The Banks-Zaks Phase

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UCSC Banks-Fischler Festival
June 2009

Everyone's first lesson in asymptotic freedom includes the equation

$$b_0 = 11 - \frac{2}{3} n_f$$

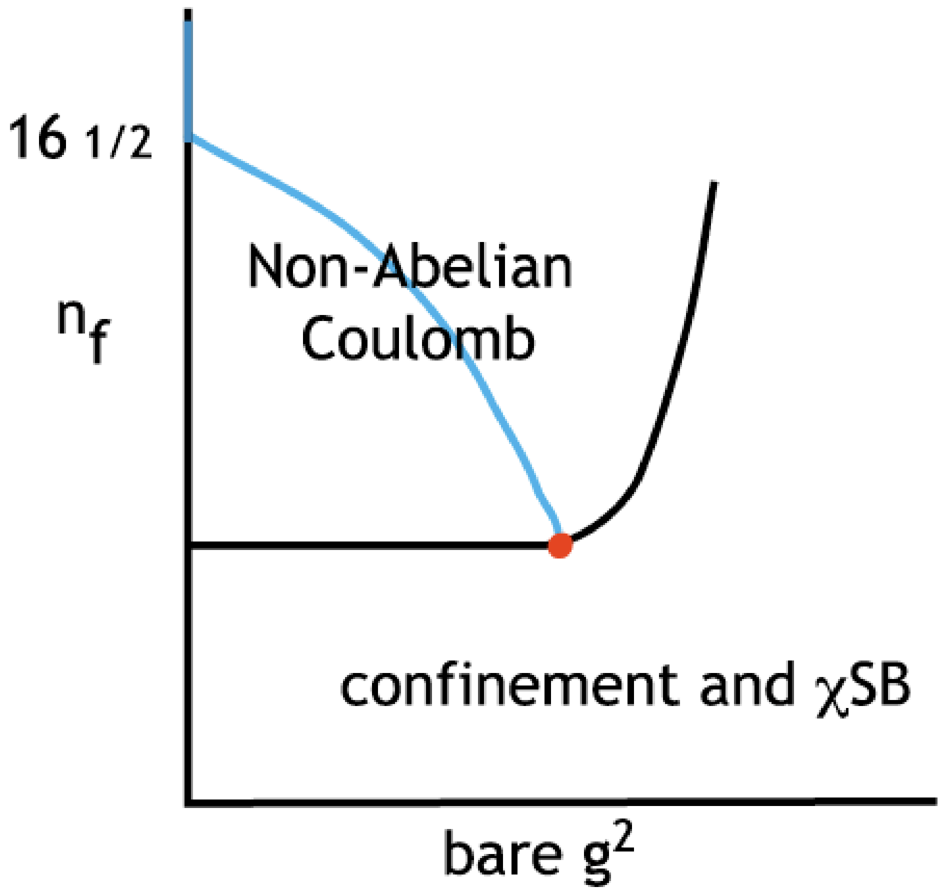
which implies that asymptotic freedom disappears at

$$n_f = 16\frac{1}{2}$$

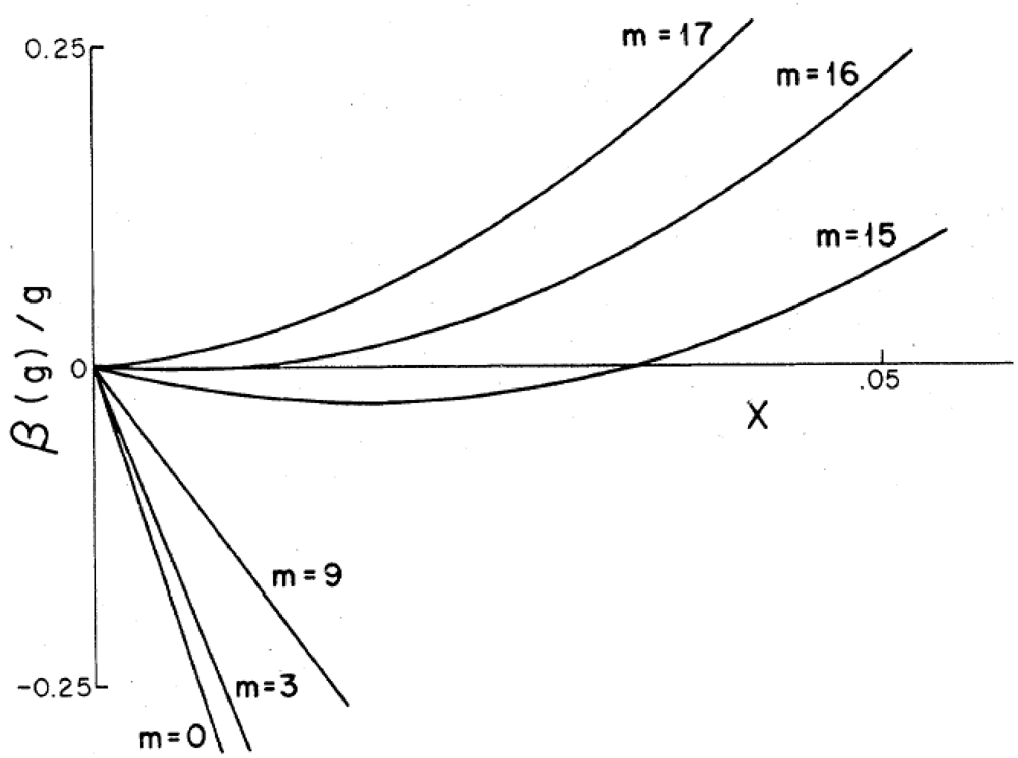
When I learn this subject, this was a curiosity which obviously had no reasonable application in physics.

But there is real physics in the region of large n_f. Here is a sketch of the phase diagram. David Kaplan will describe the critical endpoint this afternoon.

The blue line shows a line of IR attractive fixed points. Where they are at finite coupling, these are the 'Banks-Zaks fixed points'.



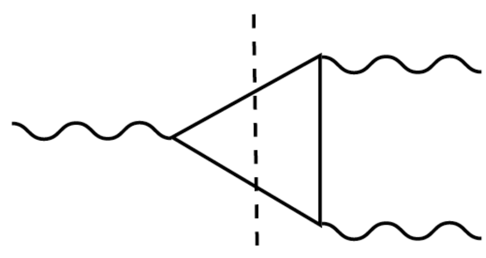
The Banks-Zaks fixed points were discovered by William Caswell. Along with Belavin and Jones, he was the first to compute the 2-loop beta function of QCD *. Here is a figure from his 1974 paper:



* Tom Banks was in the race with a clever method that did not succeed in time.

