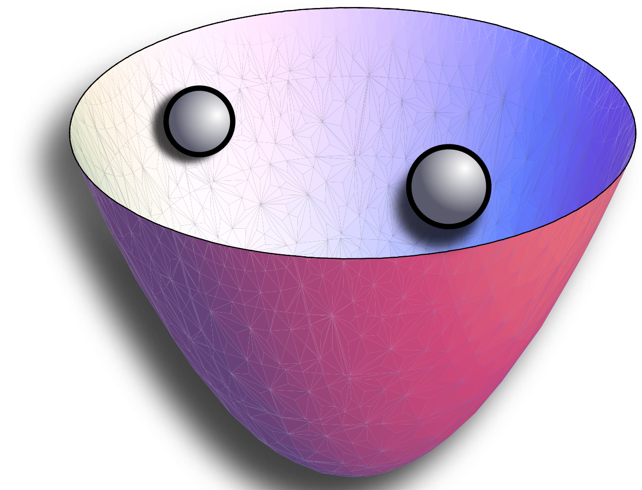

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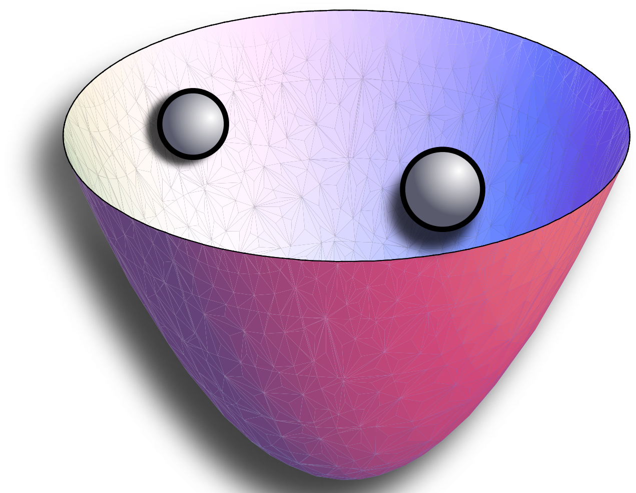

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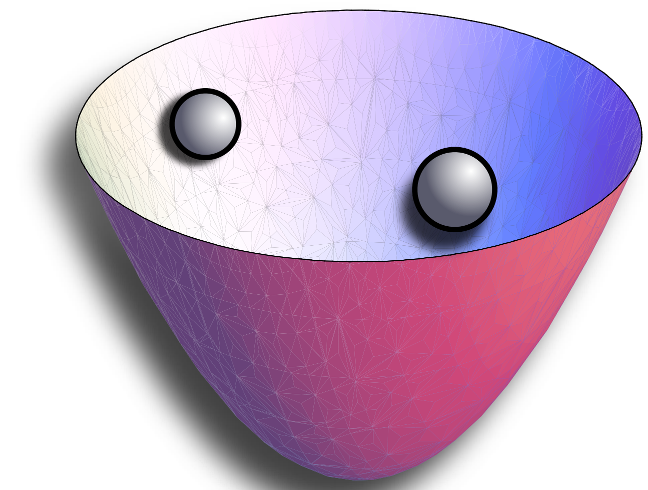

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Note: $(\Delta_+ + \Delta_-) = (d+2)$: scaling dimension of nonrelativistic spacetime.

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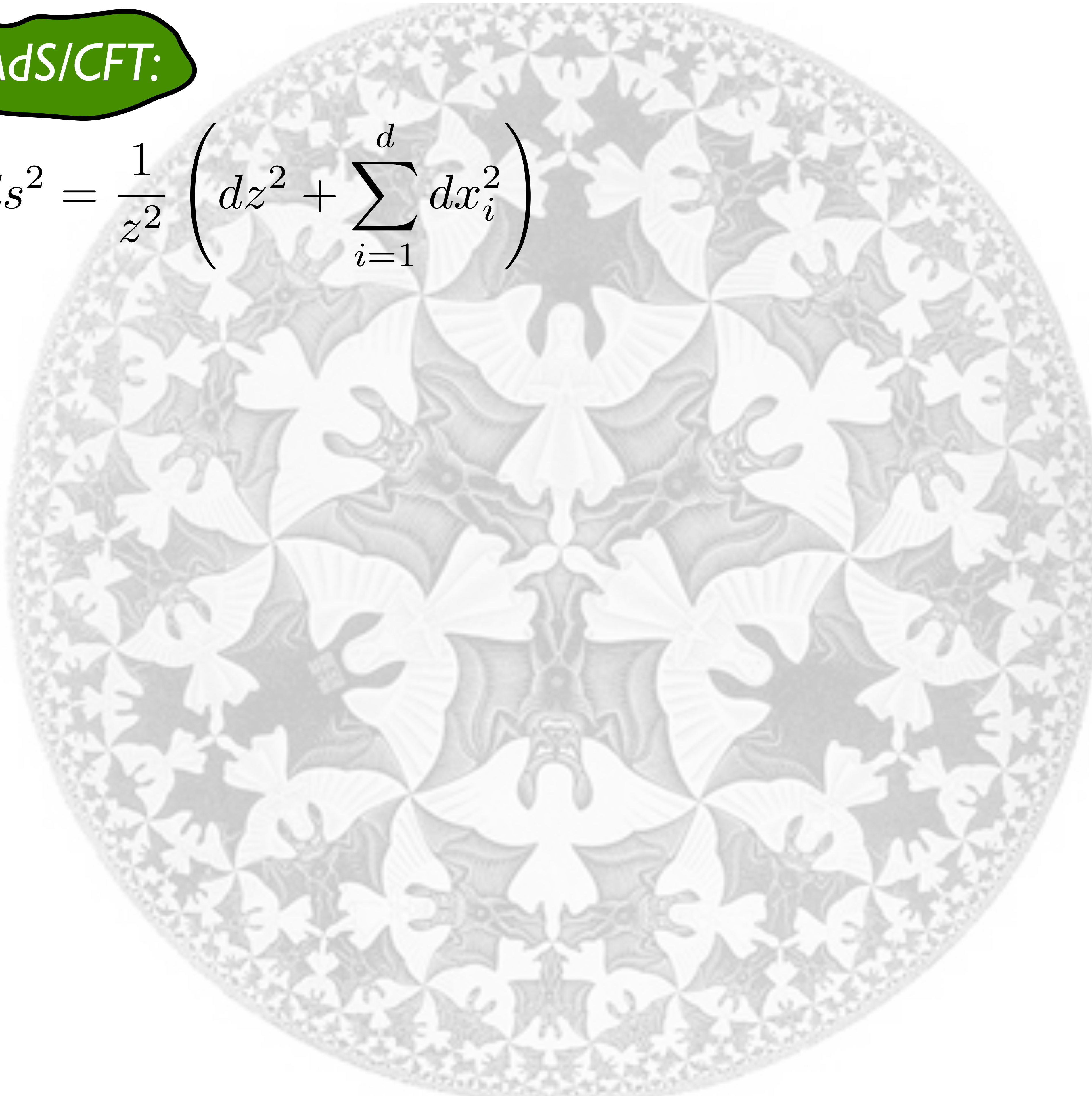

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Massive scalar in the bulk
two solutions to eq. of motion:

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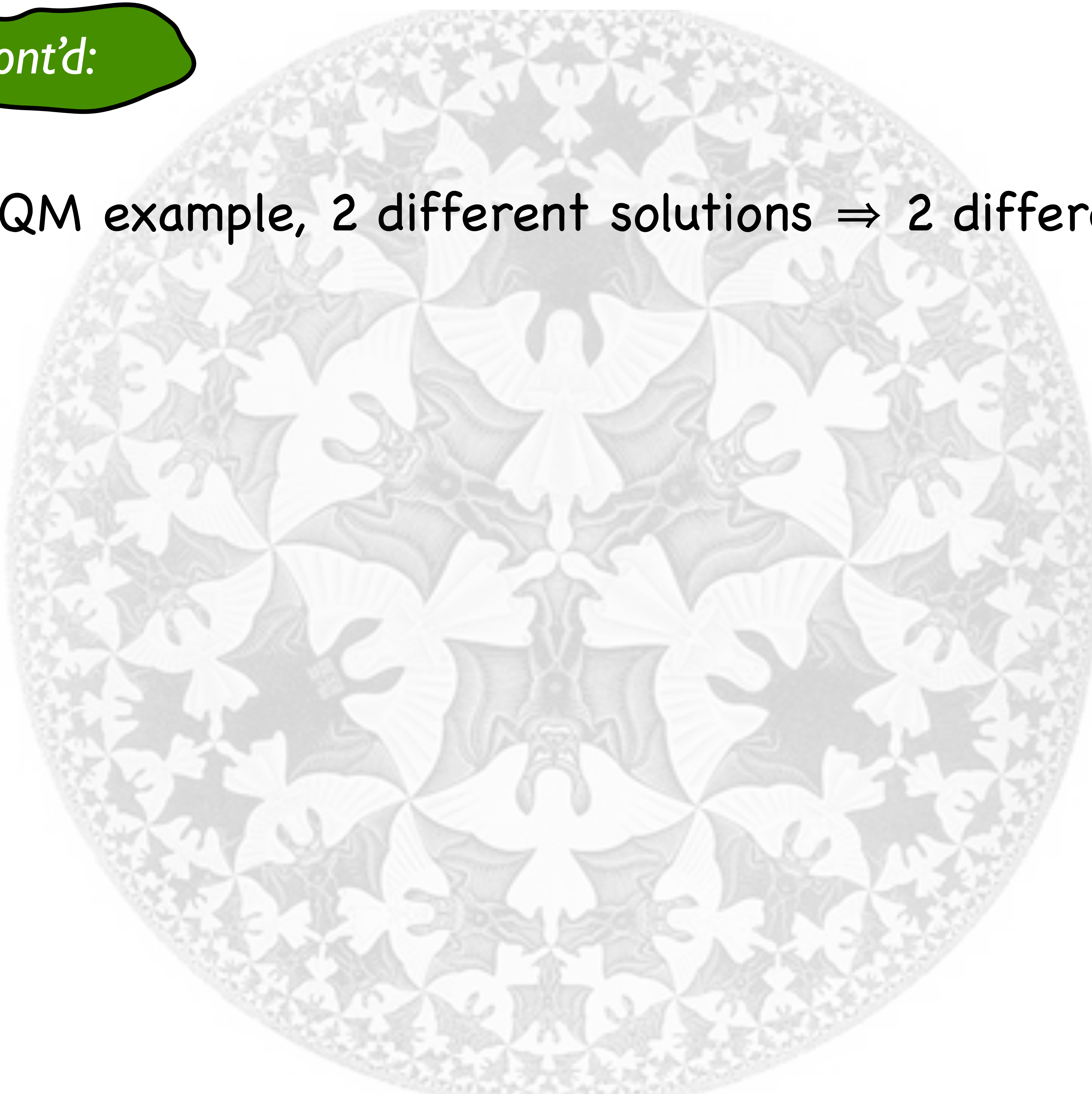
• lower bound on Δ_-	• lower bound on $\Delta^-_{\psi\psi}$

AdS/CFT cont'd:

