Gaiotto Duality and the Landscape of N=2 supersymmetric AdS_5 vacua of M-Theory

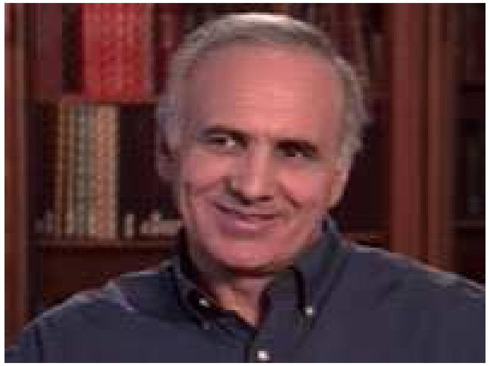
Juan Maldacena (based on Gaiotto & JM)

Tom Banks and Willy Fischler

60th birthday symposium

UC Santa Cruz, June 2009

Happy Birthday !



and thanks!

Classification of geometric/algebraic structures

- Regular solids
- Lie Groups
- Conformal field theories in 2 dimensions with c<1.

- String vacua? AdS_4 vacua ?
- Conformal field theories in 3d
- Conformal field theories in 4d with large amount of supersymmetry.

- N=4 susy theories in four dimensions
- N=2 superconformal theories in four dimensions and AdS_5 vacua.

  - Classification of such theories.
  - Classification of corresponding geometries.
  - Dictionary between the two.

Field theory ↔ geometry

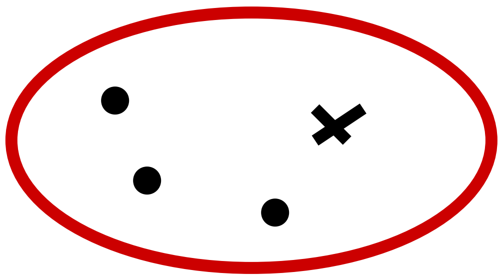
Outline

- Gaiotto Duality and a new picture for N=2 superconformal field theories
- Construction of the geometries
- Checks of the relationship
  - a, c
  - global symmetries and anomalies
  - BPS states.

Gaiotto Duality

- New way of thinking about the space of N=2 superconformal field theories.
- Usual: The space of couplings of the theories: $\tau_i = \frac{i}{g_i^2} + \theta_i$
- New: Moduli space of Riemann surface with punctures.

N



$\mathrm{M}_{g,n}$

S-duality

Strong → weak

Argyres-Seiberg duality

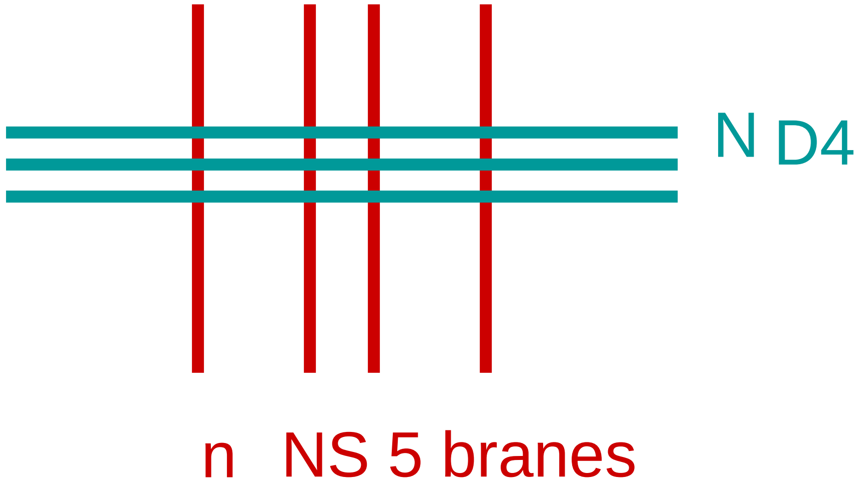
Strong → 

G-duality

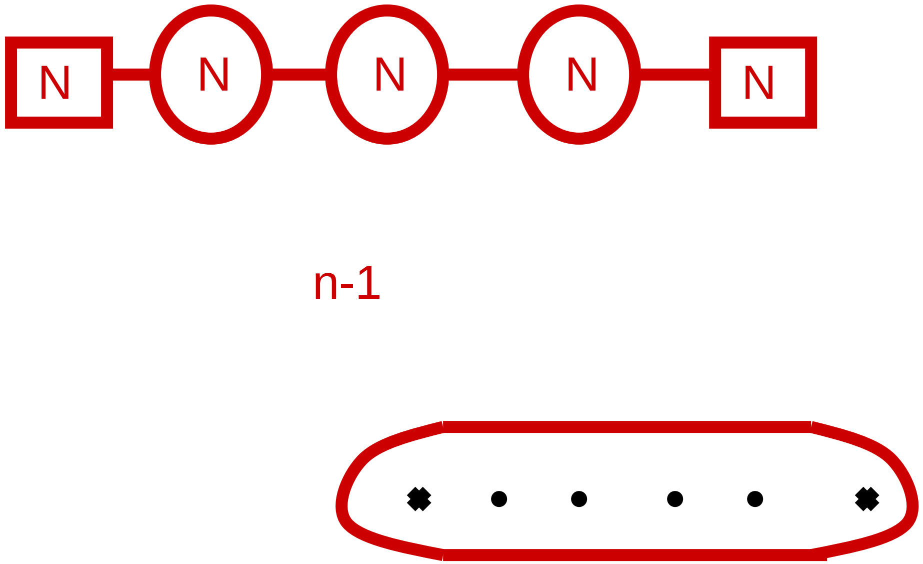
M5 brane description of the Seiberg-Witten curves