This fixed point was of little interest in 1974, but it 1980 it was another story. Everyone was talking about 'Massless Composite Fermions'.

Tom Banks had just written an amazing paper with Frishman, Schwimmer and Yankielowicz that discovered the result that the massless fermion triangle has a delta function singularity at $q^2 = 0$



$$\sim \delta(q^2)$$

and used this to give a rigorous proof of 't Hooft's new anomaly matching condition.

So it was quite remarkable to note that, near 16.5 flavors, there is a zero of the QCD beta function. Banks and Zaks argued that this behavioir was real and not an artifact by setting up a well-defined perturbative expansion in

$$\epsilon = 33/2 - n_f$$

In this expansion,

$$\frac{g_*^2}{(4\pi)^2} = \frac{2\epsilon}{321} + \cdots$$

Banks and Zaks argued that this fixed point represented a new phase of QCD, similar in some respects to the zero coupling region, but also having conformal invariance and nontrivial scaling as a consequence of the fixed point.

The story of this new phase of QCD languished for a long time.

Three developments of the 1990's brought it back.

You are very familiar with the first of these:

In 1994, Seiberg discovered his duality of supersymmetric Yang-Mills theory. This included a nontrivial solution of the 't Hooft anomaly matching conditions based on the massless fermions in the multiplets of a dual gauge theory:

$$q \qquad \overline{q} \qquad T \qquad \lambda$$

This construction gave a conformally-invariant theory with nontrivial scaling dimensions in the region (for SU(3)):

$$\frac{9}{2} = \frac{3}{2}N_c < n_f < 3N_c = 9$$

The second was the setback for technicolor models that came from the interpretation of the precision electroweak experiments.

It was already known from Holdom's work that a realistic technicolor model required large anomalous dimensions to suppress dangerous flavor-changing operators.

The precision electroweak results gave another challenge, that

$$S < 0.1$$

where S is a quantity that, in a technicolor theory, can be extracted from the spectrum of vector and axial vector mesons.

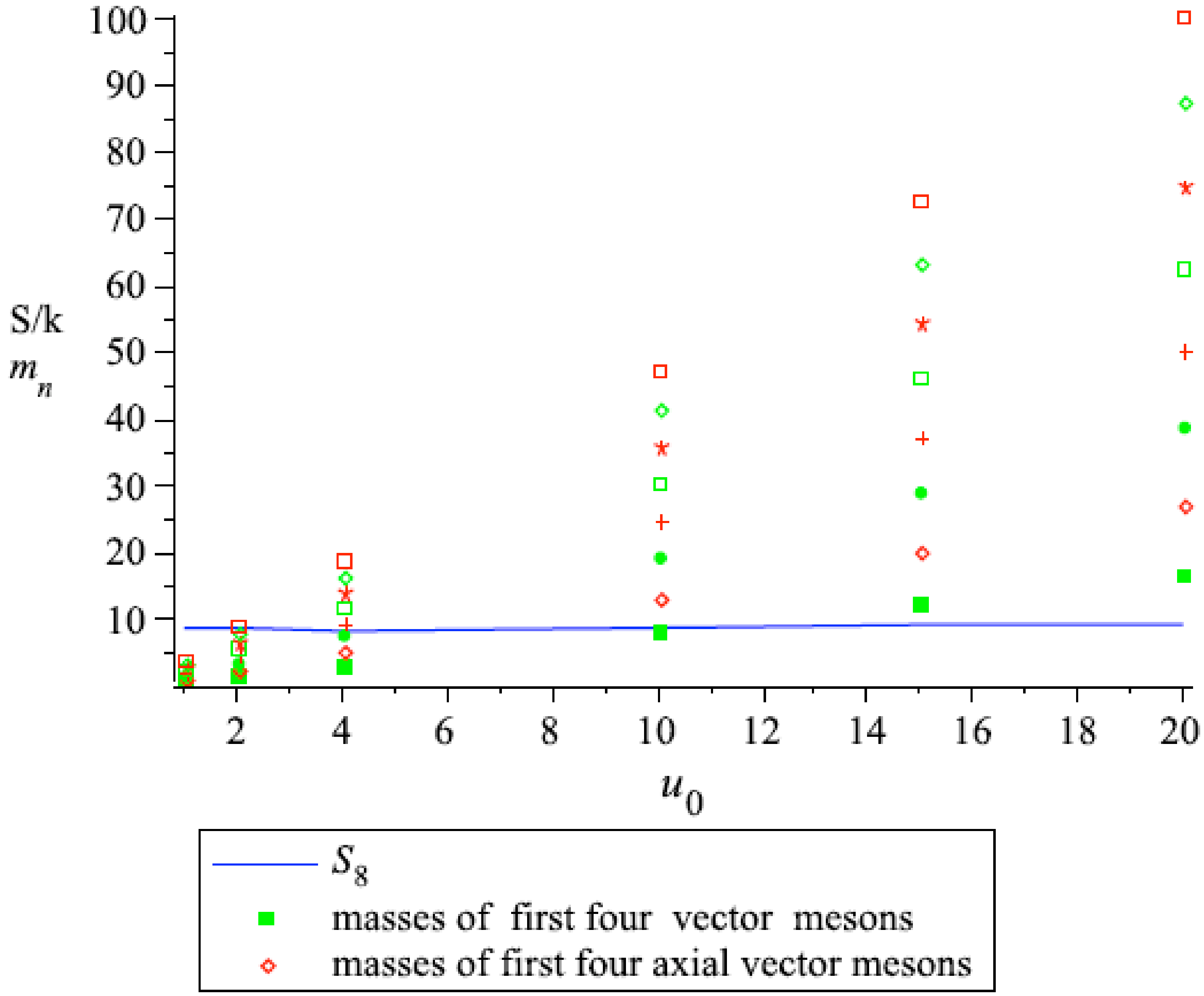
$$S = \frac{1}{3\pi} \int \frac{ds}{s} [R_V(s) - R_A(s)]$$

In QCD, this sum rule is dominated by the ρ meson and gives much too large a value.

But, consider building a technicolor theory by starting with a model whose parameters put it into the conformally-invariant phase and softly breaking conformal invariance.

It is very tempting to say that changing the short-distance behavior of the theory can change the spectrum of vector and axial vector mesons in such a way as to give a more complete cancellation in S.

