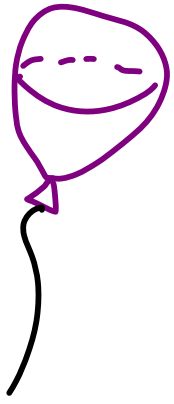




Toward 4d quantum gravity
in string theory

Work in Progress with Joe Polchinski

Banks/Fischler fest UCSC 6/15/09



Happy



$$\text{Log}_{10}\left(\frac{M_p^2}{\Lambda}\right) \text{ th}$$

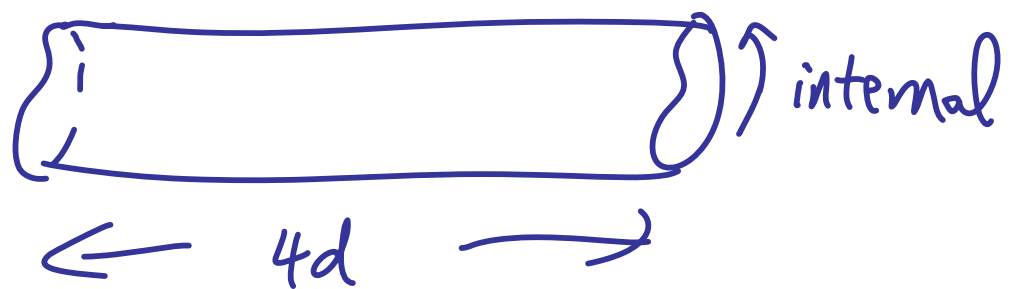
Birthday, Tom + Willy!

Don't worry! With TeV-scale

SUSY, $\text{Log}_{10}\left(\frac{M_p^2}{\Lambda}\right)$ is the

new $\text{Log}_{10}\left(\frac{M_{\text{SUSY}}^2}{\Lambda}\right)$!

We'd like a non-perturbative formulation of 4d physics:



with a hierarchy of energy scales

$$m_{\text{string}} \gg \frac{1}{L_{4d}}$$

From "Landscape" (Banks)

The landscape predicts a plethora of AdS flux vacua at weak string coupling, with AdS radius ranging from ~ 10 to N^{bs} times the string scale. The CFT duals to these states have not yet been found. The search for them leads to a large number of $2+1$ CFTs which have no dual interpretation in terms of large radius SUGRA, or weakly coupled string theory on manifolds whose size is of order string scale. It seems to me that the continued search for the CFT duals of AdS landscape states is the most likely avenue for finding a rigorous justification for part of the landscape picture or for falsifying it.

From "The Incredible Shrinking Torus" (Fischler, Halyo, Rajaraman, ^{Susskind})

For d equal to or greater than 6 we find that we are unable to dualize the theory into any form in which it can be reliably analyzed. Any attempt to dualize it to weak or intermediate coupling produces ultra-small compactification radii. Applying T-duality to increase the compactification radii inevitably leads to ultra-large coupling. From the field-theoretic side this may be connected with the lack of superconformal fixed points in space-time dimension greater than 6. What it means on the string or M-theoretic side is unclear. Perhaps it signals some sort of non-perturbative breakdown

of toroidal compactification. Resolving this point could have important implications for 4-dimensions.

Outline

- The problem & previous attempts
- Our strategy
- Consistency conditions (cf singularities)
- Candidate Examples

$$\text{AdS}_{\substack{5 \\ 4}} \times \text{Small}_{\substack{5 \\ 6}} \Leftrightarrow \text{CFT}_{\substack{4 \\ 3}}$$

Stabilized compactifications \Leftrightarrow IR limit of concrete brane systems (w/ SUSY)

- Generalizations toward dS

AdS/CFT (and previously, BFSS matrix theory) formulate some backgrounds non-perturbatively, but did not (yet) get down to 4d :


BFSS : 11d \leftrightarrow N D0-brane Q.M.

4d (max susy) \leftrightarrow D7-branes on T^7

\hookrightarrow codim 2 \rightarrow log potential $\rightarrow C_{N \leq 24}$

AdS/CFT: ① $AdS_2 \times S^2 \times CY$ small \checkmark
 \hookrightarrow want $L_{AdS} \rightarrow \infty$
 \hookrightarrow IR divergences in AdS_2

AdS/CFT (2) $AdS_4 \times \left\{ \begin{array}{l} S^7 \quad (M) \\ S^2/\mathbb{Z} \\ CP^3 \quad (IIA) \end{array} \right.$


 $L_{\text{internal}} \sim L_{\text{AdS}}$

No hierarchy of scales in Freund-Rubin compactifications.