witten

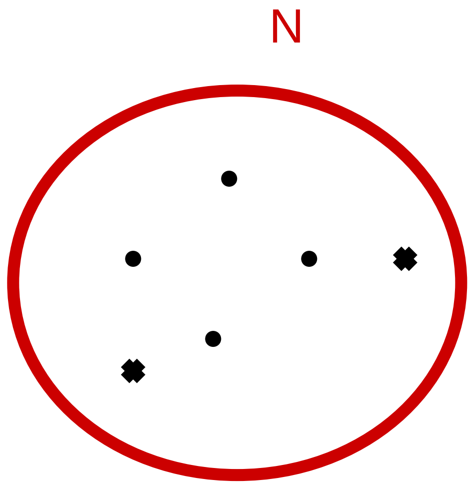
Brane construction



Quiver theory



Gaiotto picture

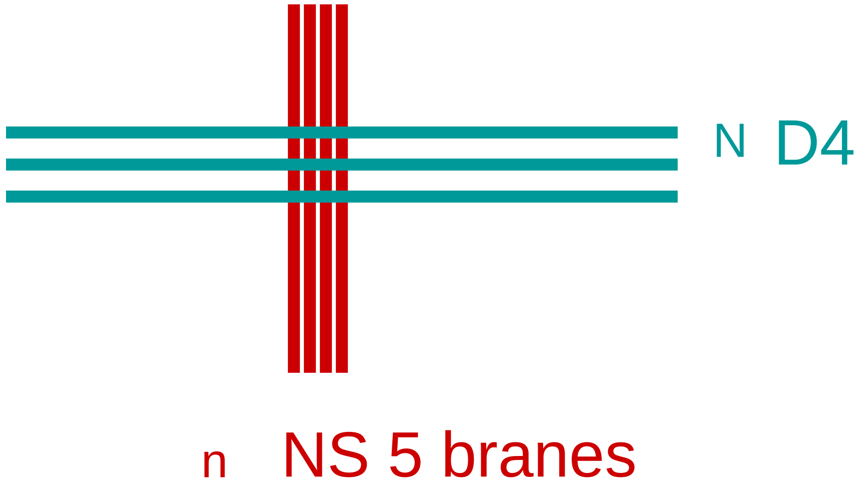


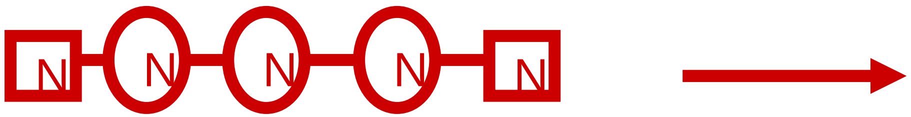
- We have N branes wrapping a Riemann surface
- There are different types of ``punctures''
- Simplest puncture: Transverse M5 inserted at a point on the Riemann surface.
- Others arise from ``collisions'' of elementary ones. We have P(N) different types.
- One field theory check (inspiration) is the study of the Seiberg-Witten curve based on the M-theory fivebrane picture.

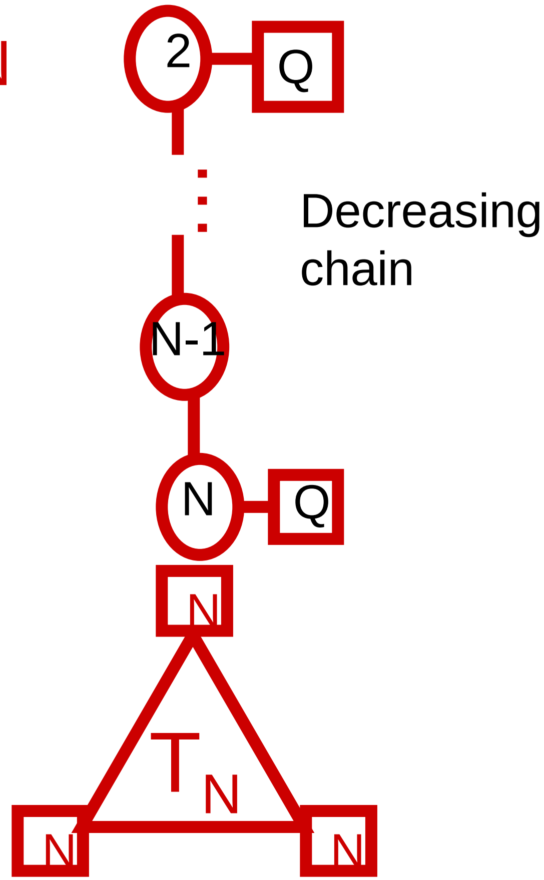
witten

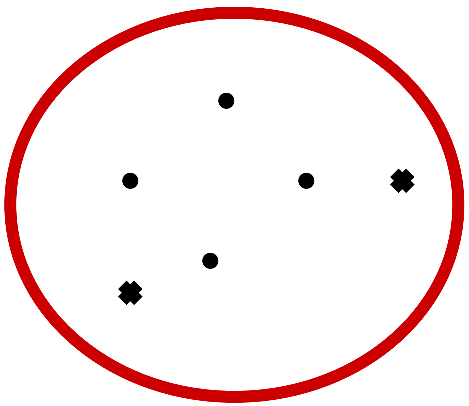
- Weak coupling limits → Degeneration of the surface

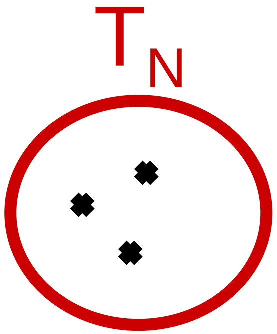
A special theory: T_N











- Theory with no coupling constant
- Has three SU(N) global symmetries.
- Sphere with three SU(N) punctures.