There is some evidence for this from AdS/CFT duality. For example, from a recent paper of Mintakevich and Sonnenschein,



Finally, in 1992, Iwasaki, Kanaya, Sakai, and Yoshie, in lattice gauge theory studies of SU(3) Yang-Mills theory mainly directed at the finite-temperature phase transition, claimed qualitatively different behavior for the cases $n_f = 4$ and $n_f = 6$.

This led them to claim that the transition to conformal behavior extended all the way down to $n_f = 6$.

These studies were done on very small lattices,

$$8^2 \times 10 \times N_t \qquad N_t = 4,\ 8$$

and with Wilson fermions, for which the order parameter $\overline{\psi}\psi$ must be carefully subtracted.

In a moment, I will review more recent lattice gauge theory attempts to locate the bottom of the conformal regime.

Stimulated by Seiberg's result, many authors tried to estimate the lower boundary of the 'conformal window' in non-supersymmetric QCD.

Gardi and Karliner applied a variety of aggressive resummation methods to the known 3 terms of the QCD beta function. They estimated $n_{cw} \sim 12$.

Miransky and Yamawaki made an estimate based on the idea that, when the value of the Banks-Zaks fixed point coupling increases, it eventually reaches the critical coupling for chiral symmetry breaking *. This gave

$$n_{cw} = N_c \left(\frac{100 N_c^2 - 66}{25 N_c^2 - 15} \right) = 11.9$$

* A concept best explained in the classic paper of Banks and Raby.

Ryttov and Sannino analyzed the problem by assuming that the beta function of nonsupersymmetric QCD takes an exact form similar to that of the NSVZ exact beta function of supersymmetric theories:

$$\beta(g) \;=\; -\frac{g^3}{(4\pi)^2}\frac{\beta_0 - \frac{2}{3}\,T(r)\,N_f\,\gamma(g^2)}{1-\frac{g^2}{8\pi^2}C_2(G)\left(1+\frac{2\beta_0'}{\beta_0}\right)}$$

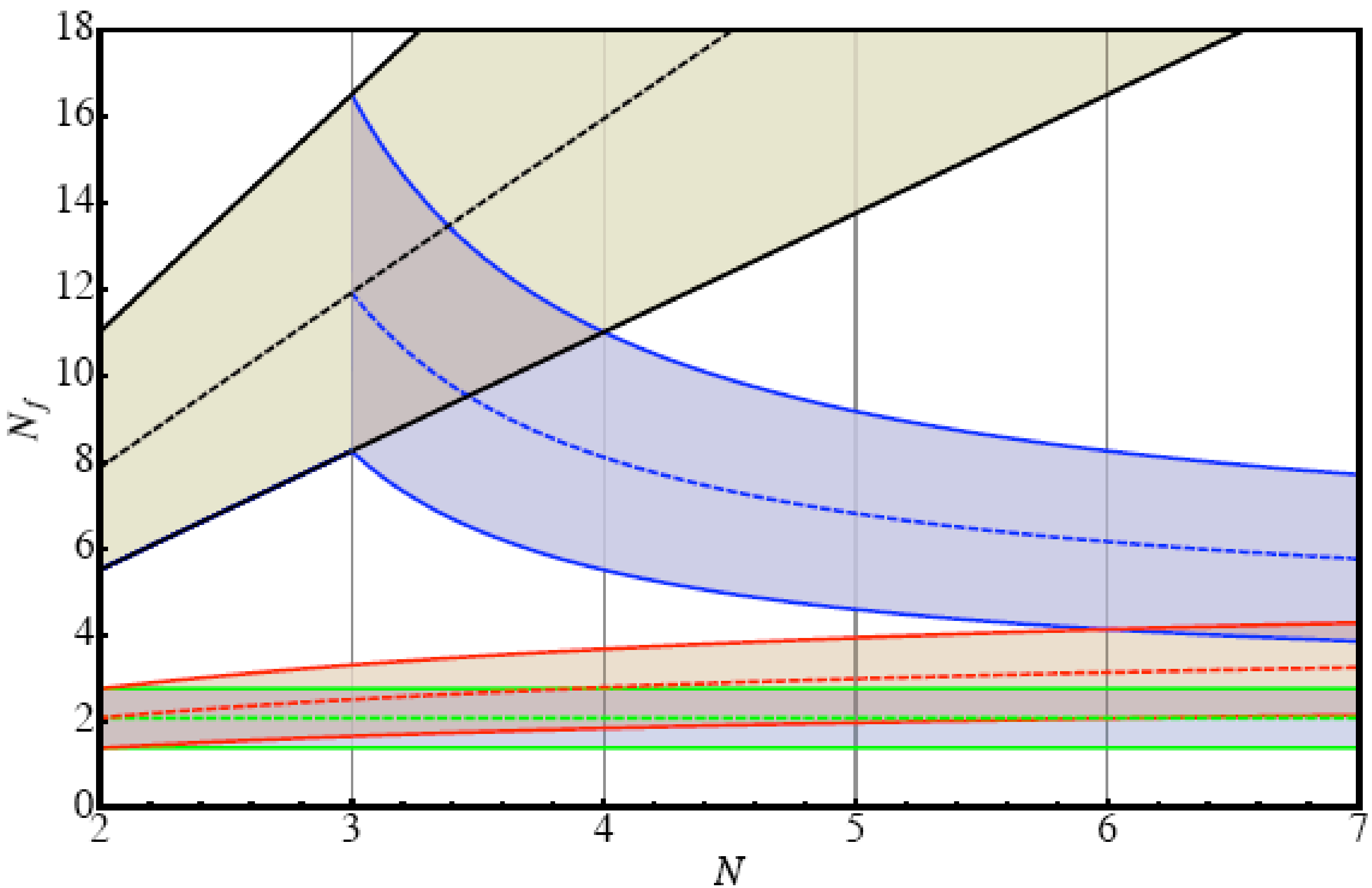
This formula does predict a conformal window. To match the known QCD beta function, we find a function for γ which, at the fixed point, becomes

$$\gamma = \frac{33-2n_f}{n_f}$$

Now imposing $d(\overline{\psi}\psi) = 3-\gamma \;>\; 1$, we find the lower boundary of the conformal window at

$$n_{cw} = \frac{33}{4} = 8.25$$

The method of Ryttov and Sannino allows estimation of the conformal window for other representations of SU(N) -- in the same handwaving way.



Finally, there is an interesting proposal of Appelquist, Cohen, and Schmaltz. These authors suggest that we should simply make a direct comparison of the IR and UV degrees of freedom, defined by

$$f_{IR} = -\lim_{T\to 0} F/(\pi^2 T^4/90) \qquad f_{UV} = -\lim_{T\to\infty} F/(\pi^2 T^4/90)$$

and insist that $f_{IR} \leq f_{UV}$

This comparison can enable us to rule out the hypothesis that we have simple quark confinement and chiral symmetry breaking. In that case, the IR degrees of freedom would be the QCD Goldstone bosons. The number of this grows like n_f^2 as n_f increases.