As with QM example, 2 different solutions ⇒ 2 different CFTs



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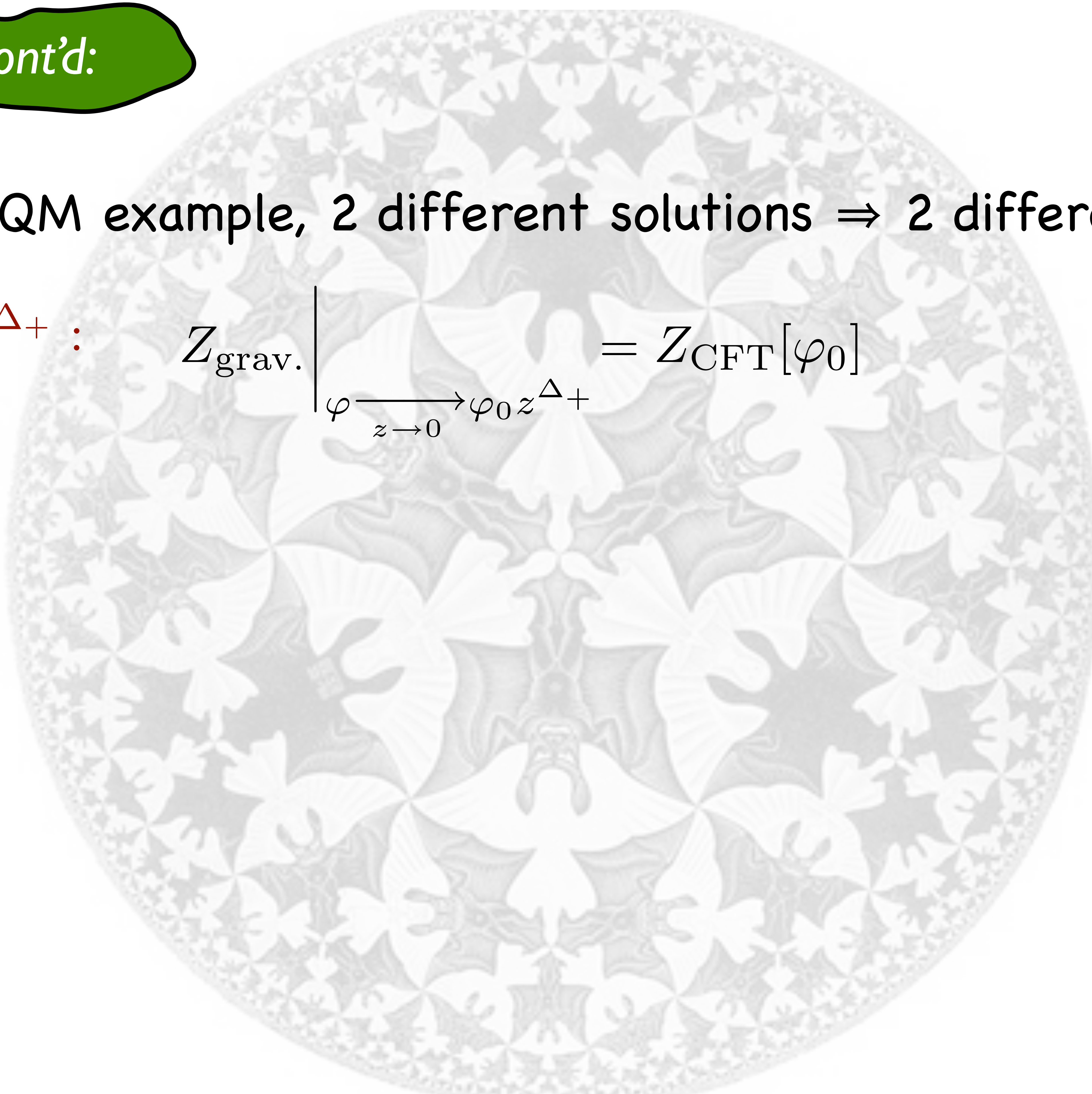

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UV fine-tuning: $m^2\varphi^2$...adds OO operator. Eg: O=$\bar{\Psi}\Psi$, OO =$\bar{\Psi}\Psi\bar{\Psi}\Psi$

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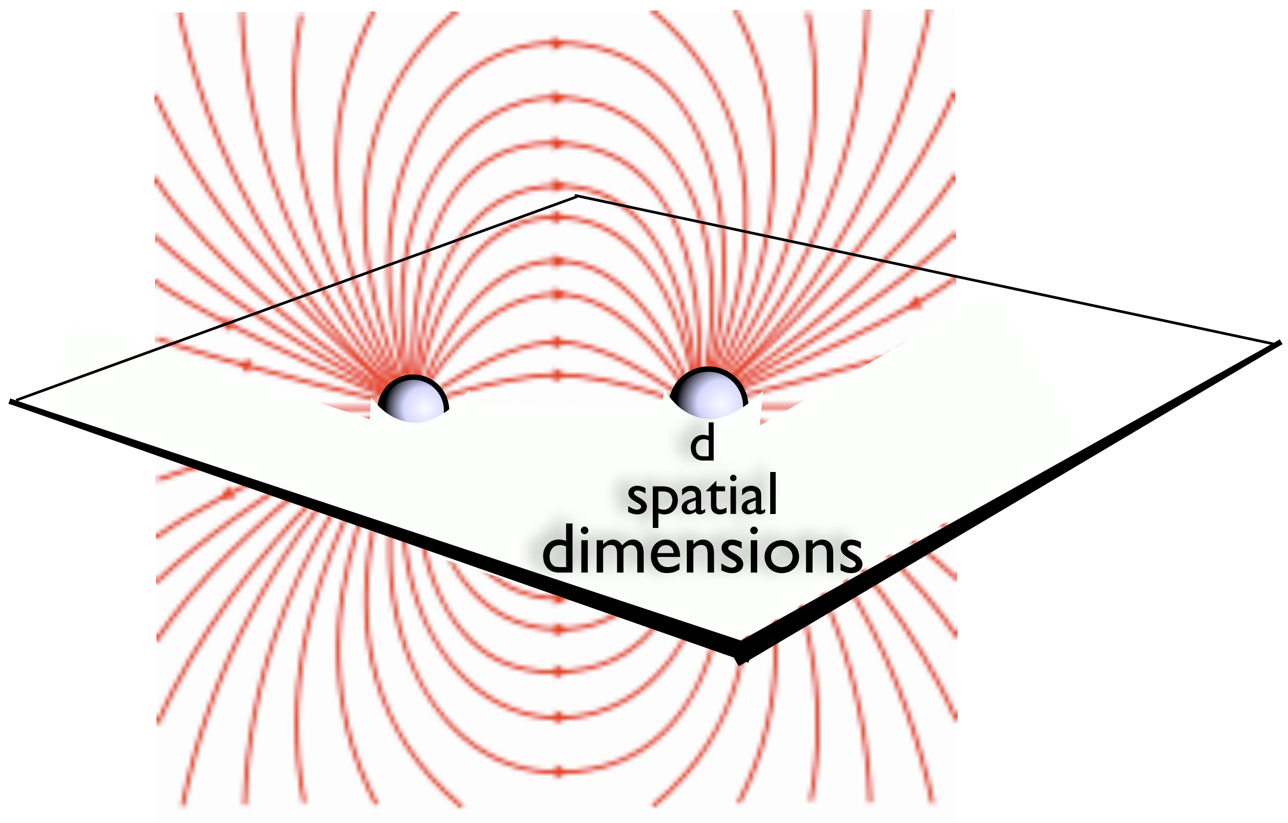
⇒ analog of δ(r) in QM example tuned to unstable UV fixed pt.

A relativistic example: defect Yang-Mills theory



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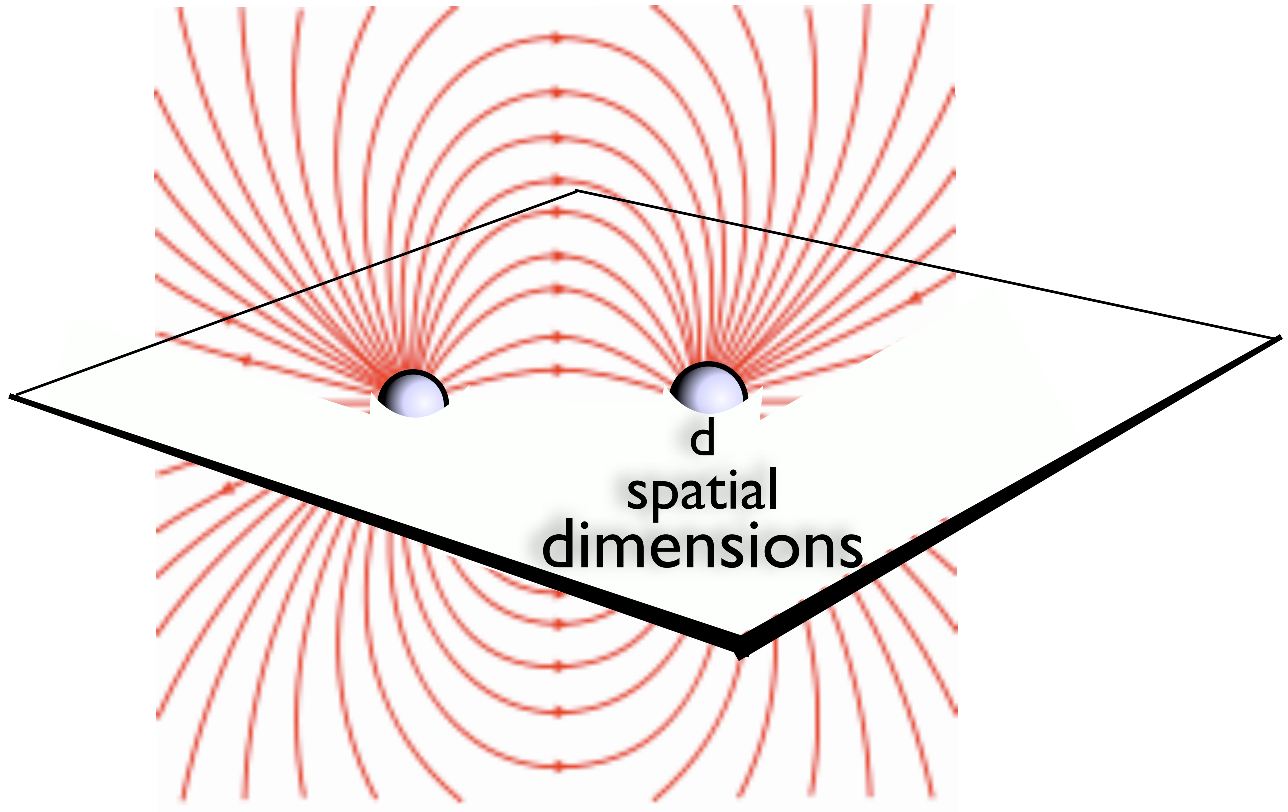

Charged relativistic fermions on a d-dimensional defect
+ 4D conformal gauge theory (eg, N=4 SYM)

$$S = \int d^{d+1}x \; i\bar{\psi}\gamma^\mu D_\mu \psi - \frac{1}{4g^2}\int d^4x \, F^a_{\mu\nu}F^{a,\mu\nu}$$