T_N

Computing a and c:

N=2,3 are special

Free theory

$$c = \frac{2n_v + n_h}{12} \ , \qquad 24(a-c) = n_v - n_h$$

Take original quiver → subtract "tail"

$$n_h^{T_N} = \frac{2N^3}{3} - \frac{2N}{3} \ , \qquad n_v^{T_N} - n_h^{T_N} = -\frac{3N^2}{2} + \frac{2N}{3} - \frac{N}{6} + 1$$

↑ leading

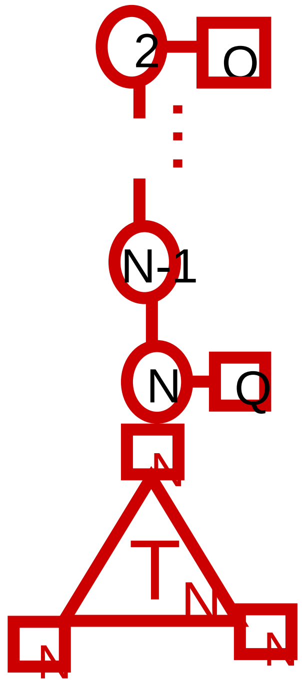
subleading

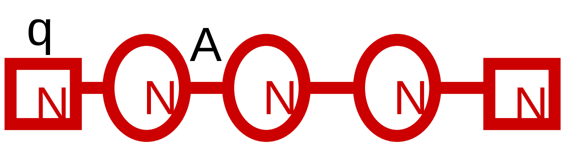
Anomalies of each SU(N) current: Same as N hypers of SU(N).

BPS states

- States carrying $U(1)_R$ charge → related to the parameters of the Coulomb branch.

- States with $SU(2)_R$ charge:
  - Global symmetries $\Delta = 2$, adjoint of SU(N)
  - Heavy state $\Delta = N-1$, fundamental of each SU(N) , (N,N,N) of SU(N)^3





$$q_i A \cdots A q_j = H_{ij} \rightarrow O_{ijk} Q^k$$

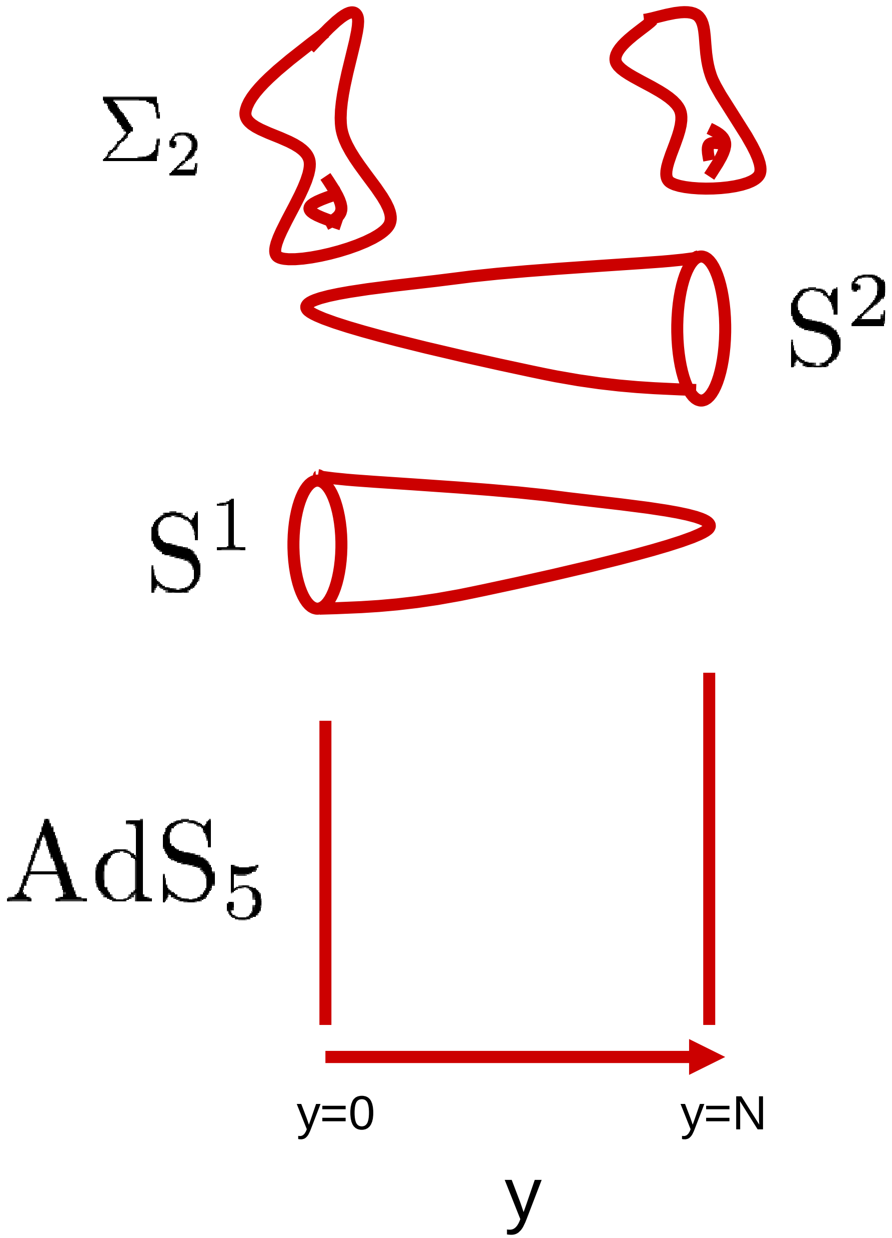
$$\Delta = N \rightarrow \Delta = (N-1) + 1$$

Geometries

- Consider this for large N
- Find the corresponding geometries

- Geometries with 4d N=2 superconformal symmetry were classified by Lin-Lunin-JM.