For supersymmetric QCD, this gives $n_{cw} = \frac{3}{2}N_c$ in agreement with Seiberg.

For nonsupersymmetric QCD, $n_{cw} = 4N_c = 12$

Can we obtain a more reliable estimate of the position of the Banks-Zaks phase ?

In the remainder of this lecture, I will review two very recent attempts to do this using lattice gauge theory.

First, I will review some results of Deuzeman, Lombardo, and Pallente (DLP). They have been trying for many years to establish a qualitative change in the physics of QCD as a function of n_f.

In principle, one can simulate QCD with n_f flavors, vary the bare coupling g, and look for a phase transition as a function of g. However, there are (at least) three difficulties.

FIrst, there is no straightforward way to vary n_f continuously. The order parameter $\overline{\psi}\psi$ is an important indicator of the properties of the phase transition. To make a zero value of $\overline{\psi}\psi$ meaningful, we should start from a theory with at least a discrete chiral symmetry. Thus, use we want to use the Kogut-Susskind fermion prescription. This prescription has fermion doubling, so that each Kogut-Susskind fermion corresponds in the continuum to 4 quark flavors.

Thus, I will show results for $n_f = 8$ and $n_f = 12$. Hopefully, the results will be different.

Second, it is not actually possible to work at zero quark mass. We have to take the quark masses finite, and even order 1 with respect to the real hadron masses, and then extrapolate to zero mass.

Finally, a lattice that can be simulated on a computer must be finite, both in the space direction and in the (Euclidean) time direction. DLP work on a lattice of size

$$16^3 \times N_t \qquad N_t = 8,\ 16$$

In particular, the time direction is finite, so we are actually at finite temperature. The lattice spacing is related to the physical length (in fm) by the renormalization group. The relation depends on the bare coupling. Thus, the temperature of a fixed lattice changes as a function of the bare coupling.

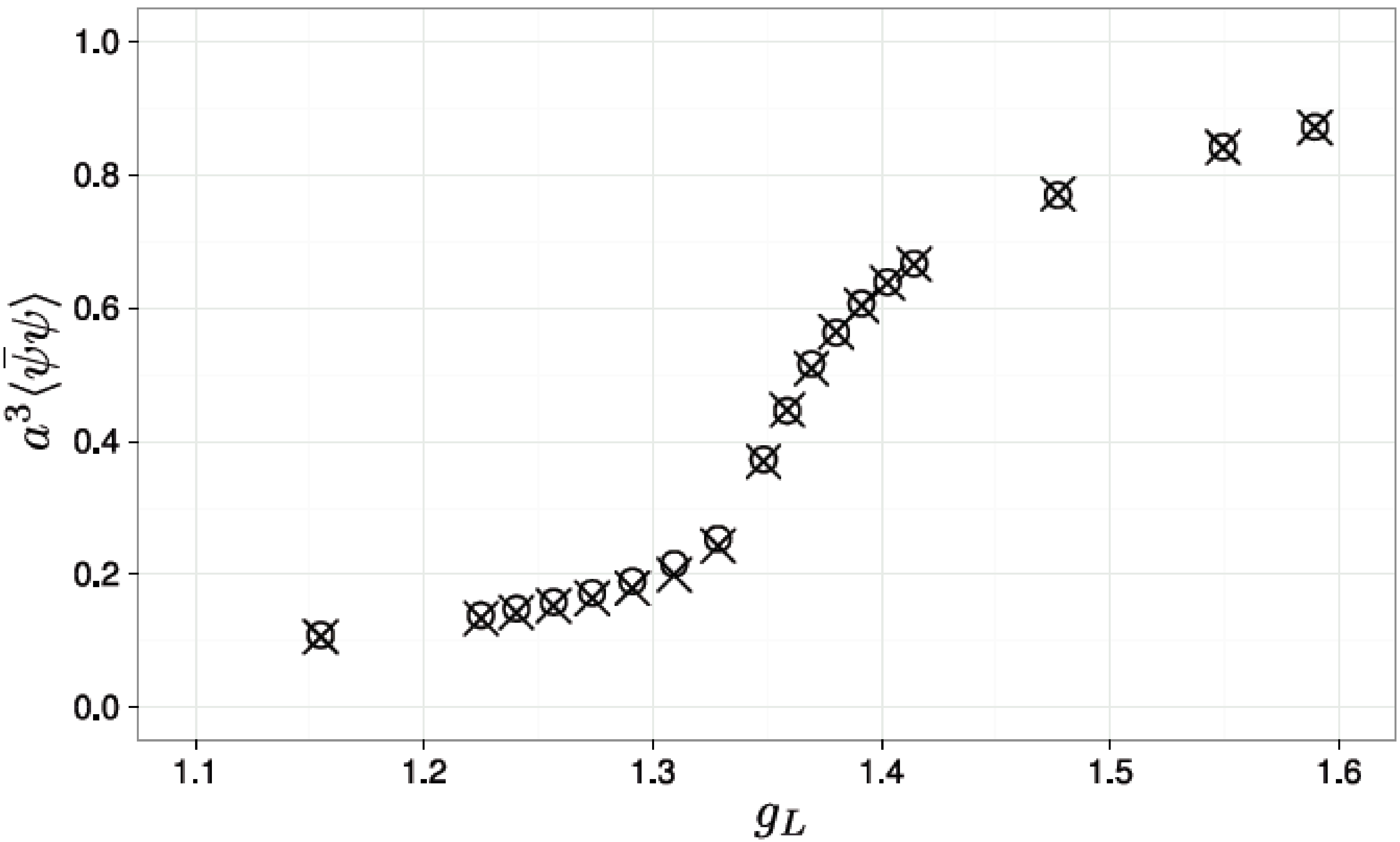
If we observe a phase transition as a function of g, this might be the deconfining phase transition that occurs at finite temperature.