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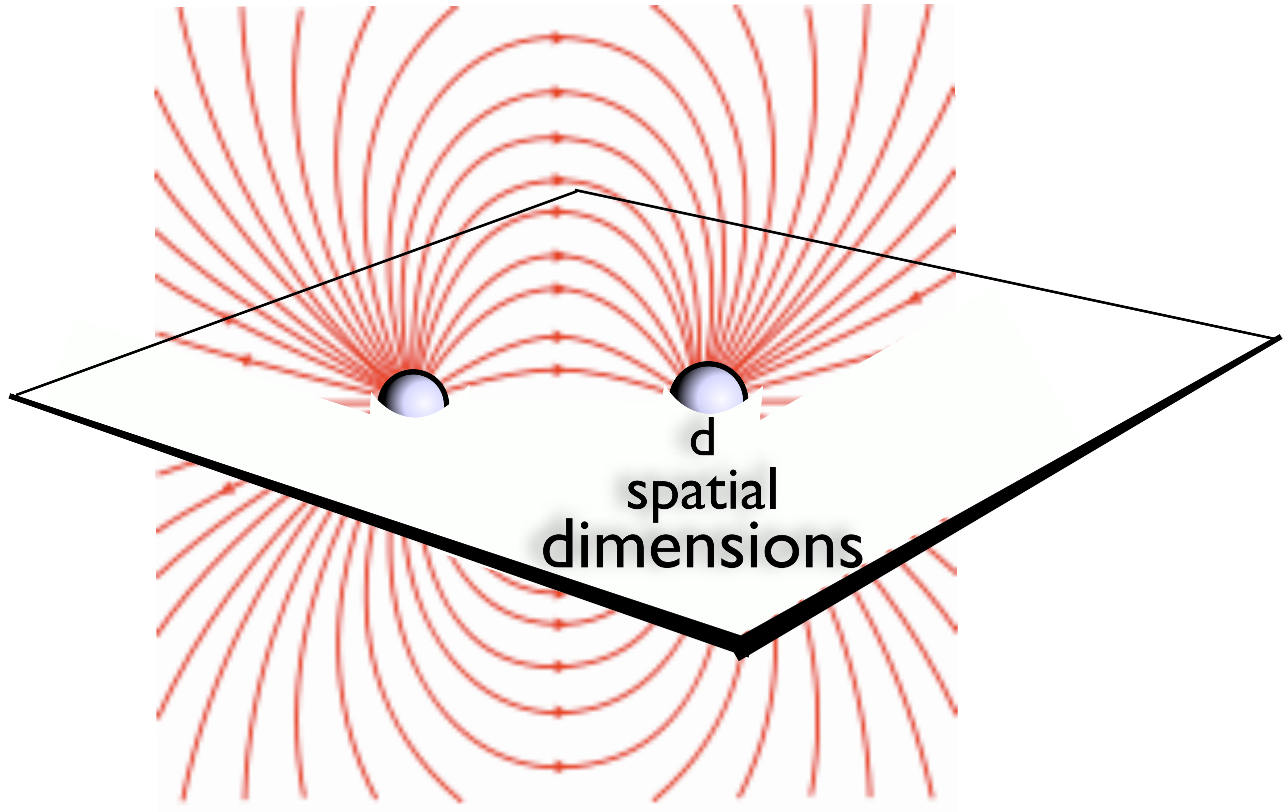
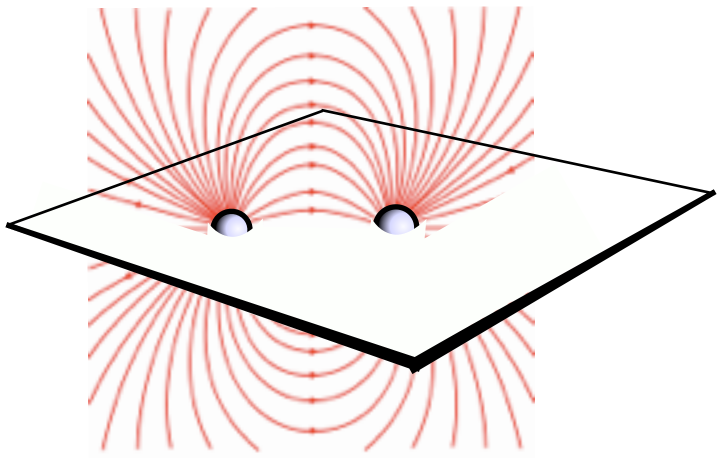


Charged relativistic fermions on a d-dimensional defect
+ 4D conformal gauge theory (eg, N=4 SYM)

$$S = \int d^{d+1}x \; i\bar{\psi}\gamma^{\mu}D_{\mu}\psi - \frac{1}{4g^2}\int d^4x \, F^a_{\mu\nu}F^{a,\mu\nu}$$

g doesn't run

g doesn't run by construction



Expect a phase transition as a function of g:

$$\langle\bar{\psi}\psi\rangle = \begin{cases} 0 & g < g_* \\ \Lambda_{\rm IR}^d & g > g_* \end{cases}$$

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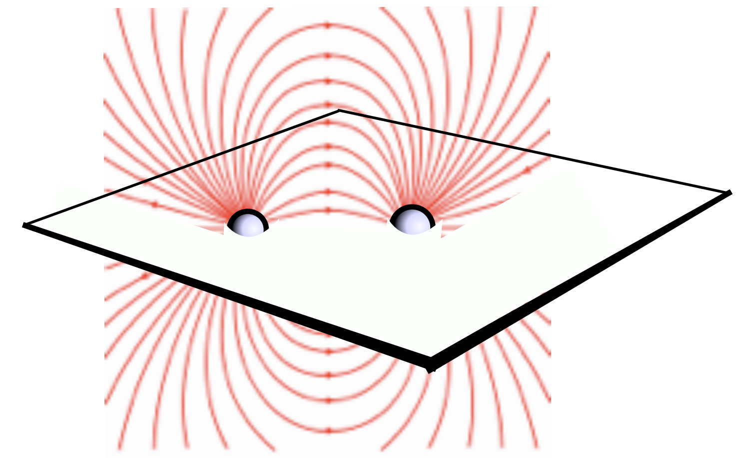

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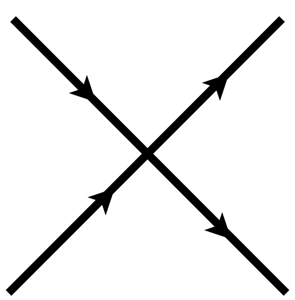

$$\Delta S = \int d^{d+1}x \, \left(-\frac{c}{2}(\bar\psi\gamma_\mu T_a\psi)^2\right)$$

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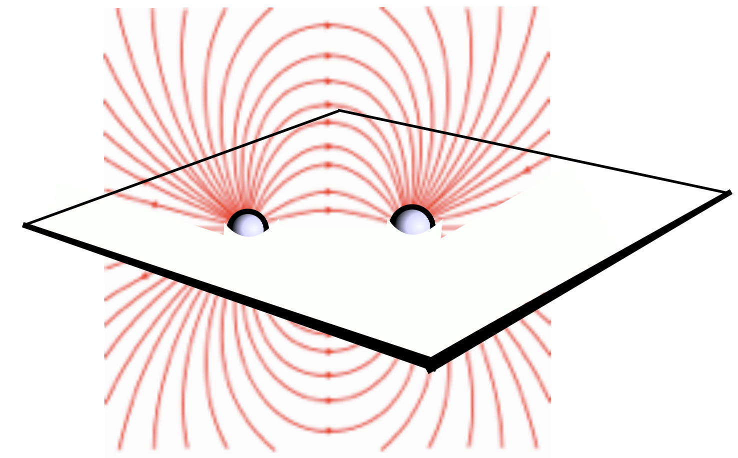

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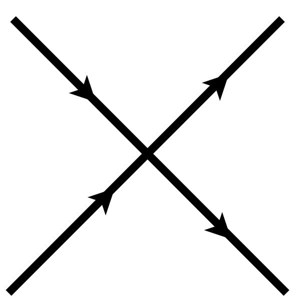

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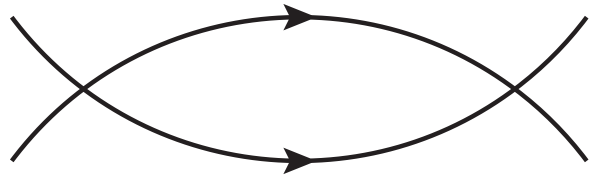
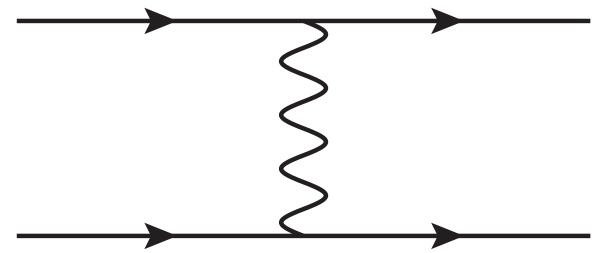
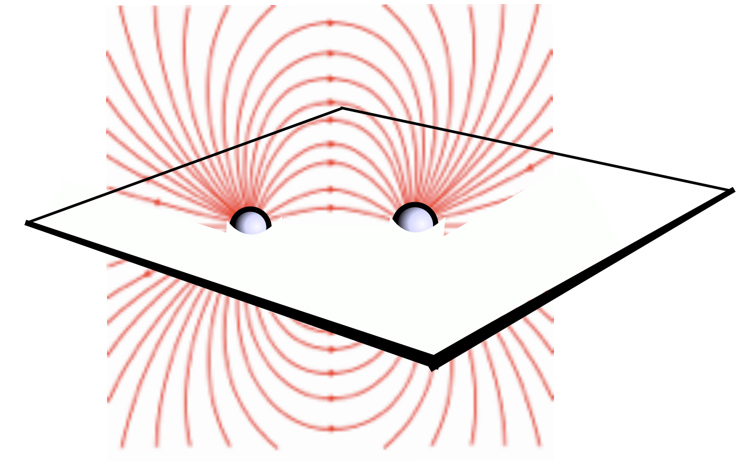
Phase transition is in perturbative regime for d=1+ε (spatial dimensions of "defect"): compute β-function

$\beta(c)$:

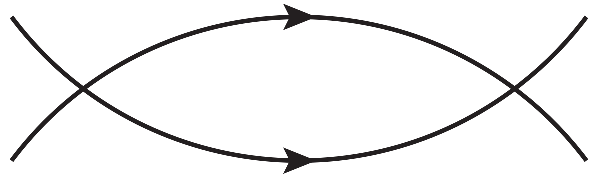


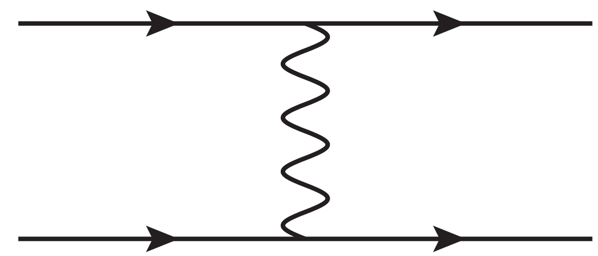


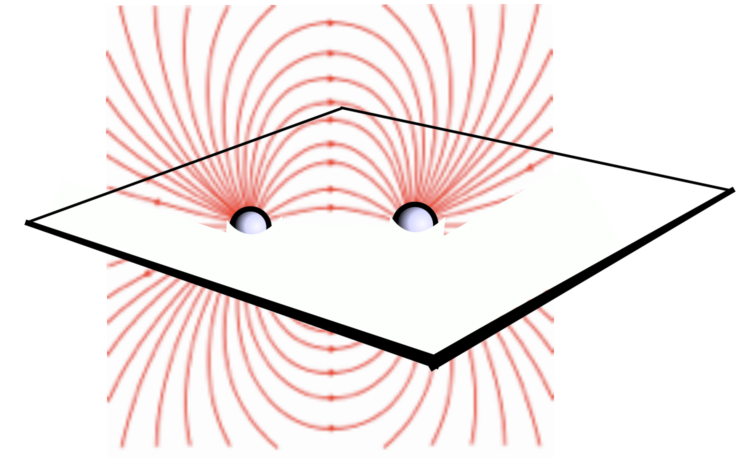


$1/\varepsilon$ pole for $d=(1+\varepsilon)$

β(c):







1/ε pole for d=(1+ε)

$$
\begin{aligned}
\beta(c) &= -\frac{g^2}{2\pi} - \epsilon c - \frac{N_c}{2\pi} c^2 \\
&= \frac{1}{2\pi}\left(\frac{\pi^2\epsilon^2}{N_c} - g^2\right) - \frac{N_c}{2\pi}\left(c - \frac{\epsilon\pi}{N_c}\right)^2
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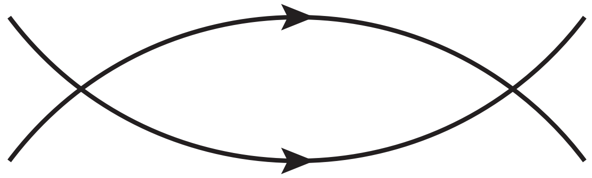

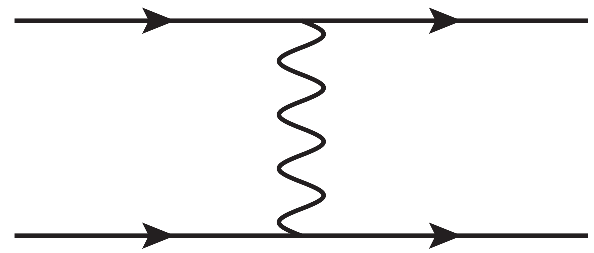

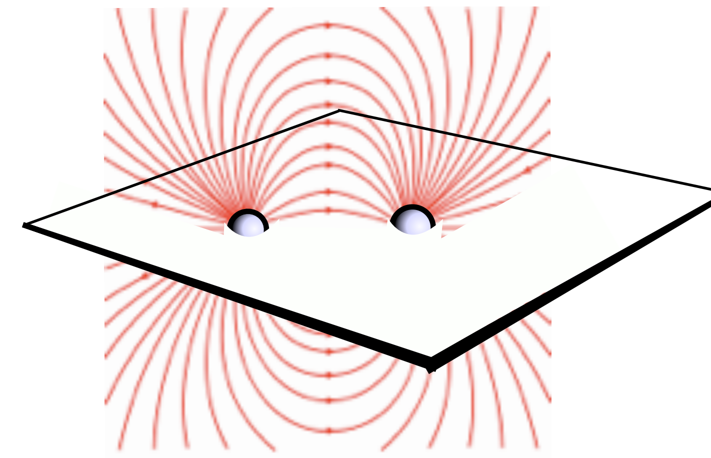

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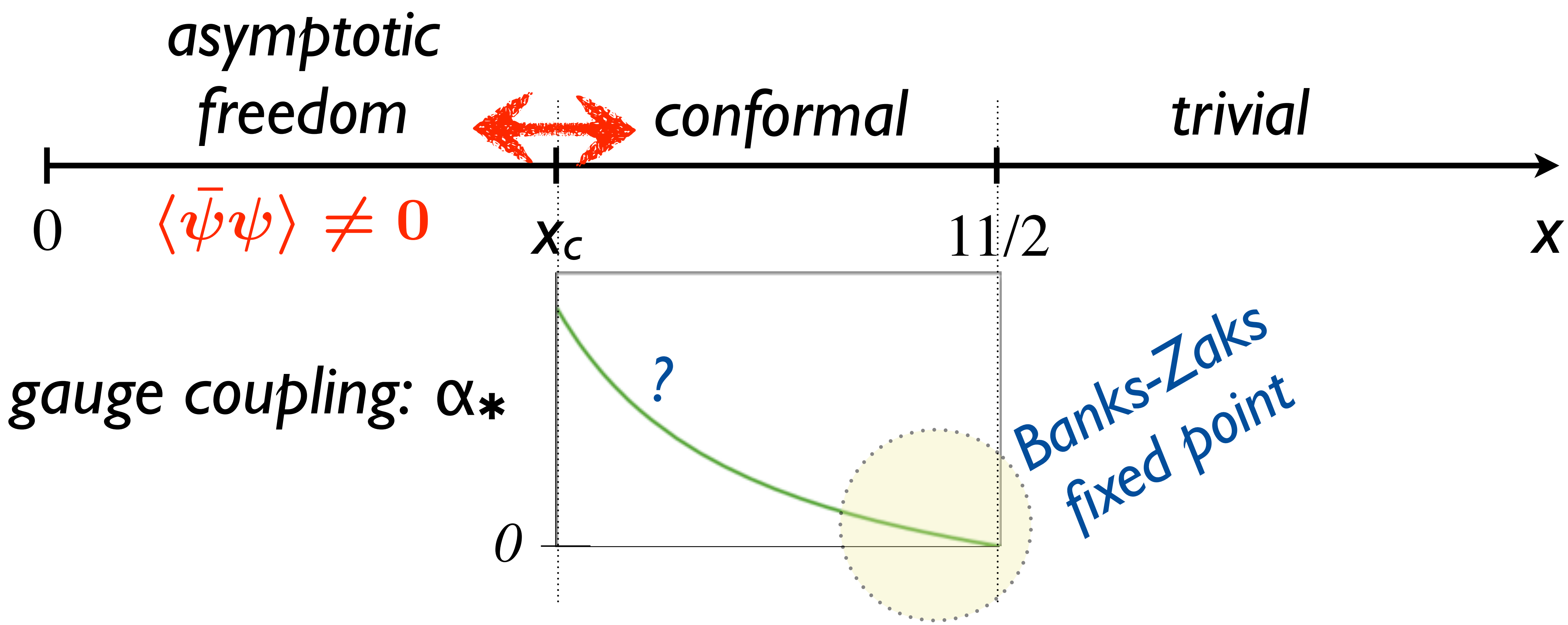
- Find BKT transition at $g^2 = g_*^2 = (\varepsilon\pi)^2/N_c$

  $\Lambda_{IR} \sim \Lambda_{UV}\, \exp[-\pi/\sqrt{(g^2-g_*^2)}]$

- Schwinger-Dyson gap eq (rainbow approx) gives qualitatively same results

Back to QCD at LARGE N_c and N_f:



Transition at $x = x_c$?

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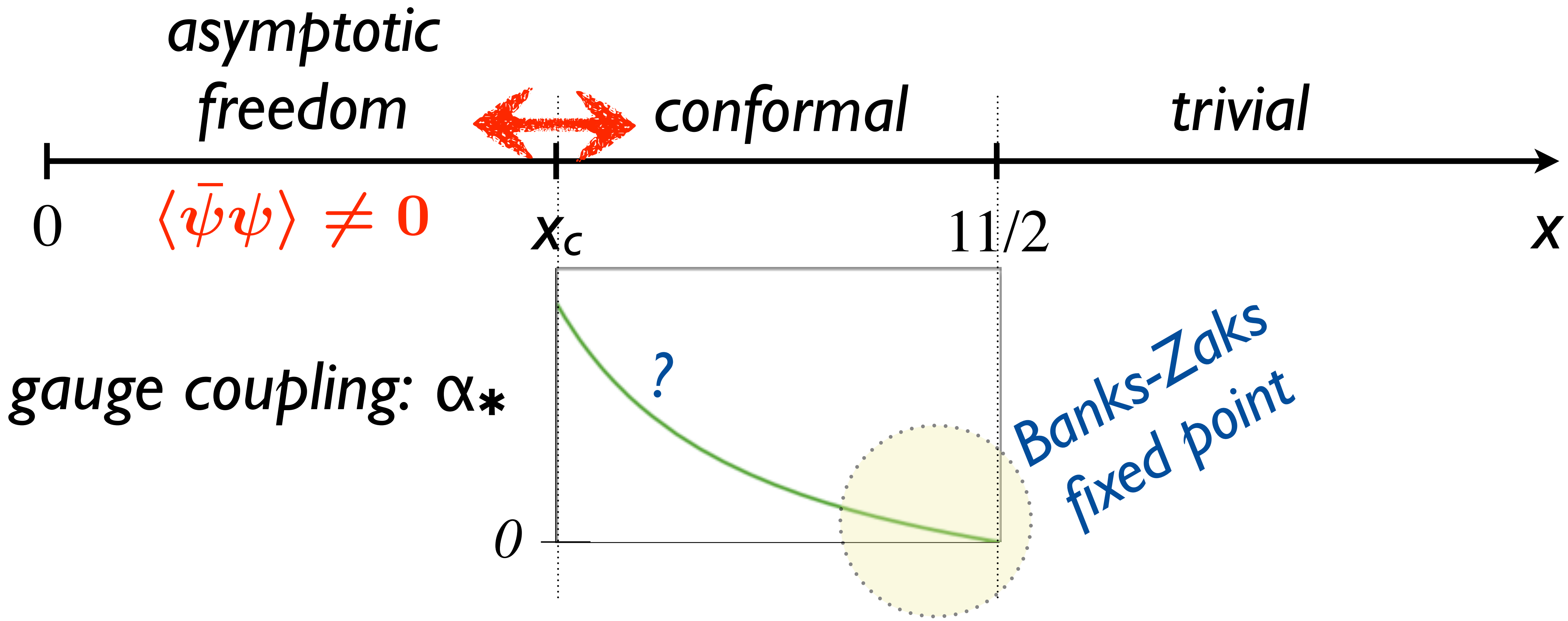

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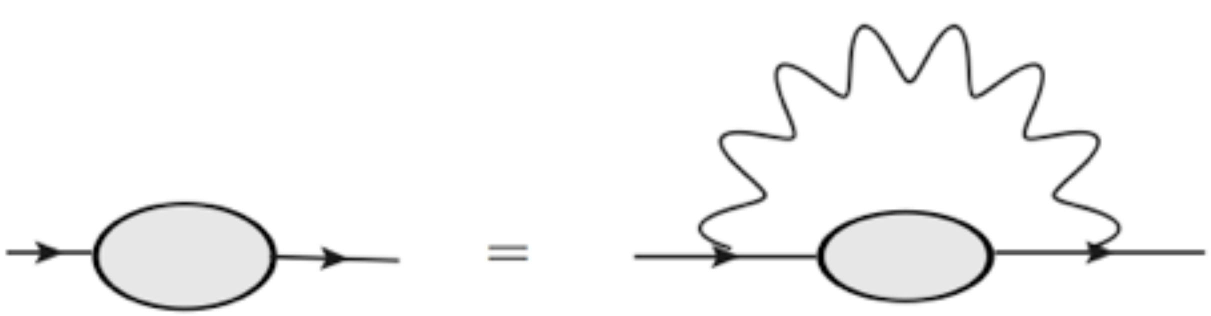