- The geometries involve solving a certain equation on a three dimensional space.



$D(y, x_1, x_2)$

$$(\partial_1^2 + \partial_2^2)D + \partial_y^2 e^D = 0$$

Toda eqn.

Bosonic symmetries

SO(2,4) x SU(2) x U(1)

- Precise boundary conditions ?

All functions in the geometry determined by D . S^1 non-trivially fibered.

Surface with no punctures

$$\Sigma_2 = H_2/\Gamma$$

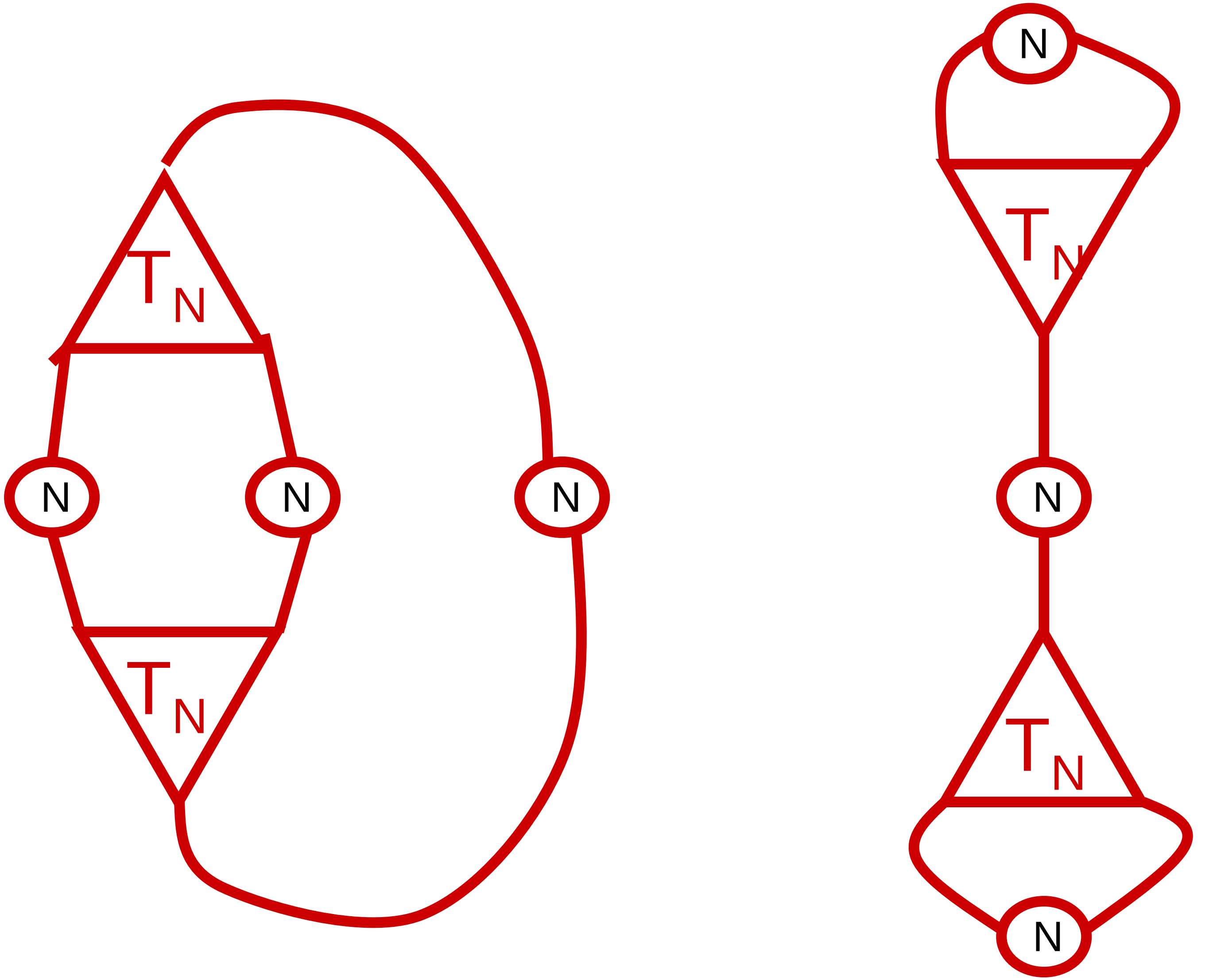
constant curvature Riemann surface

- Start with the M5 on this surface and flow to the IR.

JM & Nuñez

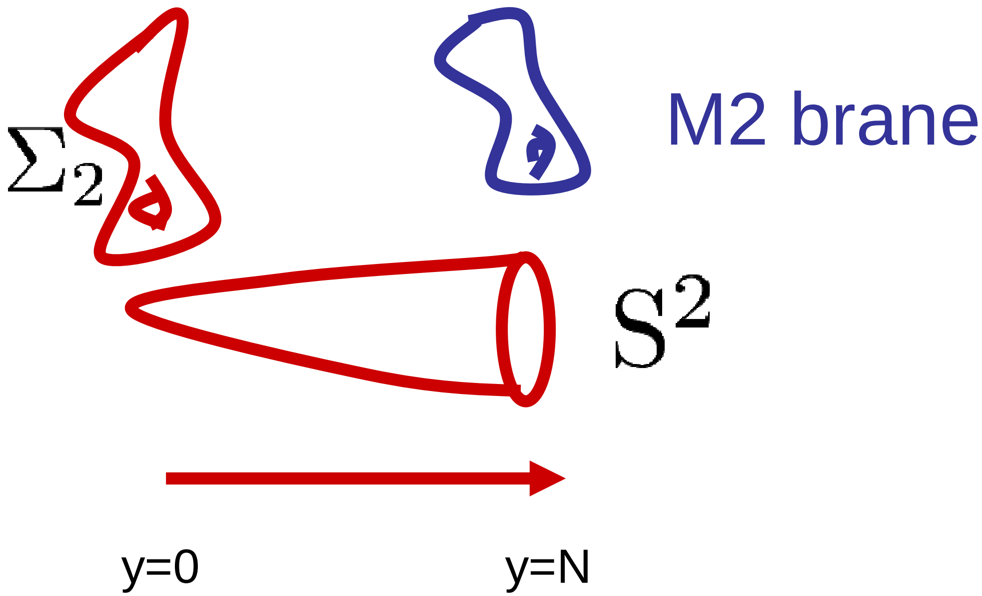
$$c = a = \frac{N^3}{3}(g-1) \qquad g \geq 2$$