At $n_f = 8$ DLP find first-order transitions at

$$1/g^2 = 4.11(1) \quad N_t = 6$$
$$1/g^2 = 4.34(4) \quad N_t = 12$$

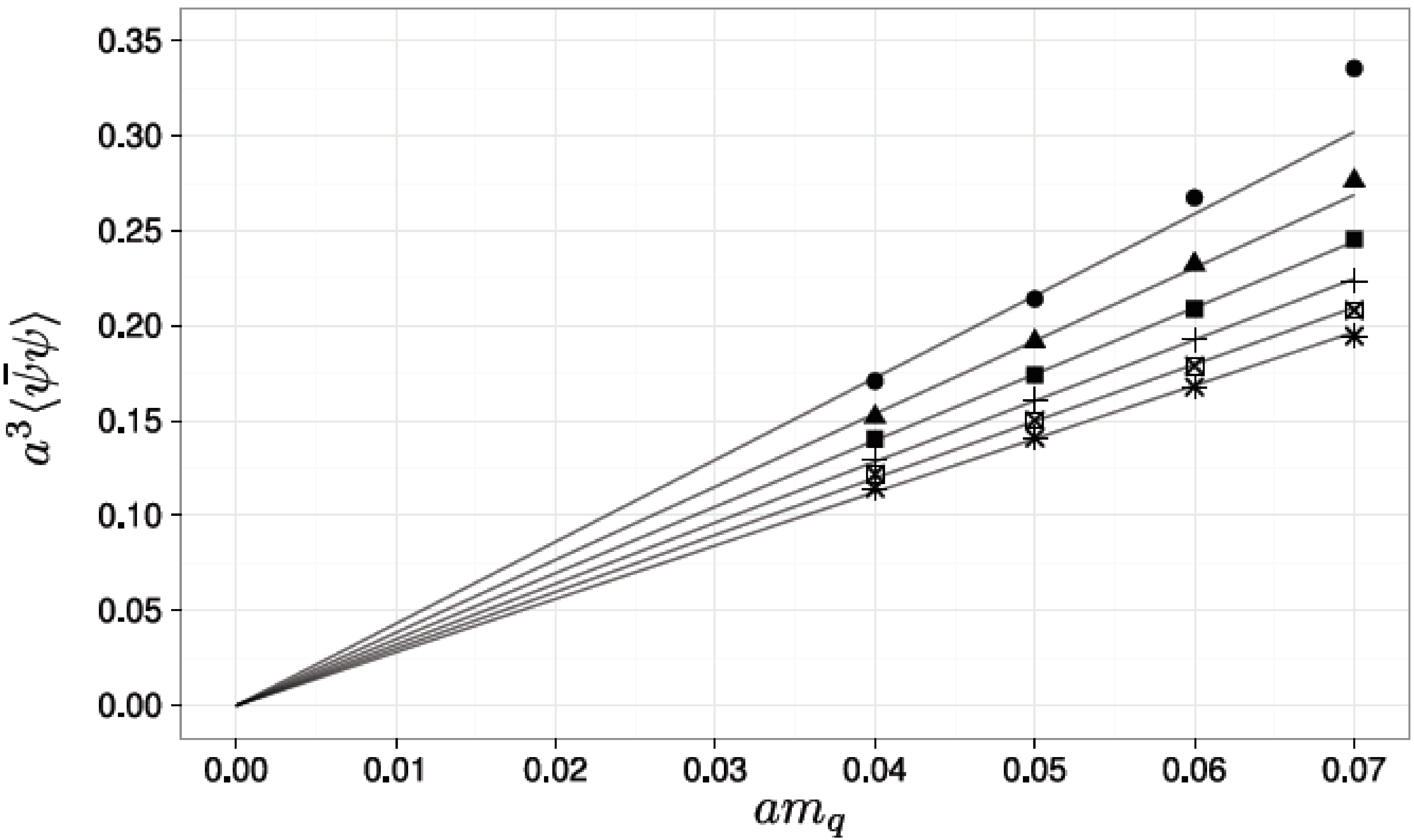
The (significant) difference is consistent with the renormalization group scaling of a critical temperature fixed in physical length.

What about $n_f = 12$? Here is the result:



$a^3\langle\bar{\psi}\psi\rangle$
1.0
0.8
0.6
0.4
0.2
0.0
1.1
1.2
1.3
1.4
1.5
1.6
g_L

The order parameter $\overline{\psi}\psi$ is observed to be nonzero below the transition, but these calculation are done at finite mass. Here is the extrapolation to zero mass:



A different set of measurements that give more direct access to the running of the coupling constant have been done (actually, somewhat earlier) by Appelquist, Fleming, and Neil (AFN).

These authors use a different setup, the 'Schrodinger Functional' formalism of Luscher, Sommer, Weisz, and Wolff. In this setup, we choose open boundary conditions in Euclidean time, both for the quarks and for the gauge fields.

AFN again use Kogut-Susskind fermions and so are limited to
$n_f = 4, 8, 12$

Heller has worked out a formalism for measuring the coupling constant renormalization in this framework.

Choose boundary conditions for the lattice gauge U variables to give a twist from $x^4 = 0$ to $x^4 = N_t$. The solution is a constant electric field through the system.