Basic reason: In 11/10d Einstein

equations $\underbrace{R_{MN} - \frac{1}{2} R G_{MN}}_{\text{Internal + 4d}} = 8\pi G \underbrace{T_{MN}}_{\text{flux}}$

all three contributions are of the same order in the solution

For future reference, let us reproduce this in the language of the 4d effective potential energy:

$$S = \int d^{10}x \frac{\sqrt{G}}{g_s^4} \left(\frac{R}{g_s^2} + F_p^2 + \dots \right)$$

→ 4d potential energy

$$U_4 = \frac{1}{g_s^2} \left\{ - \int d^6x \frac{\sqrt{G_6}}{g_s^2} R^{(6)} + \int d^6x \frac{\sqrt{G_6}}{g_s^2} F_p^2 \right\}$$

$$\sim \frac{R^6}{g_s^2 g_s^2} \left(- \frac{1}{R^2} + g_s^2 \frac{Q^2}{R^{2p}} + \dots \right)$$

where $R \equiv$ size in string units.

$$U_4 \sim M_p^4 \left(\frac{g_s^2}{R^6} \right) \left(- \frac{1}{R^2} + g_s^2 \frac{Q^2}{R^{2p}} + \dots \right)$$

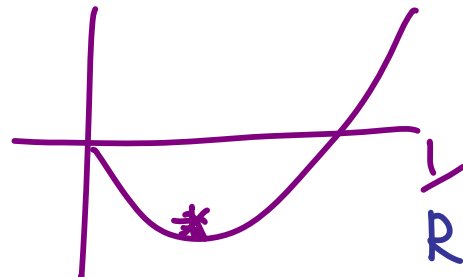
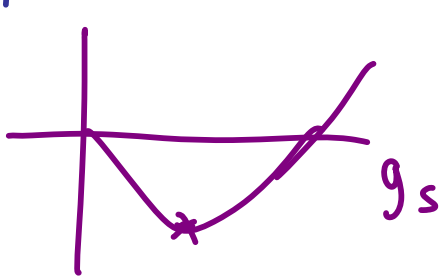
in Einstein frame

↑
 U_R

↑
 U_{F_p}

e.g. IIA on $\mathbb{C}P^3$ w/ F_6 & F_2 :

$$\frac{U_4}{M_p^4} \sim -\frac{g_s^2}{R^8} + \frac{g_s^4 Q_2^2}{R^{10}} + \frac{g_s^4 Q_6^2}{R^{18}}$$



Equivalently

$$U_4 \sim M_p^2 \left(-\frac{1}{R^2 g'} + \frac{g_s^2 Q^2}{R^{2p} g'} + \dots \right)$$

$\sim \Lambda^2 \frac{1}{R_{\text{AdS}}^2 g'}$

i.e. $\Lambda_{\text{min}} \sim \Lambda_R$ in Freund-Rubin

- n.b. CFT_3 dual to IIA/ $\mathbb{C}P^3$ is $\left\{ \begin{array}{l} \cdot \text{IR limit of } D2, D6, \text{ KK} \\ \cdot \text{strongly coupled CS} \end{array} \right.$
 or M thy / S^2/\mathbb{P}^1
 Schwarz, BL, ABJM, et seq.

In fact there is a large class of 3d CFTs obtained via RG flow from gauge theory

with flavors

Appelquist/Heinz HET
Sachdev... CNT

$$\frac{1}{g^2} = \underbrace{\frac{1}{g_0^2}}_{\text{classical}} + \underbrace{\frac{\Delta N}{E}}_{\text{1-loop screening}} + \dots$$

(In 3d $N=4$, $\Delta N = N_f - 2N_c$
and this is exact)

→ dimensionless coupling has fixed point

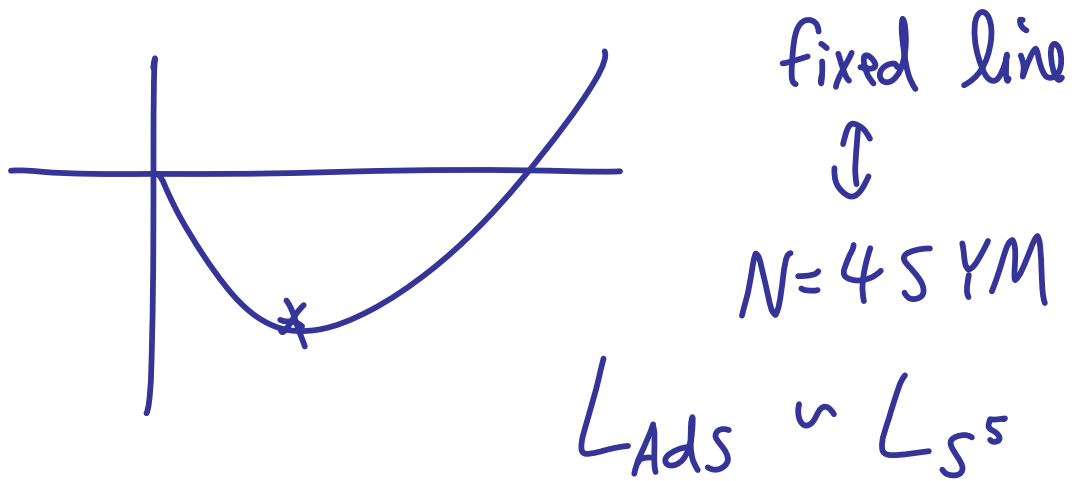
$$\frac{1}{\bar{g}^2} = \left. \frac{E}{g^2} \right|_{E \rightarrow 0} = \Delta N + \dots$$

controlled at large N_f independently of SUSY.