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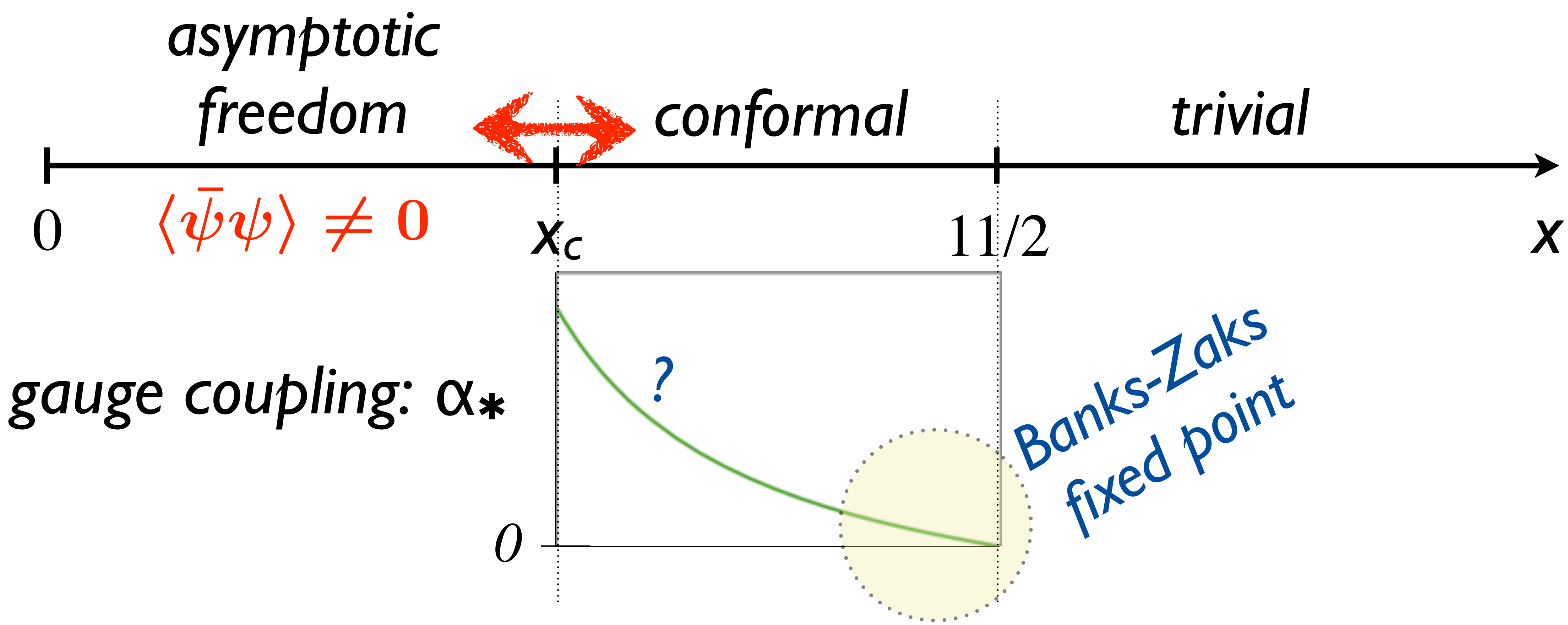

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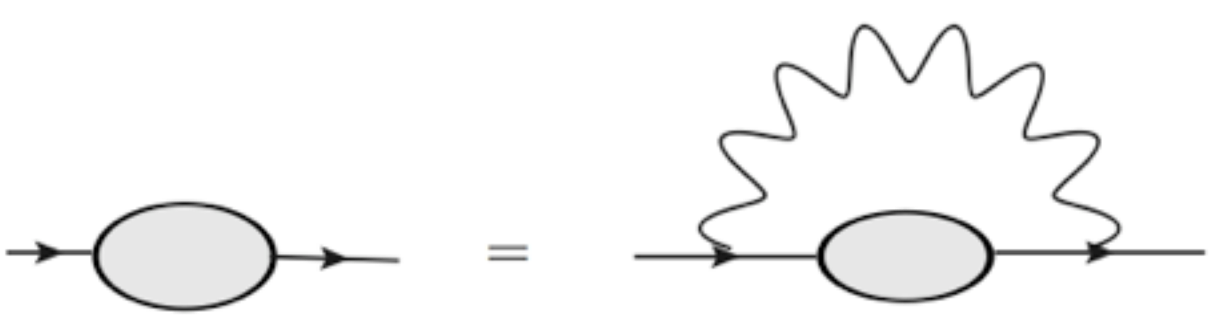

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Found: BKT scaling for $\langle\bar{\psi}\psi\rangle$...not rigorous, but qualitatively correct?

Conjecture: loss of conformality for QCD at x_c is of BKT type, due to fixed point merger.

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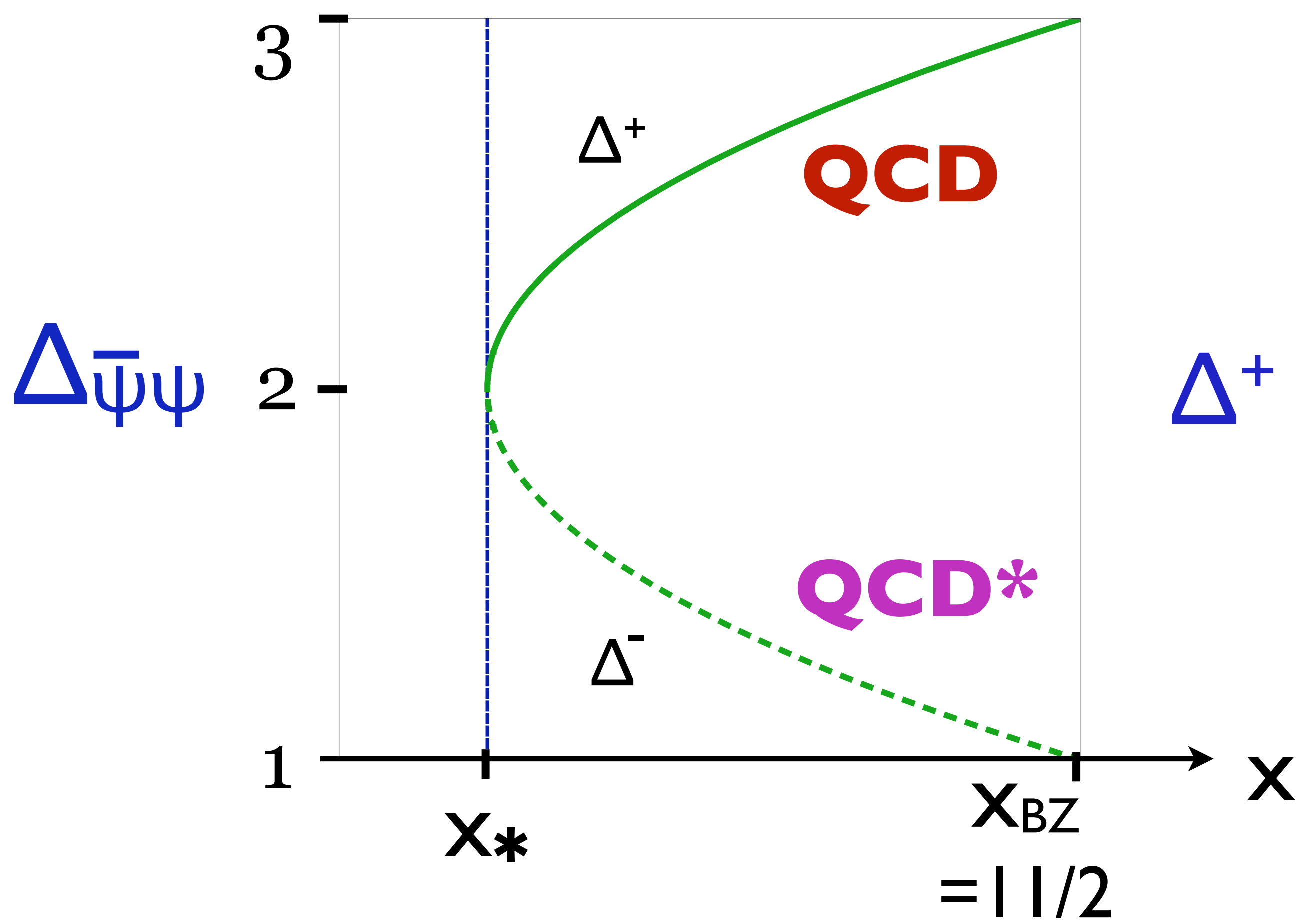

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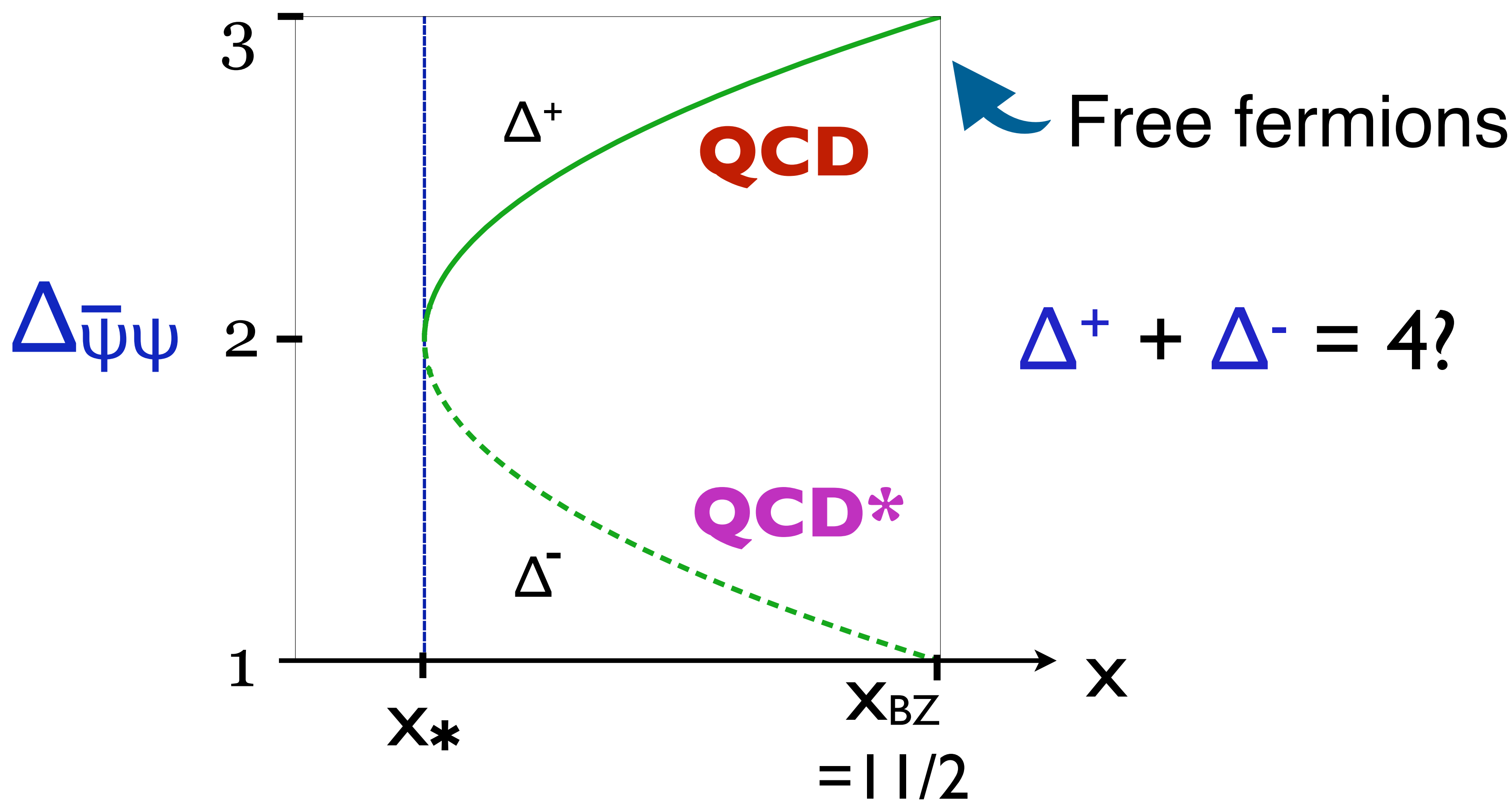