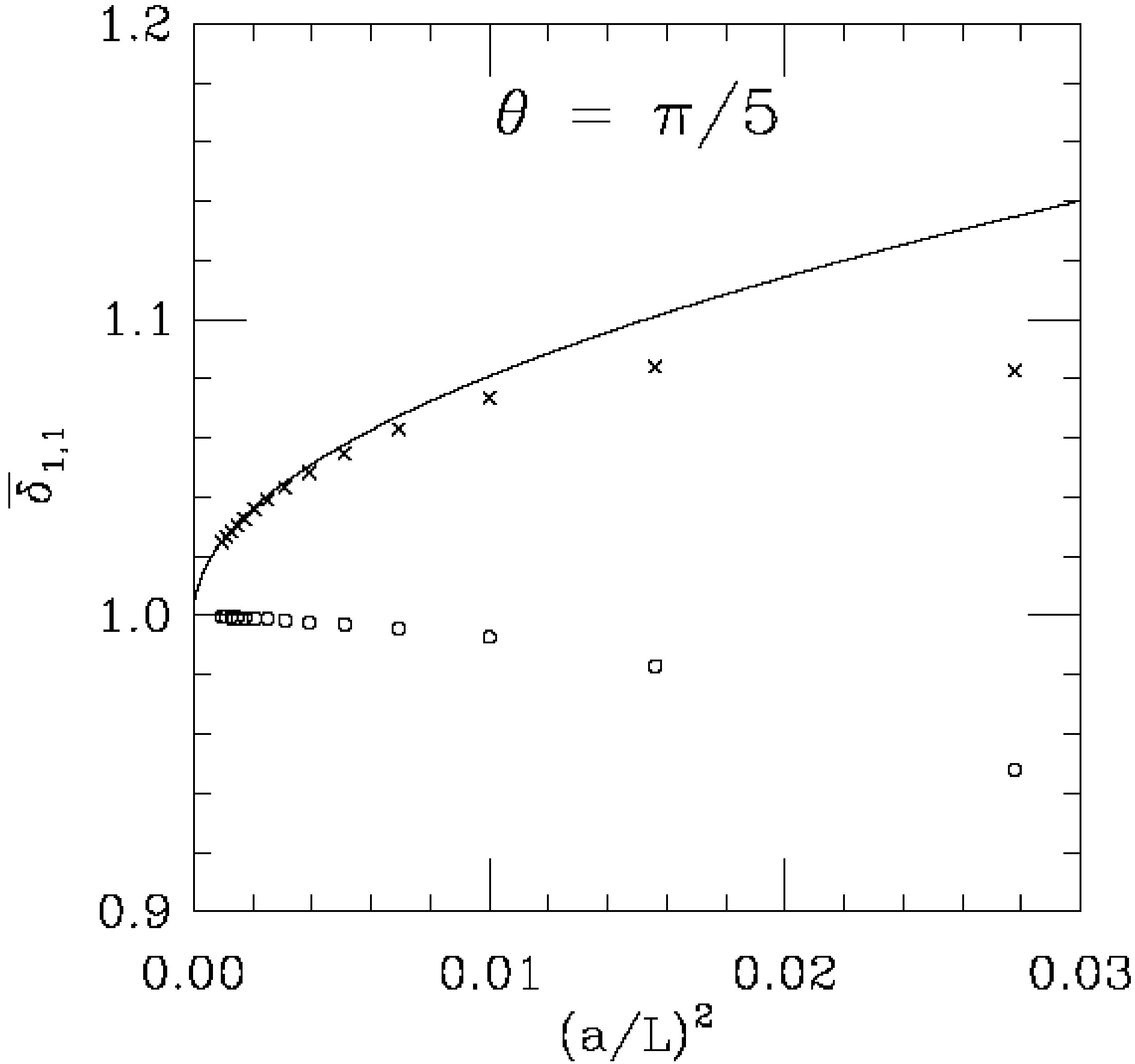
The value of the Schrodinger functional is then

$$\exp[-\frac{1}{2g^2(L)}E^2VT]$$

so we can read off $1/g^2(L)$ by taking derivatives (or differences) as a function of the twist:

$$k/g^2 = -\frac{\partial}{\partial\theta}\log Z$$

Lattice artifacts contribute to this observable. These must be understood and cancelled off.

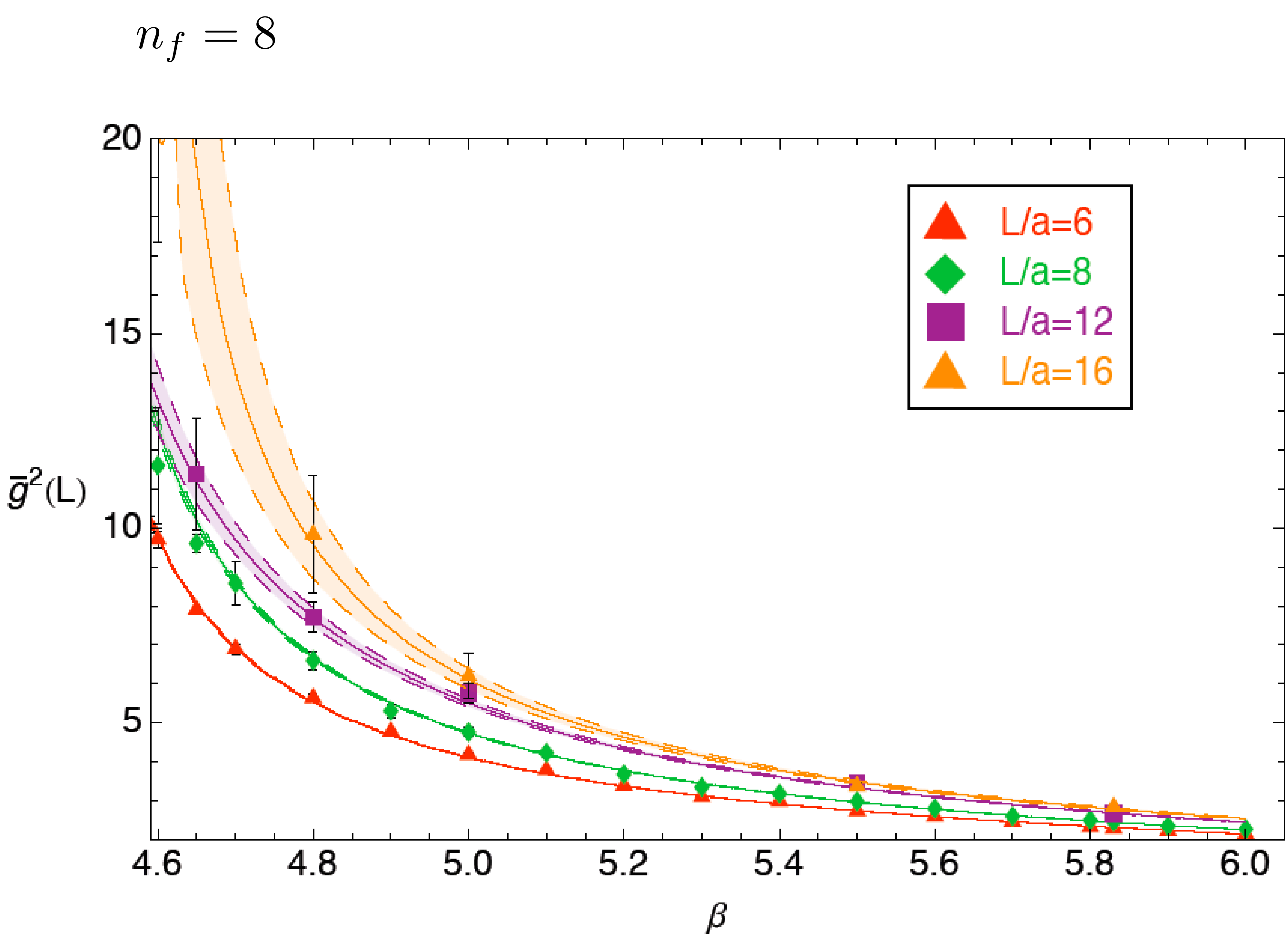


At $n_f = 16$, Heller found that the physical coupling decreases with increasing lattice size, as expected from the form of the beta function.