e.g. D2-D6, D2-D6-orbifold; Hanany-Witten

... examples: Still Freund-Rubin (or stringy)

Similarly, compactification
on S^5 w/ $F_5 \rightarrow AdS_5 \times S^5$



In general, the negative
term(s) in the potential are key

e.g. { positive curvature
0-planes
 $-3|W|^2$

Coming from the other direction,
we can construct

$$(A)dS_4 \times X_{\text{small}}$$

in an apparently large number
of ways B^2 DRS BP MSS GKP + KKLT...

suggesting a rich set of dual
CFT₃s.

- Not a priori realized as near-horizon limit of brane system
- Can read off interesting properties:

$$N_{\text{d.o.f.}} \sim L_{(A)dS}^2 M_p^2 \leq N \quad \begin{array}{l} b \leftarrow \text{betti \#} \\ \leftarrow \text{flux \#} \end{array} \quad \begin{array}{l} ES \\ B \text{ and } B \\ AAB \end{array}$$

Plan: Start from known,
Freund-Rubin dual pair:

$AdS \times S \leftrightarrow$ QFT
(at least brane construction)

add ingredients
which cancel or
nearly cancel

U_q

\leftrightarrow

additional
field content,
couplings of
QFT

stabilize the
moduli $\rightarrow AdS_4$

\leftrightarrow

CFT_3

7-branes contribute to U naively as

$$U_7 \sim M_p^4 \left(\frac{g_s^2}{R^6} \right)^2 \cdot \left(\underbrace{\tau_7}_{\text{tension in string units}} \cdot R^4 \right)$$

compare to curvature energy

$$U_{\text{curv}} \sim M_p^4 \left(\frac{g_s^2}{R^6} \right)^2 \cdot R^6 \cdot \left(-\frac{1}{R^2 g_s^2} \right)$$

\Rightarrow for $\tau_7 \sim \frac{1}{g_s^2}$, i.e. $(p, q) 7Bs$,

they compete.

cf Aharony
Fayazuddin
Maldacena

Of course $7Bs$, being codimension 2,
have large IR back reaction...

The interplay between curvature
 & 7-brane energy is accurately
 captured using the techniques
 of F-theory:

Vafa '96

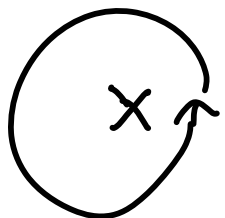
$$T^2 \rightarrow X$$

$$\downarrow$$

$$B$$



$$\tau_{T^2} = C_0 + \frac{i}{g_s} \text{ in } \mathbb{H}^2$$



D7-brane: $\tau(u) \sim \frac{1}{2\pi i} \log u$

$\tau \rightarrow \tau+1$ monodromy

(p, q) 7-brane $\tau \rightarrow \frac{(1+pq)\tau - p^2}{q^2\tau + (1-pq)}$

Plan: Start from

Freund-Rubin dual pair:

$AdS \times S \leftrightarrow QFT$
(brane construction)

add 7 -branes
which cancel or
nearly cancel

U_q

\leftrightarrow additional
 $flavors^*$
couplings of
 QFT

stabilize the
moduli $\rightarrow AdS_4$

\leftrightarrow CFT_3

* in general, both electric
& magnetic cf Douglas/Shenker,
Argyres/Douglas, Argyres-Plesser-Seiberg-Witten

Probing F -theory With Branes

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Last week, A. Sen found an explicit type I string compactification dual to the eight-dimensional F-theory construction with $SO(8)^4$ nonabelian gauge symmetry. He found that the perturbations around the enhanced symmetry point were described by the mathematics of the solution of $\mathcal{N} = 2, d = 4$ $SU(2)$ gauge theory with four flavors, and argued more generally that global symmetry enhancement in $\mathcal{N} = 2, d = 4$ gauge theories corresponded to gauge symmetry enhancement in F -theory.

We show that these $\mathcal{N} = 2, d = 4$ gauge theories have a physical interpretation in the theory. They are the world-volume theories of 3-branes parallel to the 7-branes. They can be used to probe the structure of the exact quantum F -theory solutions. On the Higgs branch of the moduli space, the objects are equivalent to finite size instantons in the 7-brane gauge theory.