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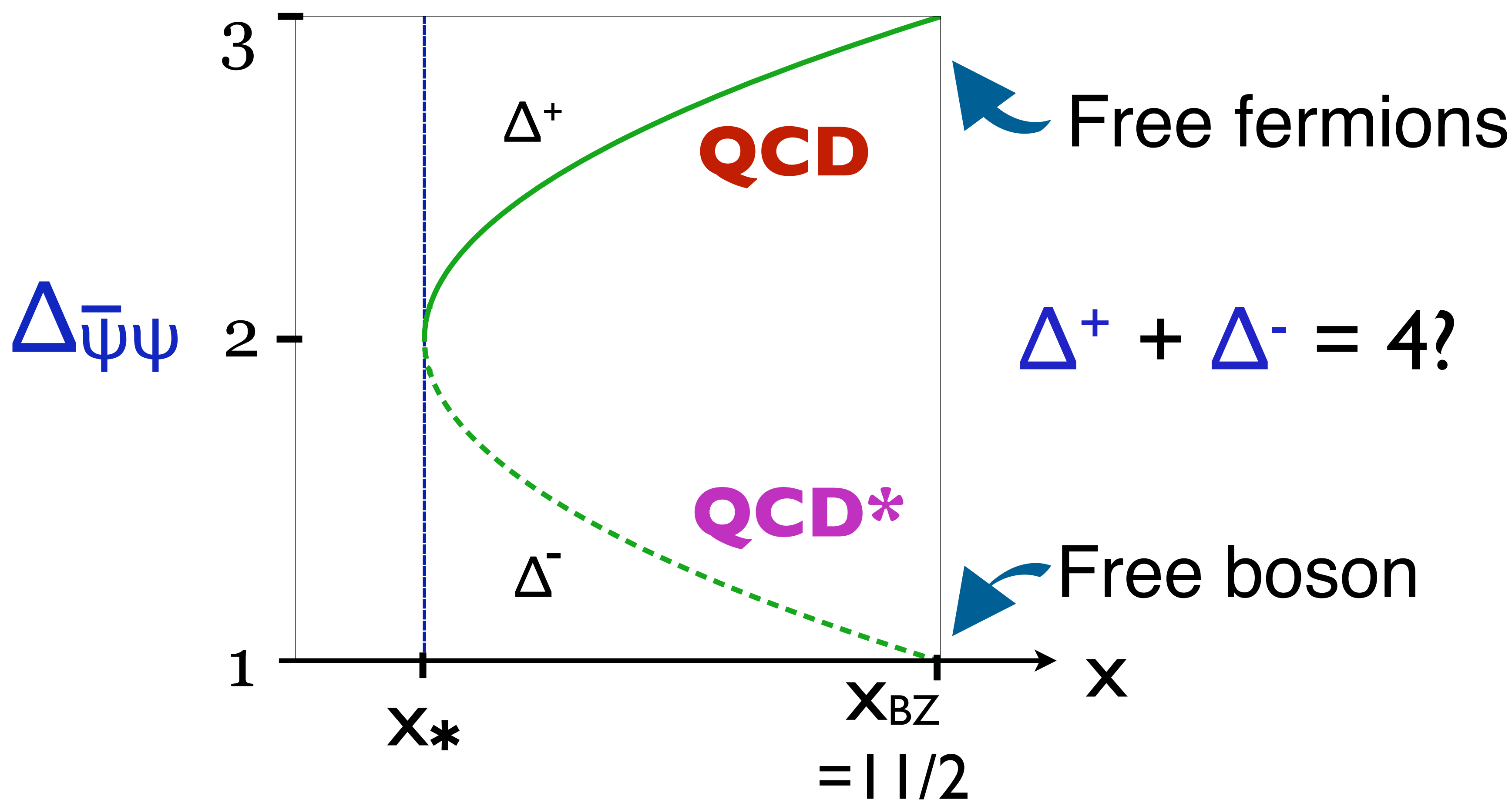

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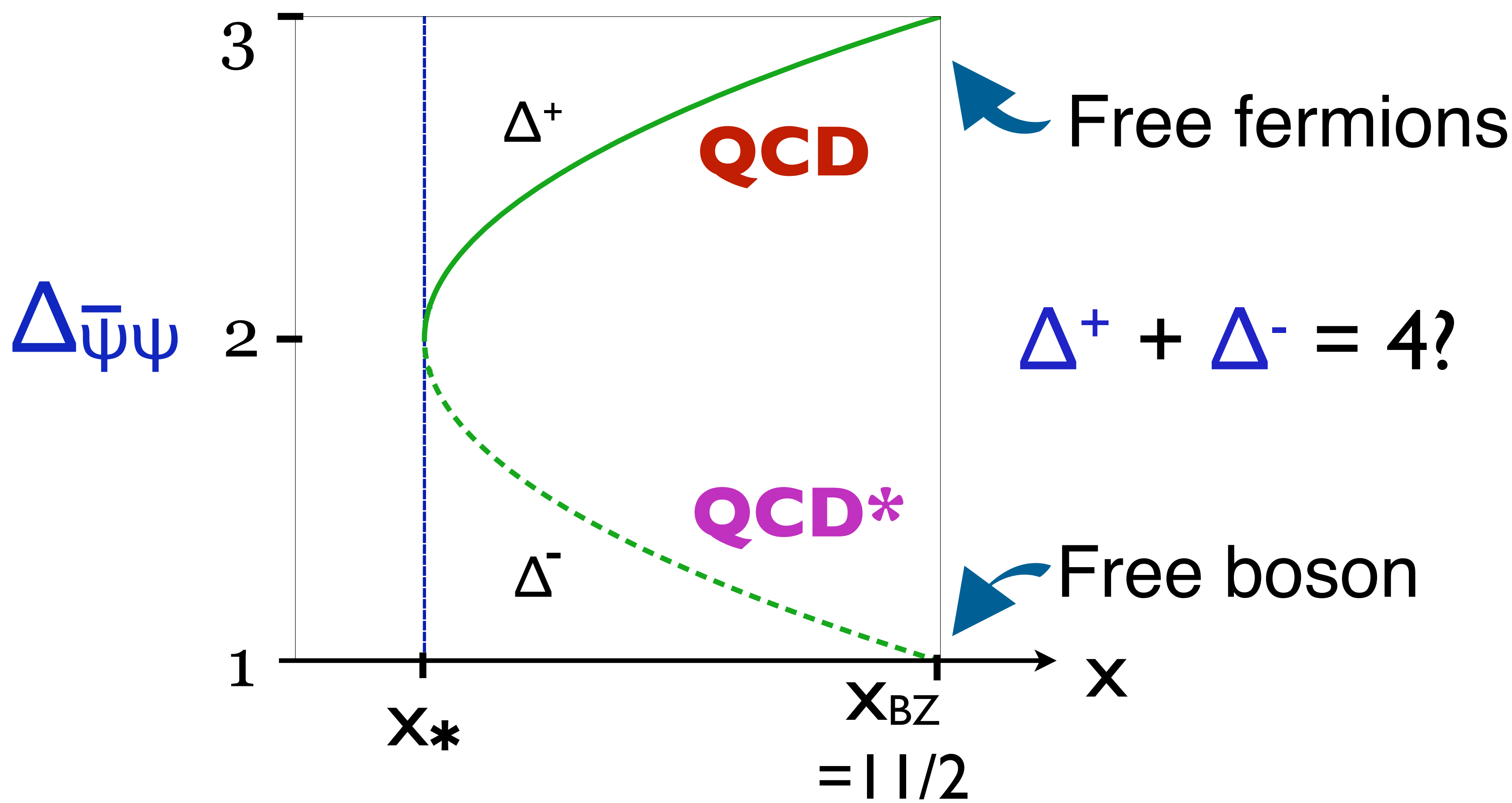

Near Banks-Zaks (IR) fixed point:

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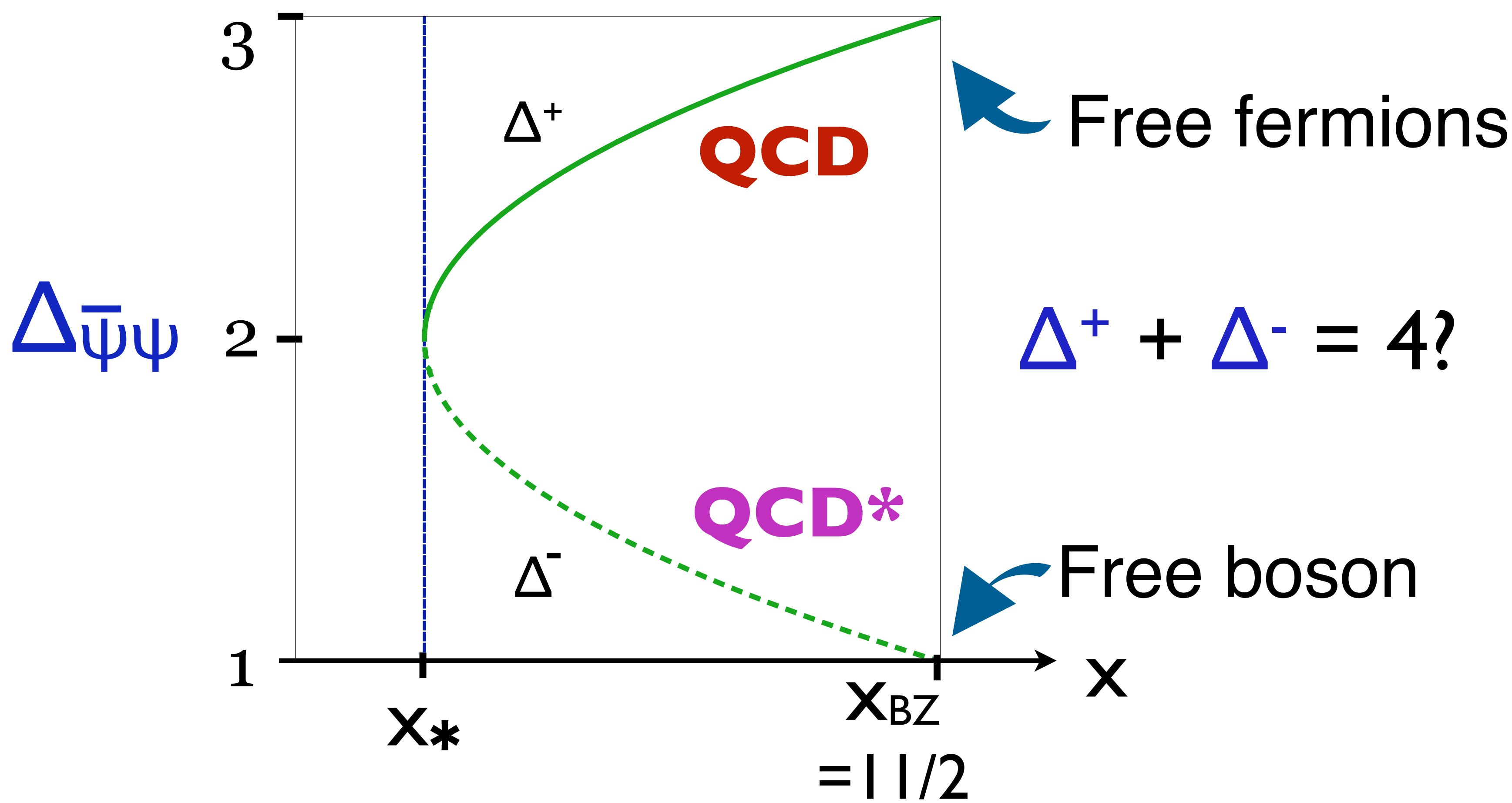

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$$\Delta^+_{\psi\bar{\psi}} = 3 - \#\, g^2 N_c$$

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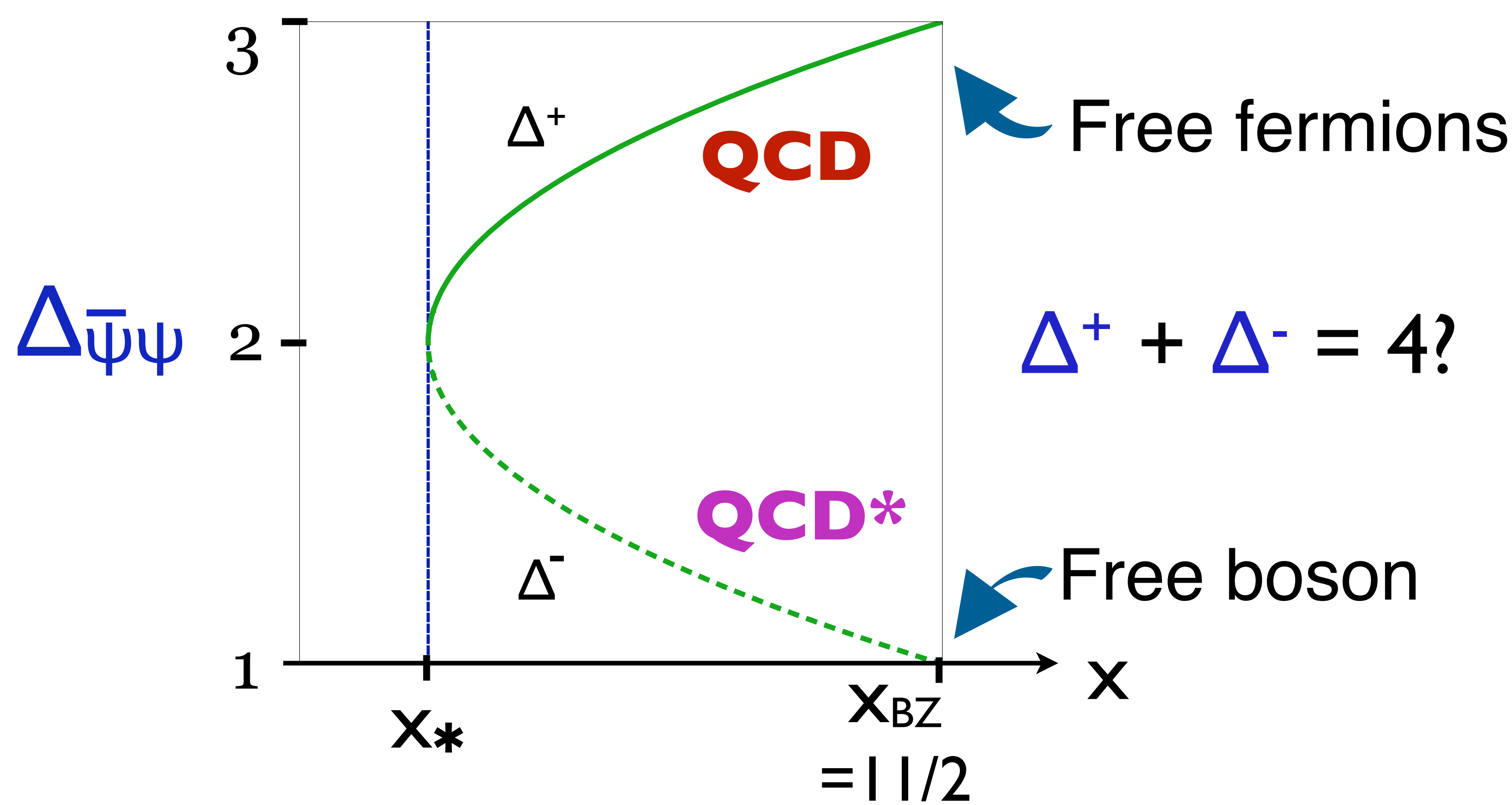

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(almost free quarks)

Partner theory QCD*:

$$\Delta^-_{\psi\bar\psi} = d - \Delta^+_{\psi\bar\psi} = 1 + \#\, g^2 N_c$$

(almost free scalar?)

WANTED

☛ Conformal theory defined at nontrivial UV fixed point to merge with QCD at $x = x_c$

LAST SEEN WITH WEAKLY COUPLED SCALAR

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Consider:

- $SU(N_c)$ gauge theory
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- $\mathbf{M_f}^2$ scalars φ, tuned to be massless
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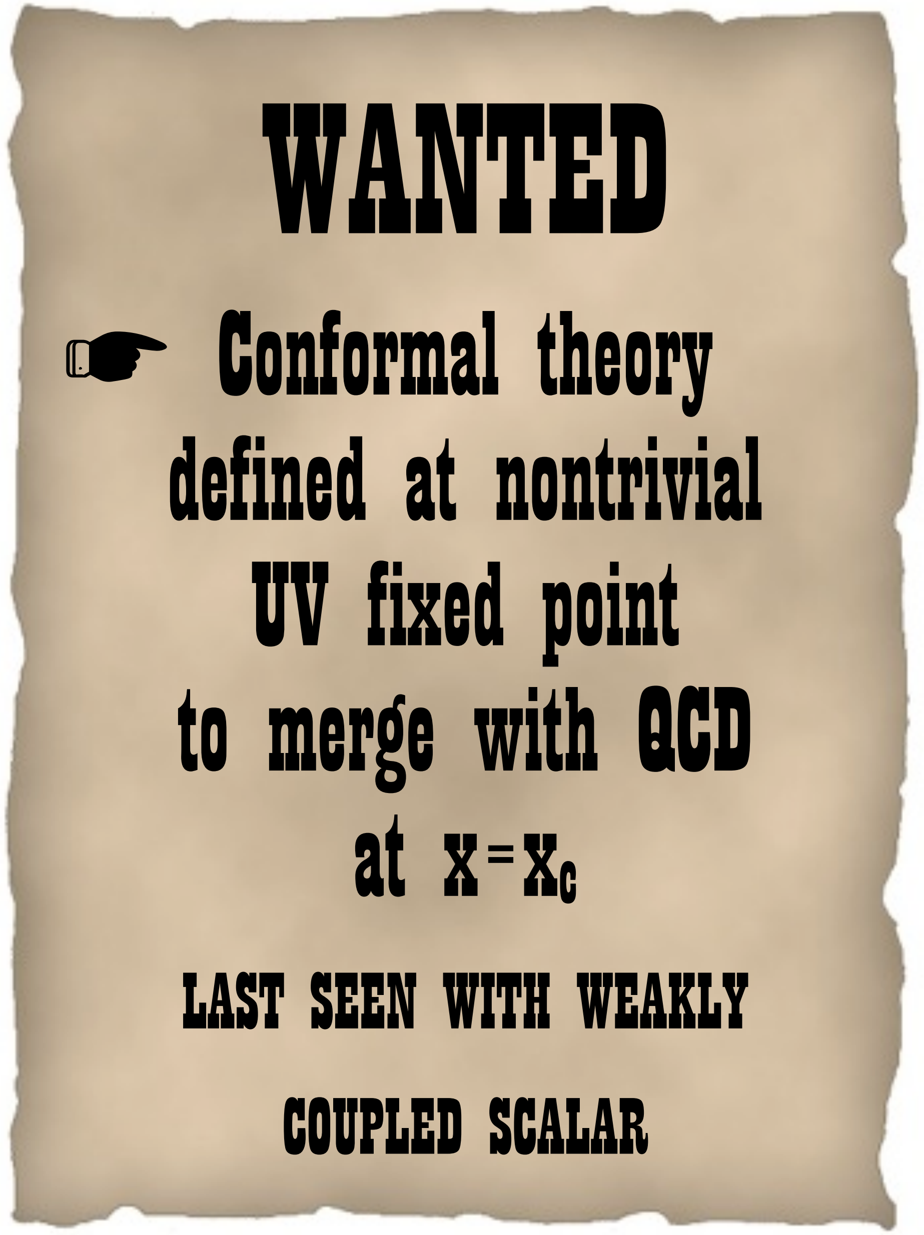