$g^2(L) =$

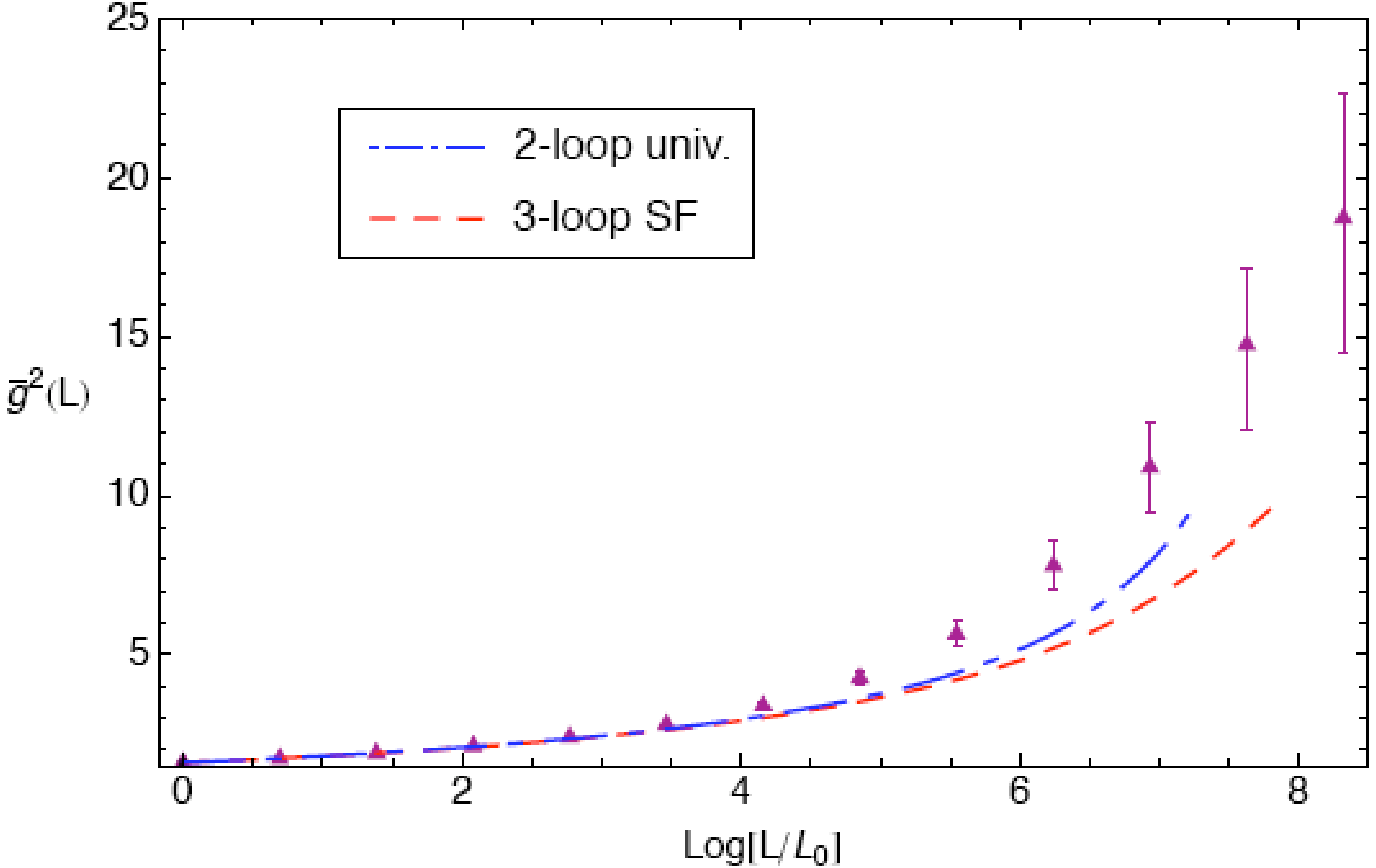
β	$L=4$	$L=6$	$L=8$	$L=12$
4.5	5.31(15)	3.50(16)	3.22(14)	2.87(12)
4.6	4.41(16)	2.97(5)	2.77(9)	
4.7	3.92(11)	2.78(4)		
4.8	3.30(6)	2.67(3)	2.37(3)	
4.9	3.07(7)	2.42(3)		
5.0	2.91(7)	2.28(3)		

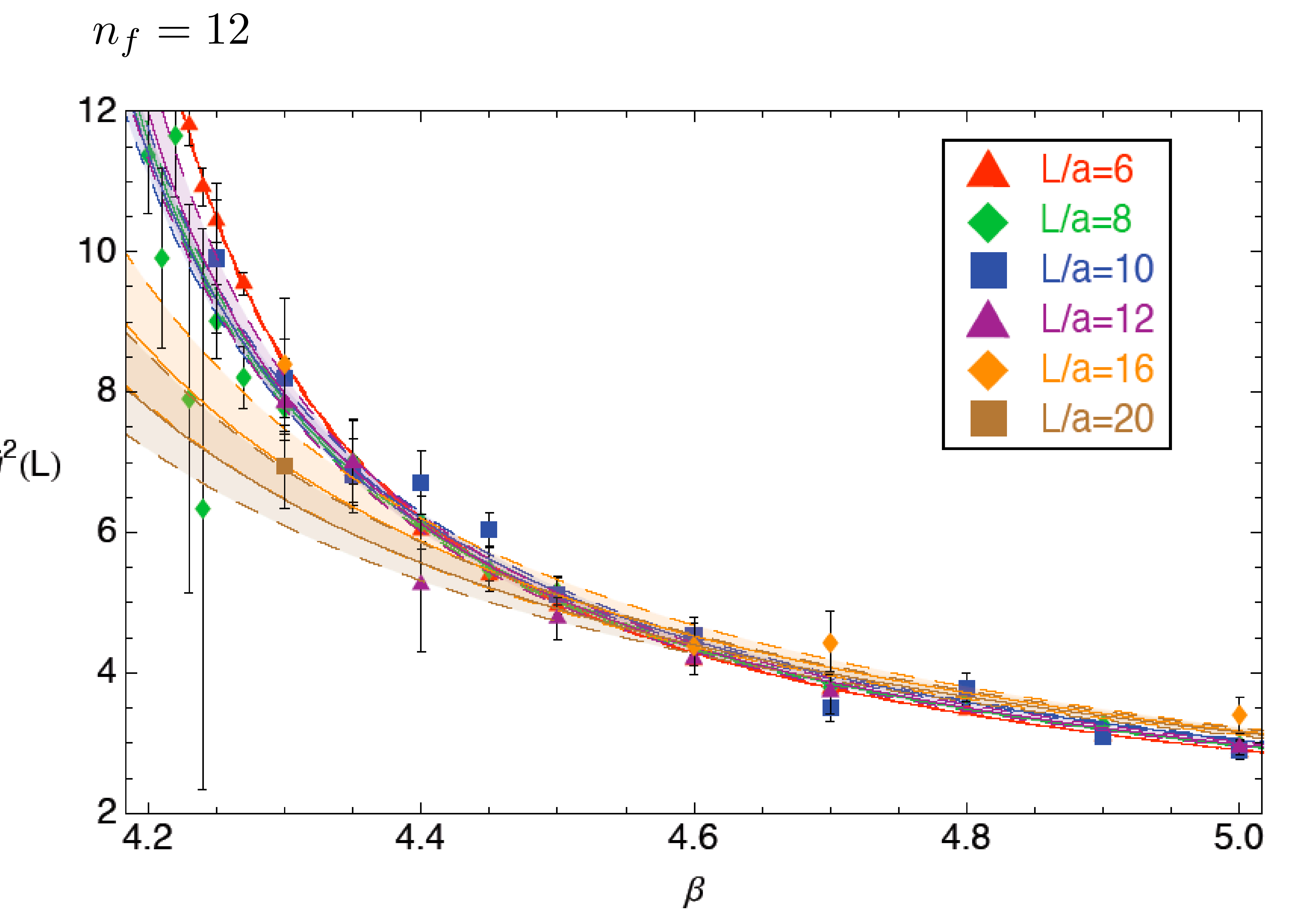
Appelquist, Fleming, and Neil applied this technology to the cases $n_f = 8, 12$. Here are some results.