May 1996

D3, D7 and Electric/Magnetic Matter

- 4d $N=2$ $SU(2)$ SYM w/ N_f hypermultiplets

Seiberg -
Witten
solution

monopole
⊗

dyon
⊗

\mathbb{Z} (Coulomb branch)

⊗ ← quark

AD/APSW : can change mass matrix

M such that

mutually nonlocal
matter is light.

monopole
⊗

dyon + quark
⊗

⊗

- In brane constructions (Sen, Banks, Douglas, Seiberg, ...)
- $u \leftrightarrow$ D3 position
- $\otimes \leftrightarrow$ 7B position
- $\Upsilon_{YM} \leftrightarrow \Upsilon_{IB}$

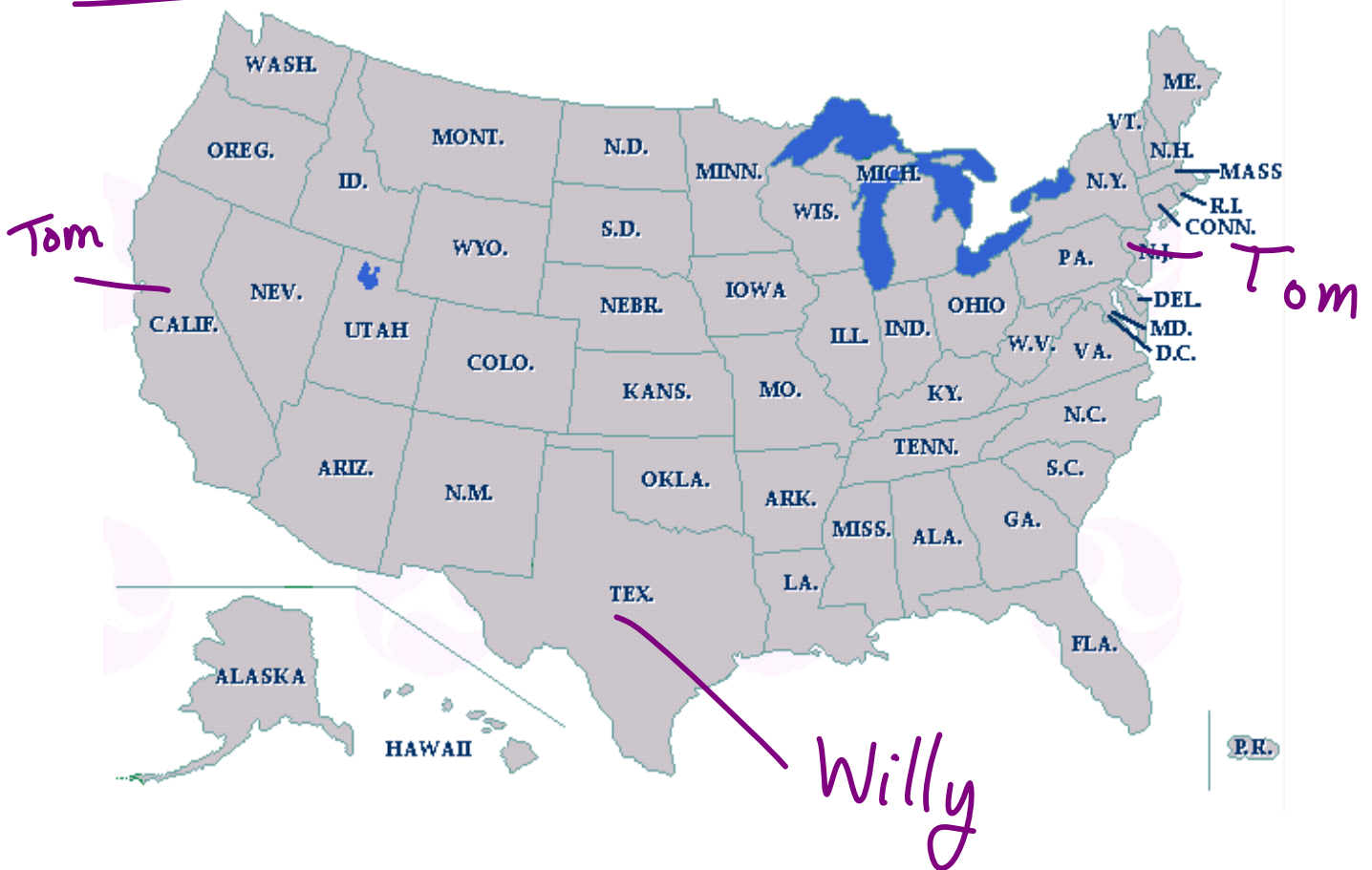
→ Can we unify

"West Coast" & "East Coast" physics? *

landscape

QFTs from
3-7 systems

Existence proof:



* apologies to rest of world

The T^2 varying over B

can be described as

Vafa
Morrison-Vafa
Kachru Intriligator
Morrison Vafa

$$y^2 - x^3 - x f(u) z^4 - g(u) z^6 = 0$$

↑ ↗
coordinates on B

i.e. as a degree 6 hypersurface
in $WP^2(2,3,1)$.