3 (g-1) coupling constants (parameters in Γ)



genus g= 2 , 3(g-1) = 3

BPS states



$$O_{ijk}O^{ijk}$$ (for g =2)

Conformal weight determined the the $SU(2)_R$ charge

$$\Delta = 2(N-1)(g-1) = 2N(g-1) - 2g + 2$$

Flux on Σ_2 x S^2

fermion zero modes

Central Charge

- Add the ones from T_N plus the vector multiplets

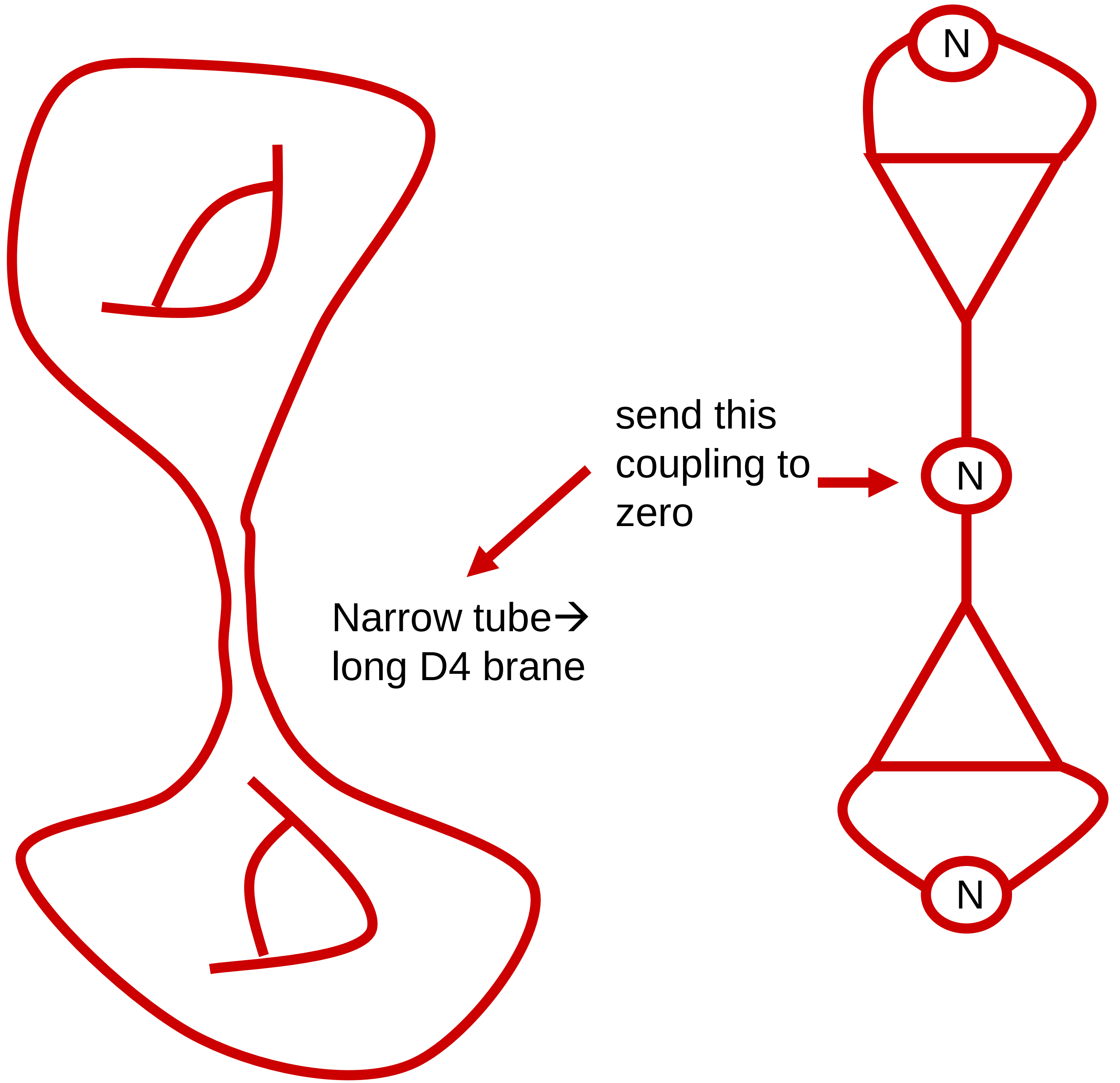
$$c \sim (c_1 N^3 + c_2 N + c_3)(g-1) \ , \qquad a - c \sim (N-1)(g-1)$$