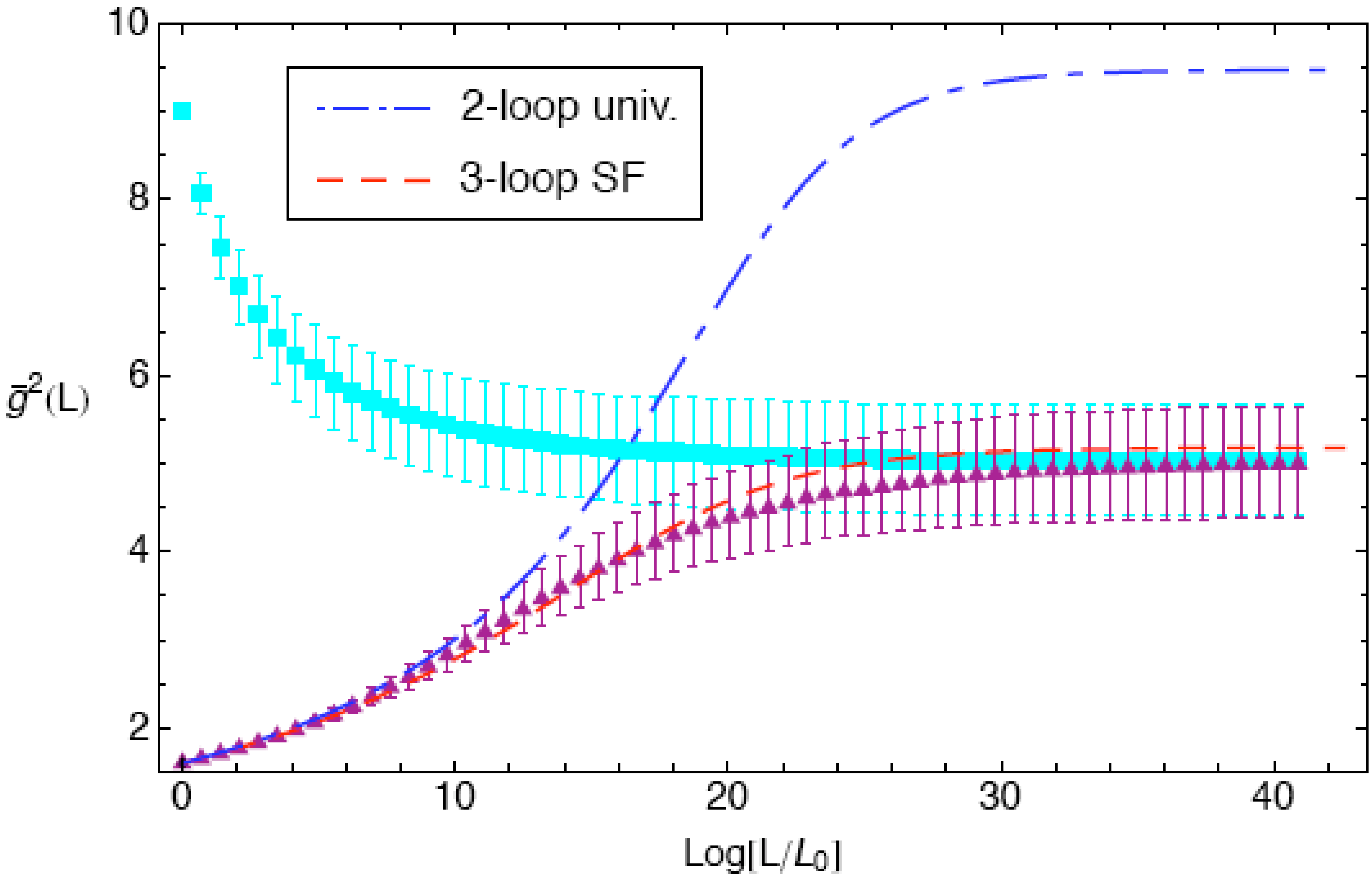
$n_f = 8$
20
15
10
5
$\bar{g}^2$(L)
4.6
4.8
5.0
5.2
5.4
5.6
5.8
6.0
β
L/a=6
L/a=8
L/a=12
L/a=16

$n_f = 8$




$n_f = 12$
12
10
8
6
4
2
g^2(L)
4.2
4.4
4.6
4.8
5.0
β
L/a=6
L/a=8
L/a=10
L/a=12
L/a=16
L/a=20

$n_f = 12$



Thus, there is now strong evidence from lattice gauge theory that the bottom of the Banks-Zaks phase in QCD occurs in the range $n_f = [8, 12]$, and probably not very close to either end.

The next challenge is to obtain higher resolution by understanding better the evaluation of fraction powers of the quark determinant.

Over his career, Tom Banks has introduced many fascinating ideas into quantum field theory and particle physics.

The Banks-Zaks phase diagram is only one example.

A way of measuring the fascination of these ideas is the amount of effort that many members of our community have put into exploring their consequences, and in making the original visions more precise.

Here, and in many other places, you have given us the spark !

Thank you, Tom !