For a Kähler base B , one can
formulate the T^2 fibration $T^2 \rightarrow X$
 \downarrow
 B

as a hypersurface in $B \times WP^2(2,3,1)$,

and as the target space of a

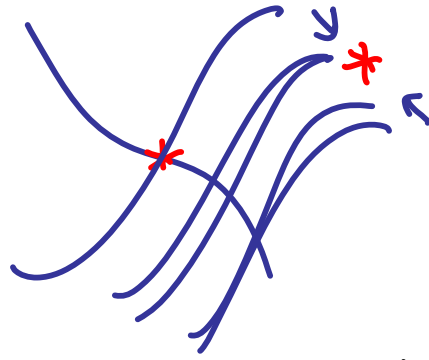
(2,2) gauged linear σ -model (GLSM) written

\mathbb{Z} -branes live at the locus

$$\Delta = 27g^2 + 4f^3 = 0$$

Singularities

- Some allowed (e.g. enhanced gauge symmetries)



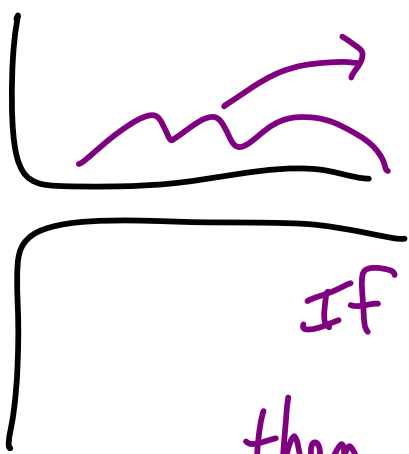
- Some not allowed

A criterion for allowed singularities:

cf W, SW '90s

In the GLSM, singularities arise from extra non-compact branches of scalar field space

bulk of geometry



compute (using GLSM)

\hat{C}_{throat} :

$$\text{IF } \hat{C}_{throat} \geq \hat{C}_{bulk}$$

then truly singular. (otherwise

linear dilaton \rightarrow mass gap in throat)

compute (using GLSM)
 \hat{C}_{throat}
 IF $\hat{C}_{throat} \geq \hat{C}_{bulk}$
 then truly singular. (otherwise
 linear dilaton \rightarrow mass gap in throat)

This agrees with known cases ...

e.g.

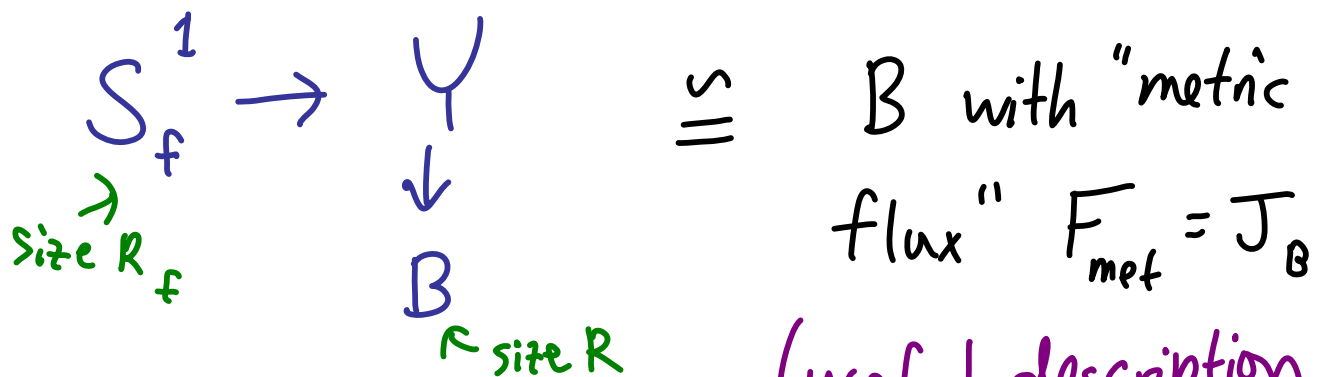
$$\int d^2\theta P \left(y^2 - X^3 - x \overset{r=0}{f(u)} z^4 - g(u) \overset{r=U}{z^6} \right)$$

(D-term)

branch with $\langle p \rangle \sim \langle z \rangle$; $Y = X = 0 = U$
 $\hat{C} = 1$ $\hat{C}_Y = 0$ (massive) $\hat{C}_X = \frac{1}{3}$
 $\hat{C}_U = 1 - \frac{2}{n}$