Classical gravity

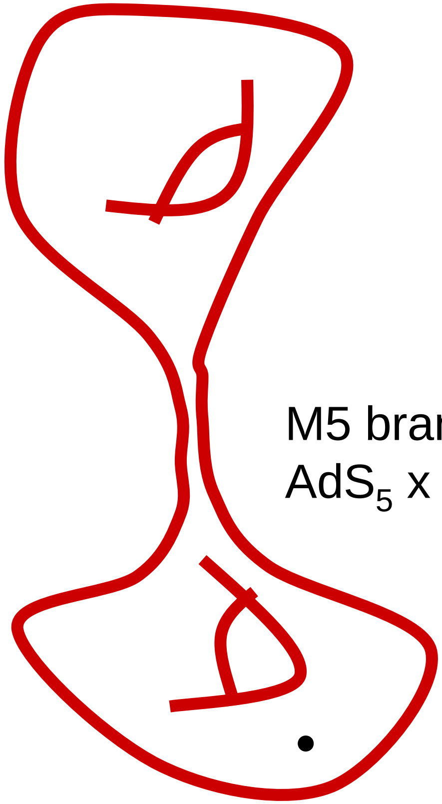
R⁴ correction

Decoupled U(1) CM

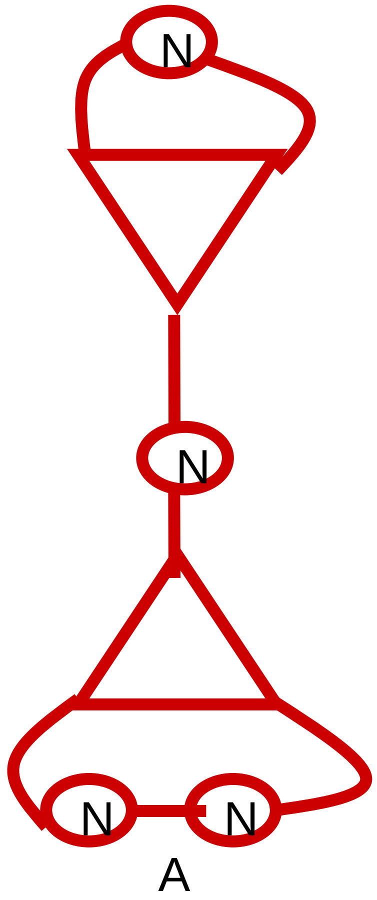
Precise match

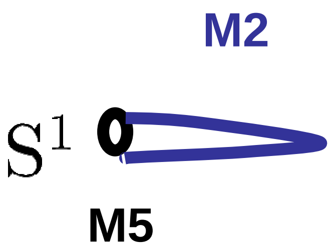

N
send this
coupling to
zero
N
Narrow tube→
long D4 brane
N

Adding a puncture



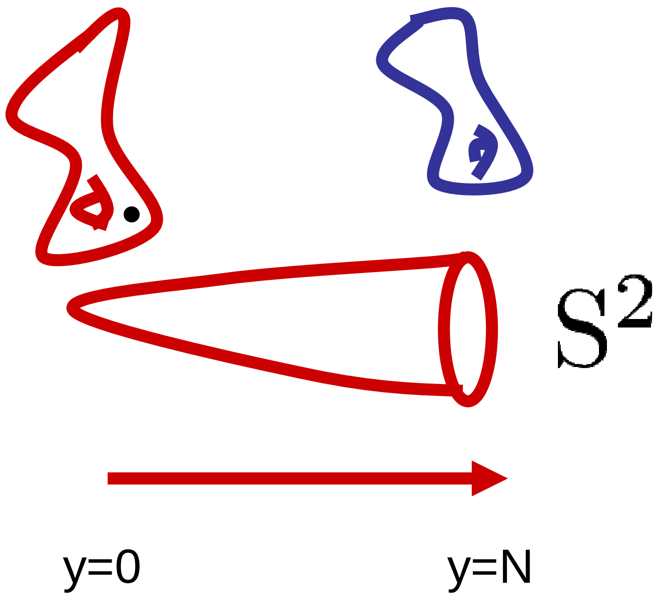
M5 brane wrapped on
$AdS_5 \times S^1$ and at y=0





M2 brane ending on M5 brane ← Det(A)

M5 two form field → vector field in AdS ← U(1) global symmetry



$$\mathrm{O}_{ijk} O^{ijm} A^k_m$$

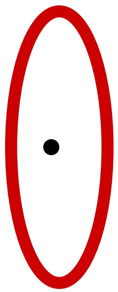
M2 → Bigger dimension $\Delta = 2(N-1)(g-1) + K$

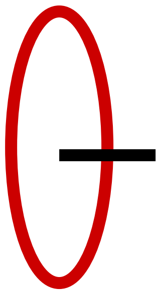
Number of punctures
Extra flux

Full non-linear solution near the puncture

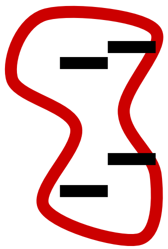
$AdS_7 \times S^4$ because $R^6 \rightarrow AdS_5 \times S^1$

H^2





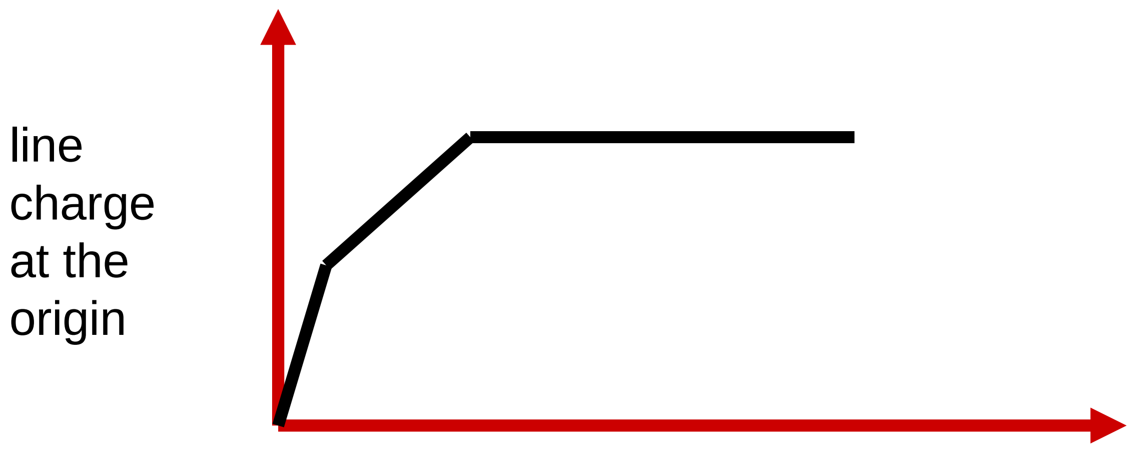
Line source for Toda equation



Solving the Toda equation
we get a smooth solution,
in principle.