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Find analog of Banks-Zaks pt. for:

$$\text{iff } M_f \leq \frac{5}{2\sqrt{11}} N_f \simeq .75 N_f$$

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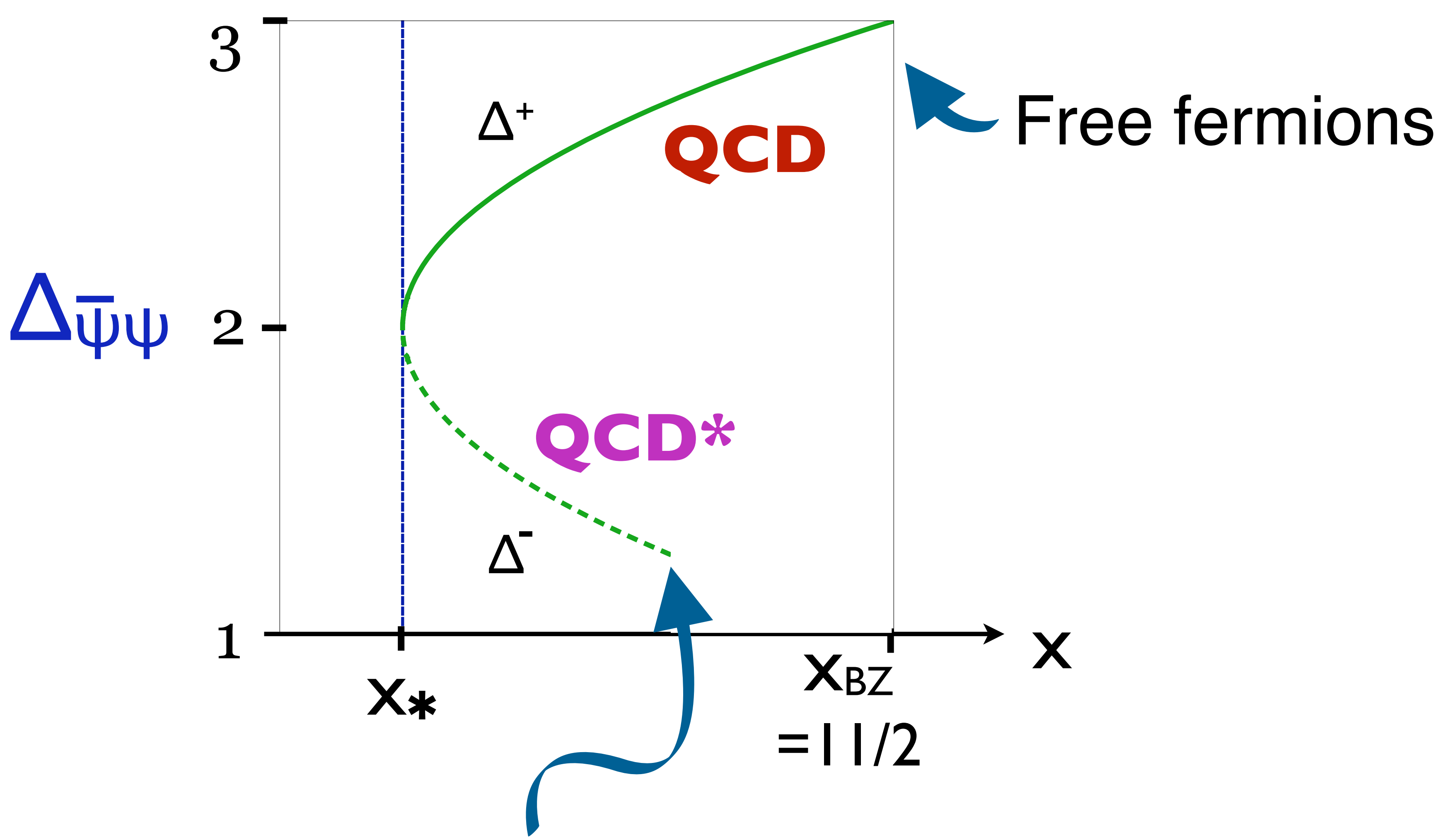
..but QCD* needs full flavor symmetry. Possibly only at stronger coupling?

QCD* ?



UV fixed point starts at strong-ish coupling?

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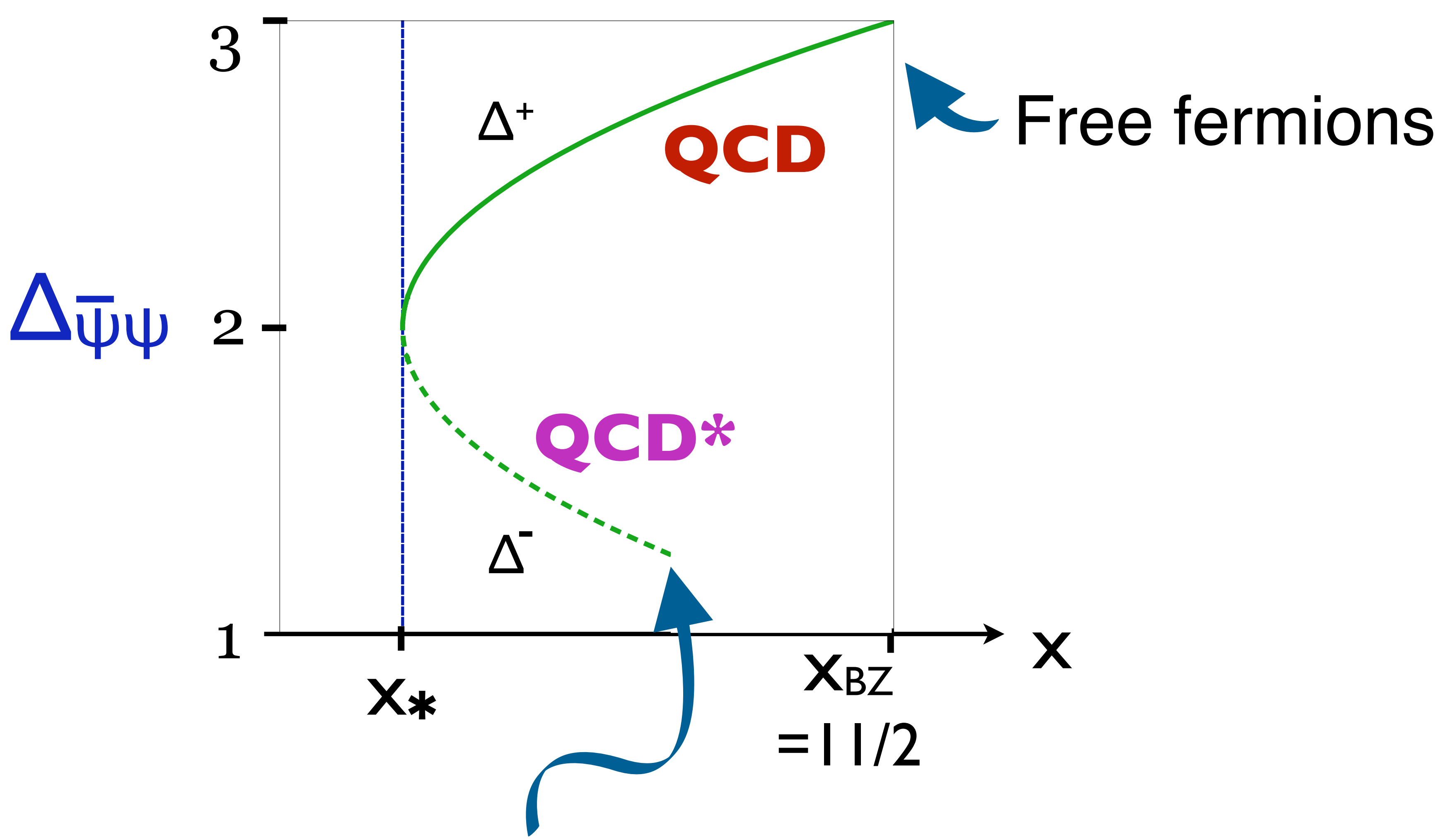

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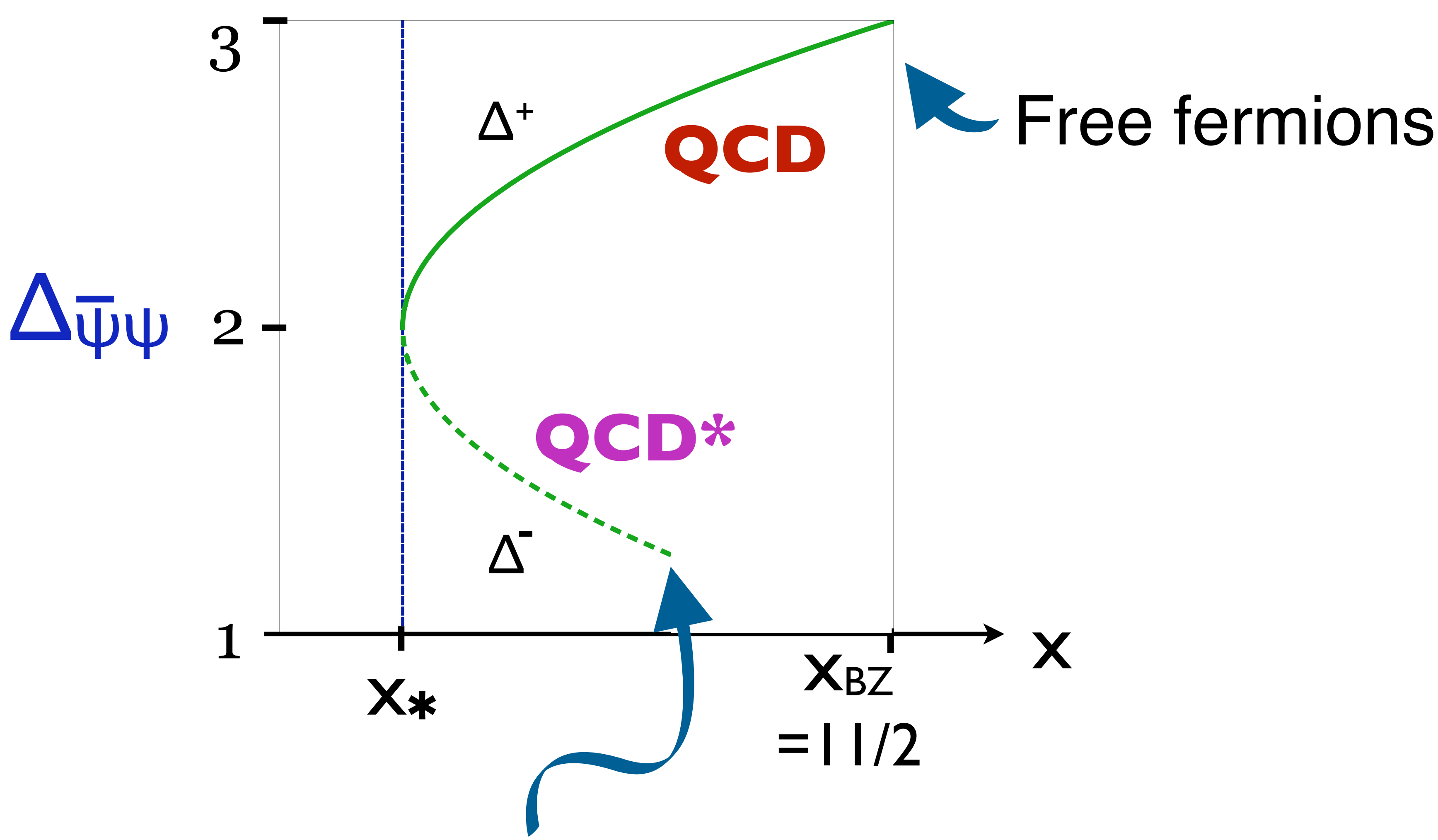

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V. Implications for QCD with many flavors? Is there a pair of conformal QCD theories? What is QCD*?
Finding QCD* should be on field theory / lattice QCD "to-do" list.


E Paradiso ab Angelo pelluntur. Genes. III.