\Rightarrow singular if $n \geq 6$ ✓

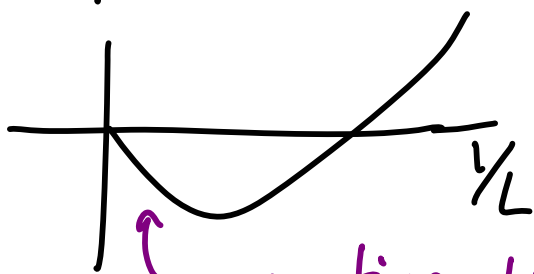
... and can be applied widely



Two classes of candidate examples:

① 7-branes fully cancel curvature energy: e.g. 36 7-branes on $\mathbb{C}P^2$

$$U_R + U_7 = 0 \quad (\text{F-theory on CY})$$



negative term from O-planes

$$R_{\text{Ads}} \gg R \gg R_f$$

② 7-branes nearly cancel $U_R^{(B)}$
 $Y \times S^1 \times \text{AdS}_4$

$$U \sim M_p^4 \left(\frac{1}{R^4 R_6 R_f} \right) \left(\frac{R_f^2}{R^4} - \frac{\varepsilon}{R^2} + \frac{N_c^2}{R^8 R_f^2} + \frac{Q_1^2}{R_\perp^2} \right)$$

$g_s \sim 1$



enforce
with e.g.

E_n 7-branes

\hookrightarrow stable minimum with

$$R_f \ll R \ll R_{\text{AdS}}$$

Related (simpler) AdS_5 model:

$$U \Big|_{g_s \sim 1} \sim \frac{M_5^5}{(R^5 R_f)^{2/3}} \left(\frac{R_f^2}{R^4} - \frac{\Sigma}{R^2} + \frac{N_c^2}{R^8 R_f^2} \right)$$

* Are 7-brane moduli tachyonic?

On S^5



allowed tachyon for $R \ll R_{\text{AdS}}$
but what about $R \ll R_{\text{AdS}}$?

Note that 7-brane moduli

$$y^2 - x^3 - x \overbrace{f(u)} z^4 - \overbrace{g(u)} z^6 = 0$$

are flat directions in the CY
and nearly flat for $U_2 \propto \frac{\epsilon}{R^2}$

To get started, consider

$$Y = S^5 \quad (\text{topologically})$$

Start from the $\mathbb{C}P^2$ model:

(2,2) chiral multiplets U_1, U_2, U_3

$U(1)$ Gauge symmetry

$$(U_1, U_2, U_3) \cong e^{2\pi i \varphi} (u_1, u_2, u_3)$$

$$\Rightarrow D^2 = \left(|u_1|^2 + |u_2|^2 + |u_3|^2 - R^2 \right)^2$$

$\hookrightarrow D=0$ alone gives S^5

φ parameterizes S^1 fiber

$$S^1_f \rightarrow \begin{matrix} S^5 \\ \downarrow \\ \mathbb{C}P^2 \end{matrix}$$

$$ds^2_{S^5} = d\sigma_{\mathbb{C}P^2}^2 + R_f^2 (d\alpha + A)^2$$

$$dA = J$$

Gibbons, Pope

To add the 7-branes, want a T^2 fibration over $B = \mathbb{C}P^2$

Gauged Linear σ -model becomes

		u_1	u_2	u_3	X	Y	Z	P
T^2	$U(1)$	0	0	0	2	3	1	-6
$\mathbb{C}P^2$	$U(1)$	1	1	1	g_x	$\frac{3}{2}g_x$	0	$-3g_x$

$$S_w = \int d^2\sigma d^2\theta P \left(y^2 - x^3 - f(u) x z^4 - g(u) z^6 \right)$$

Now, $\sum_{\text{fields } I} g_I = 0$ is the Calabi-Yau condition (ensuring anomaly-free $U(1) \times U(1)$ R-symmetries appropriate to (2,2) SCFT) written

The running of R^2 in

$$D^2 = \left(|u_1|^2 + |u_2|^2 + |u_3|^2 + g_x |x|^2 + \frac{3}{2} g_x |y|^2 - |p|^2 - R^2 \right)^2$$

is $M \frac{\partial R^2}{\partial M} \sim \sum_I g_I$

Now $\sum g_I = 3 - \frac{1}{2} g_x$

and the degree of $G = y^2 - x^3 - fxz^4 - gz^6$

is $\deg_G = \deg_g = 3g_x = 18 - 6 \sum g_I$

Fully canceling curvature energy

means $\sum g_I = 0 \Rightarrow \deg_g = 18$

$\Rightarrow \deg \Delta = 36 \Rightarrow 36$ 7-branes

(This agrees with naive result from $U_2 \sim \dots (\gamma_2 \times \text{vol}) \sqrt{}$)

- The 7Bs are extended along $U(1)$ fiber

As next step, generalize to cases where we do not fully cancel the curvature energy. Consider

T^2 fibration over $B = \underline{W}P^2$

Gauged Linear σ -model

		u_1	u_2	u_3	x	y	z	P
T^2	$\left\{ \begin{array}{l} U(1) \\ \times \\ U(1) \end{array} \right.$	0	0	0	2	3	1	-6
WP^2	$\left\{ U(1) \right.$	<u>w_1</u>	<u>w_2</u>	<u>w_3</u>	0	0	$w_0 - \sum w_i$	0

$$S_w = \int d^2\sigma d^2\theta P \left(y^2 - x^3 - f(u) x z^4 - g(u) z^6 \right)$$

Again $\beta_{R^2, \text{full}} \sim \sum q_i = w_0$

in the full system including the 7Bs.