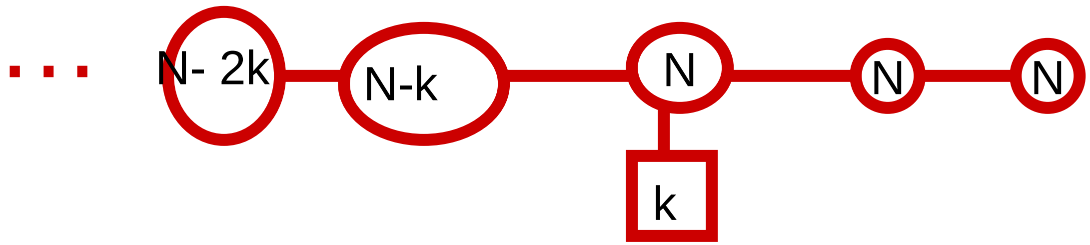
General punctures

- U(1) symmetry around the puncture
- Toda simplifies to a simple Laplace equation in 3d with U(1) symmetry
- Solution specified by a line charge along the axis of symmetry.



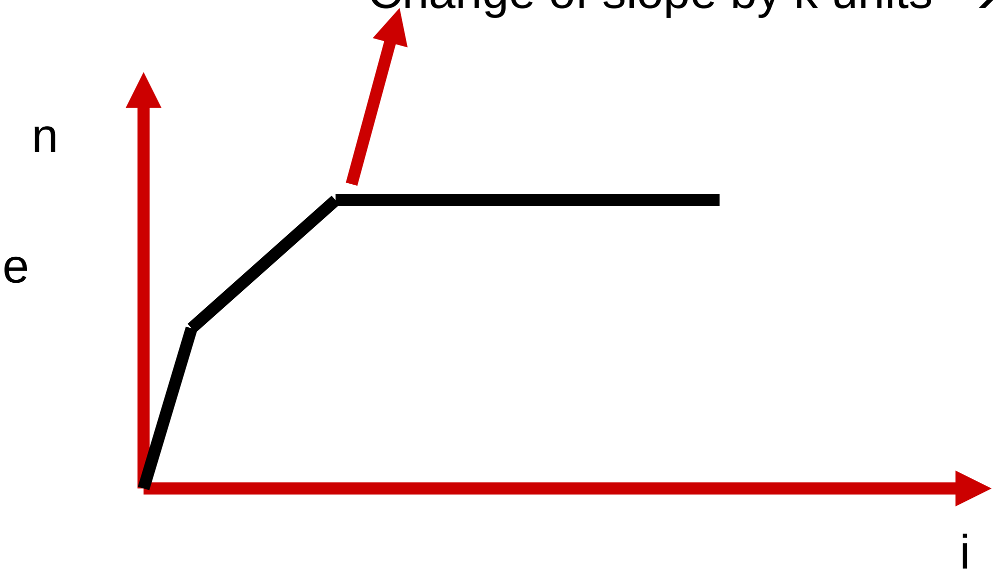
Lines given by a list of integers n_i

In correspondence with ways to end a quiver



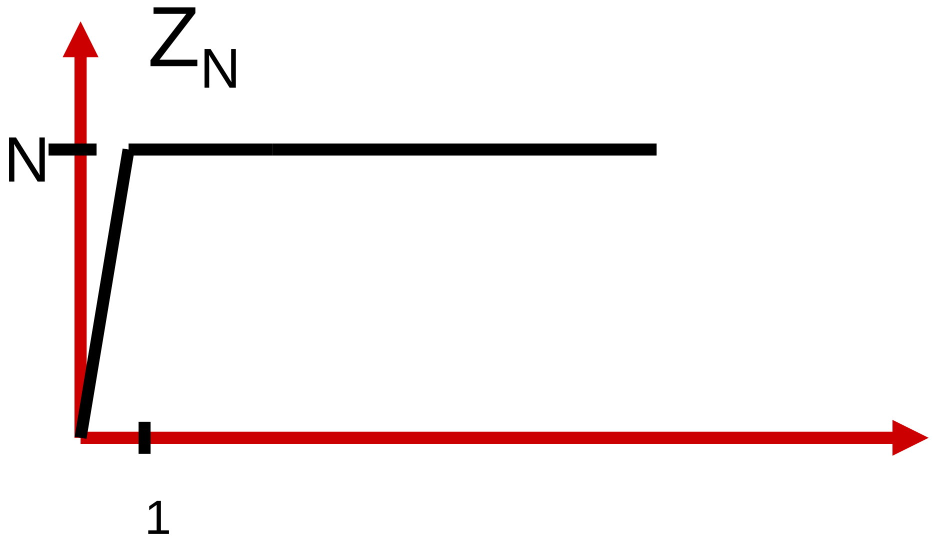
Change of slope by k units → Z_k singularity

rank
of gauge
group



wrapped on
$AdS_5 \times S^2$

volume of sphere→
anomaly of SU(k)