Again $\beta_{R^2, \text{full}} \sim \Sigma g = W_0$
in the full system including the 7Bs.

For WP^2 alone,

$$\beta_{R^2, wp^2} = W_1 + W_2 + W_3$$

\Rightarrow IF $W_0 \ll W_1 + W_2 + W_3$
then we almost cancel the
curvature.

$$\rightarrow \mathcal{U}_{R, \text{full}} \sim M_p^4 \left(\frac{g_s^2}{\text{Vol}} \right)^2 \cdot \text{Vol} \cdot \left(-\frac{\Sigma}{R^2} \right)$$

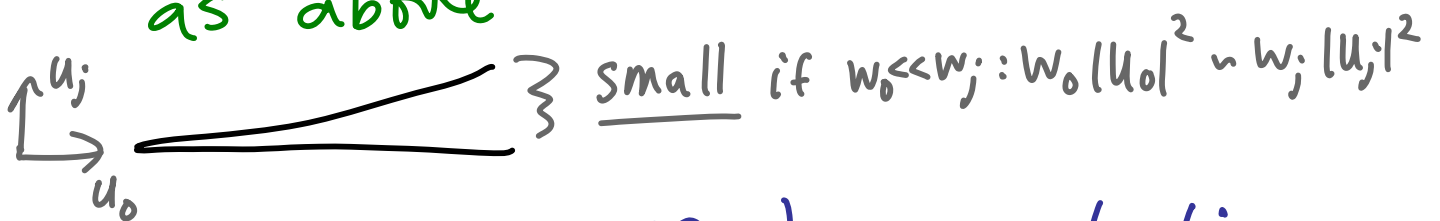
with $\left[\Sigma \sim \frac{W_0}{W_1 + W_2 + W_3} \right]$

(using the NLSM result $\beta \sim R_{MN}$)

We can describe the full brane system (whose low-energy limit is the QFT) as F-theory on the following noncompact CY_4 :

u_0	u_1	u_2	u_3	X	y	z	P
0	0	0	0	2	3	1	-6
$-w_0$	<u>w_1</u>	<u>w_2</u>	<u>w_3</u>	0	0	$w_0 - \sum w_i$	0

$\sum w = 0$ overall; cross-section geometry as above



- This + N_c D3-branes at tip ($u_0 = u_j = 0$) is the brane construction. Altogether preserves 4 supercharges.

* What about the singularities of $WP^2(w_1, w_2, w_3)$?

$$(u_1, u_2, u_3) \cong (\lambda^{w_1} u_1, \lambda^{w_2} u_2, \lambda^{w_3} u_3)$$

$$\rightarrow \text{for } \lambda = e^{\frac{2\pi i}{w_1}} \in U(1)$$

$(u_1, 0, 0)$ is a (generally non-SUSY)

$\mathbb{C}^2 / \mathbb{Z}_{w_3}$ orbifold fixed point.

By itself^{*}, this has twisted tachyons which condense, smoothing it out

Adams
Polchinski
ES ...
.. Morrison..

- ★ Don't Panic! • Must include 7-branes
can restore SUSY
- and S^1_{fiber} : Removes $U(1)$ projection entirely
- ★ full brane construction is supersymmetric

* Singularities of 7-branes:

compute (using GLSM)
 \hat{c}_{throat}
 IF $\hat{c}_{throat} \geq \hat{c}_{bulk}$
 then truly singular. (otherwise linear dilaton \rightarrow mass gap in throat)

w_0	w_1	w_2	w_3	z
-1	1	n	n	-2n

\rightarrow non singular but anisotropic

w_0	w_1	w_2	w_3	z
$-w_0$	$w \cdot \delta$	w	$w \cdot \delta$	$w_0 - 3w$

isotropic but singular $g \sim \sum_i u_i$ $\begin{matrix} I & 18 + \frac{3w_0}{\delta} \\ I & u_2^{-2I} \\ I & u_3 \end{matrix}$ $\begin{matrix} I - \frac{3w_0}{\delta} \\ \delta \end{matrix}$
 high degree \rightarrow high \hat{c}

Generalizations (\approx isotropic, nonsingular)

More fields + more conditions

e.g.

u_0	u_1	u_2	u_3	u_4	z	P_2
$-w_0$	$w-\delta$	$w-\delta$	$w+\delta$	$w+\delta$	w_0-2w	$-2w$

with superpotential

$$\int d^2\theta \left\{ P(y^2 - x^3 - \dots - z^6) g \right\}$$

lower degree than above
nonsingular, isotropic

$$+ P_2 \left(\lambda_1 u_1 u_3 + \lambda_2 u_1 u_4 + \lambda_3 u_2 u_3 + \lambda_4 u_2 u_4 \right)$$

u_0	u_1	\dots	u_D	z
$-w_0$	w_1	\dots	w_D	$w_0 - \sum w$
0	Q^a_1	\dots	Q^a_D	$-\sum Q^a$

$a=1 \dots D-3$
extra $U(1)$
conditions

The string coupling

7-branes corresponding to mutually non-local flavors have $g_s \sim 1$

⊗ e.g. $T \sim e^{\frac{i\pi}{3}}$ $f(u) = 0$ branch
Dasgupta/Mukhi

In a (near-) SUSY background,

the (approximate-) moduli are

- R^2 (GLSM D-term)
- polynomial coefficients in f, g

(Other modes of the metric + dilaton have KK-scale masses $\sim \frac{1}{R\sqrt{g_i}}$.)

In the LSM, $f + g$ are superpotential couplings \Rightarrow don't run even at $\mathcal{O}(\epsilon)$
 \Rightarrow expect $|m^2| \leq \mathcal{O}(\epsilon^2)$

Sen Limit

In F theory, \exists limit (Sen)

$$\begin{aligned} f &= -3h^2 + \epsilon \eta \\ g &= -2h^3 + \epsilon h \eta - \epsilon^2 \frac{\chi}{12} \end{aligned} \quad \epsilon \rightarrow 0$$

for which $g_s \rightarrow 0$. i.e. all the (p, q) 7-branes boil down to $O7$ -planes + $D7$ -branes

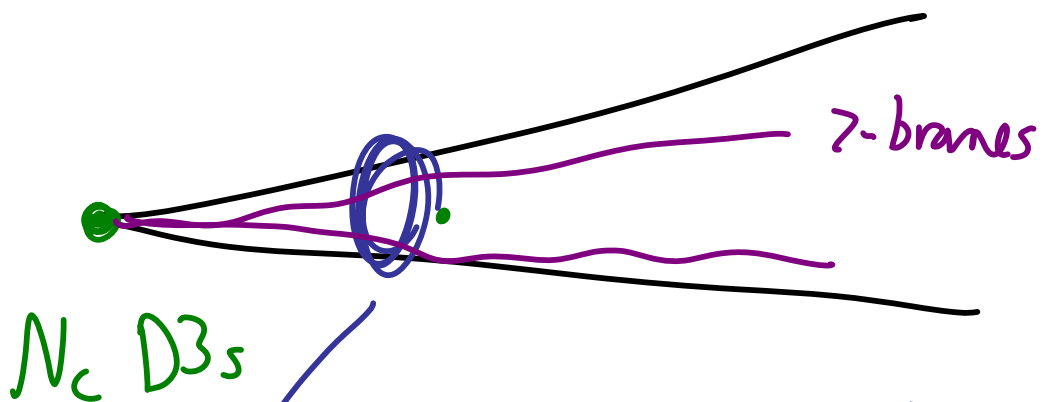
Such examples, if they can also be stabilized (including g_s), would be purely electric on the QFT side.

Number of degrees of freedom:

AdS_5 :

$$N_{d.o.f.}^{(7Bs)} \sim M_5^3 R_{AdS}^3 \sim \frac{L}{\epsilon^3} \cdot N_{d.o.f.}^{(no\ 7Bs)}$$

$$R_f \sim \epsilon R_{AdS}, \quad R^2 \sim \epsilon R_{AdS}^2$$

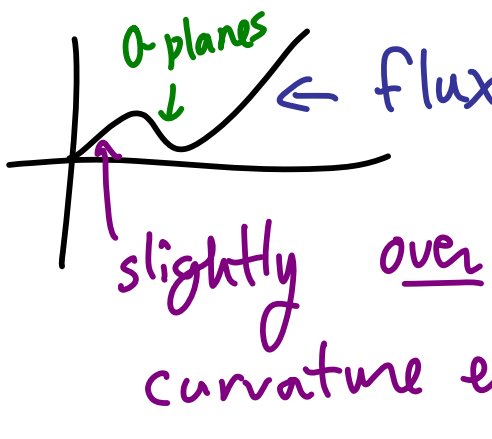


→ # of light states

$$\sim \left(\frac{L}{\epsilon^{\frac{1}{2}}}\right)^4 \cdot \frac{1}{\epsilon} \sim \frac{1}{\epsilon^3}$$

Future directions

- QFT content & couplings from brane system

• dS_4 : 
slightly over-cancel curvature energy

→ Now no tachyons are allowed.

- GKP + KKLT use 7-branes (F-theory) and have 5-form flux. Interpret as above?