Maximal puncture → SU(N) symmetry

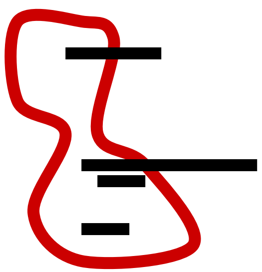
Locally: H^2/Z_N

T_N → Hyperbolic triangular domain with three Z_N singularities.

$$\mathrm{H}^2/\Gamma$$

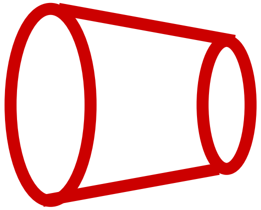
Full Solutions

- Choose N
- Give a Riemann surface
- Specify the different types of punctures and their locations.
- Solve the Toda equation with this data
- Obtain a smooth solution, up to Z_k singularities at some of the punctures.

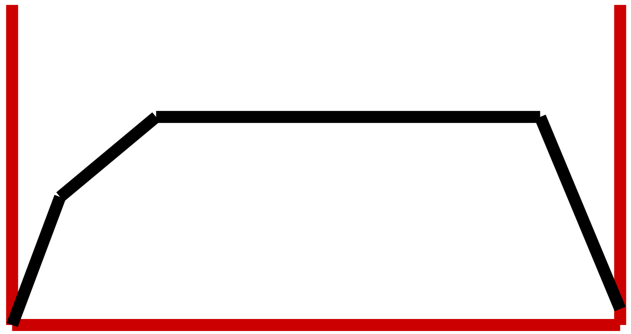


- We do not need to find the full solution to compute the spectrum of BPS states.
- We can read off the topology and the fluxes without knowing the full solution.

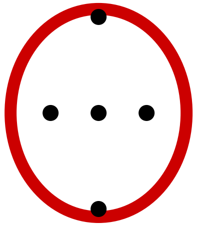
Sphere



Size ~ - 2 N + K → K > 2 N

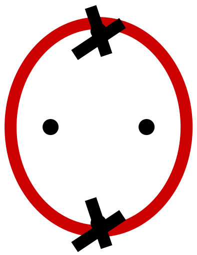


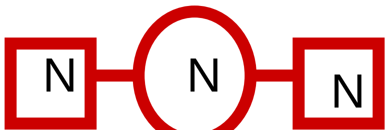
Approximate
U(1) symmetry



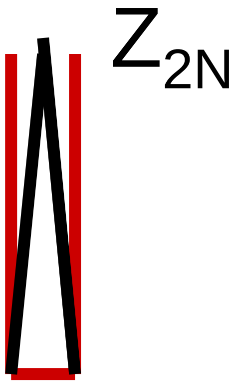
Extreme
case

Geometry
smooth but
highly curved





SU(N) with 2N flavors



Torus

Kachru-Silverstein

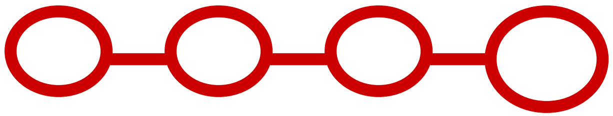
- $AdS_5 \times S^5 / Z_k$
- T-dualize and lift to M-theory
- Gives a 2 torus with smeared punctures
- Localize punctures→ get a smooth solution

Classification of conformal SU(N) quiver theories

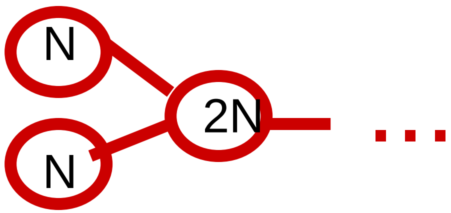
Classification in terms of A D E dynkin diagrams.

Gaiotto Witten

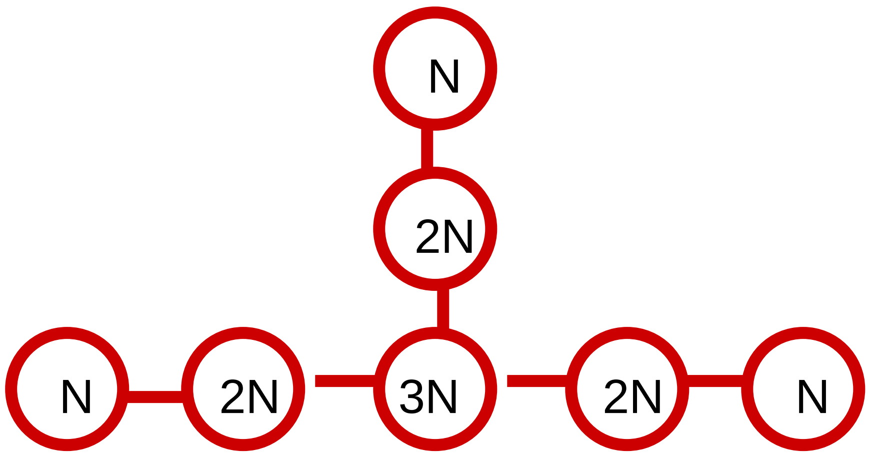
A



D



E



Conclusions

- We discussed a large class of N=2 superconformal field theories
- We found a precise correspondence with the gravity dual
- We gave the prescription for how to construct smooth gravity solutions for all of them. (but we did not find them explicitly)
- Even the case of SU(N) with 2N flavors appears as an extreme example
- The topology of the solution is specified by the choice of punctures and Riemann surface.

Future

Tachikawa
Benini, Benvenutti

- Extend to SO-Sp quivers
- Other N=2 theories not covered here. eg Argyres-Douglas theories
- Generalize to N=1 theories in 4d
- 3-d superconformal field theories.