More On Superstring Perturbation Theory

Edward Witten

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(2) The integrals one has to study are only conditionally convergent in the infrared region and need to be treated carefully. For this, it is very helpful to use the *supersymmetric* version of the Deligne-Mumford compactification of moduli space.

It is a little dry to explain these points in the abstract, and anyway I have done so several times already. Instead, today I will explain these points in the context of a concrete model which has long been known to be an interesting test case for superstring perturbation theory.

The model was first studied by Dine, Ichinose and Seiberg (Nucl. Phys. B293, 253 (1987)) and Atick, Dixon, and Sen (Nucl. Phys. B292, 109 (1987)), The model was first studied by Dine, Ichinose and Seiberg (Nucl. Phys. B293, 253 (1987)) and Atick, Dixon, and Sen (Nucl. Phys. B292, 109 (1987)), with further insight by Green and Seiberg (Nucl. Phys. B299, 559 (1988)), The model was first studied by Dine, Ichinose and Seiberg (Nucl. Phys. B293, 253 (1987)) and Atick, Dixon, and Sen (Nucl. Phys. B292, 109 (1987)), with further insight by Green and Seiberg (Nucl. Phys. B299, 559 (1988)), following a field theory analysis by Dine, Seiberg, and EW (Nucl. Phys. B289, 589 (1987)).

The model is not terribly exotic. The basic example is compactification of the $SO(32)$ ten-dimensional heterotic string to four dimensions on a Calabi-Yau manifold K, with the spin connection embedded in the gauge group in the usual way.

The holonomy group of the Calabi-Yau manifold K is $SU(3)$, and if one embeds $SU(3)$ in the gauge group of the $E_8 \times E_8$ heterotic string, using the subgroup

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one gets a model in which the four-dimensional gauge group is $E_6 \times E_8$. This, with minor modifications, can be used to make semirealistic models of particle physics.

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One can calculate the low energy spectrum of the model by the same methods used for the possibly more familiar case of $E_8 \times E_8$, and it turns out that for a generic choice of the Calabi-Yau manifold K, the $U(1)$ is anomalous. There are various anomalies, but there is always a $U(1)$-gravity-gravity anomaly with a coefficient proportional to $\mathrm{Tr}\, Y$, where Y is the $U(1)$ generator and the trace is taken in the space of massless chiral superfields.

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In string theory, the anomaly is canceled by the Green-Schwarz mechanism. But what this means in the present case is as follows: At one-loop, the Green-Schwarz interaction $\int_{\mathbb{R}^4\times K} B\wedge \mathrm{Tr}\, F^4$ is generated, where B is the usual B-field and the integral is over Minkowski spacetime times K. Assuming that $p=\int_K \mathrm{Tr}_{SU(3)} F^3$ is non-zero, we induce in four dimensions an interaction $p\int_{\mathbb{R}^4} B\wedge F$. The effect of this interaction is to cause the $U(1)$ photon to become massive.

To understand in a possibly more familiar way the mechanism for this mass generation, we can dualize B to a periodic scalar field a, which actually is the imaginary part of a chiral multiplet

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In other words, the one-loop Green-Schwarz interaction causes the field a to transform non-trivially under $U(1)$ gauge transformations, and the chiral multiplet $Z = \exp(-Y)$ acquires a $U(1)$ charge of p.

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Since the potential energy has a term

$$\frac{1}{g_{\mathrm{st}}^2} D^2,$$

where $D = \frac{p}{\mathrm{Re}\, Y} + \sum_i e_i |\phi_i|^2$, it follows that a one-loop expectation value of D will give masses to the ϕ_i – without shifting the masses of their fermionic partners. At two-loop order, there will be a non-zero cosmological constant or more precisely a non-zero potential energy for the dilaton field.

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(2) Then we will discuss the two-loop "cosmological constant," or rather dilaton potential.

(3) Finally we will try to understand the essential difference between spacetime supersymmetry and bosonic gauge symmetries that it makes it possible for supersymmetry to be spontaneously broken by closed-string loop effects.

Simple considerations of chirality and gauge-invariance prevent the relevant massless charged chiral fermions from getting one-loop masses; we would like to understand how the charged bosons ϕ_i that are massless at tree level manage to get such masses.

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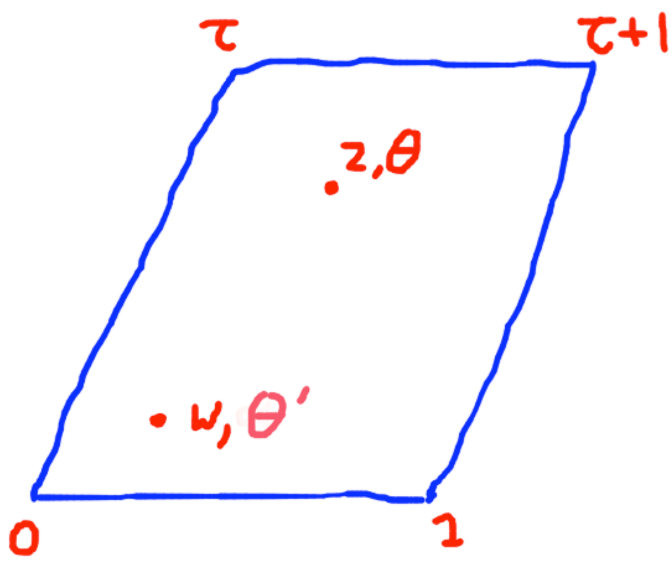

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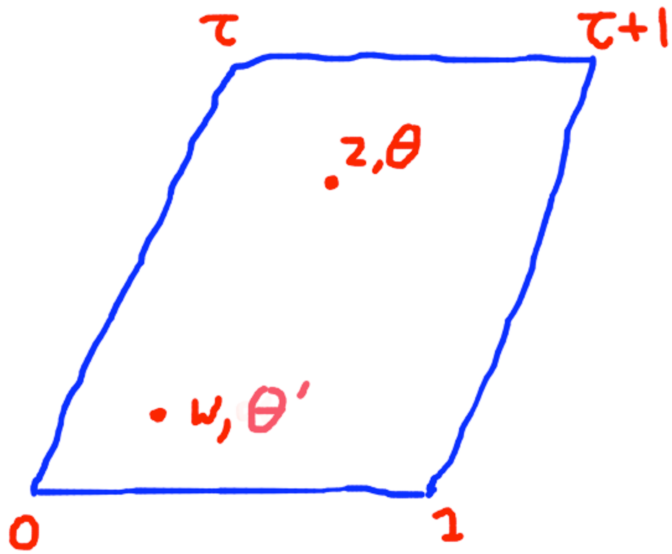

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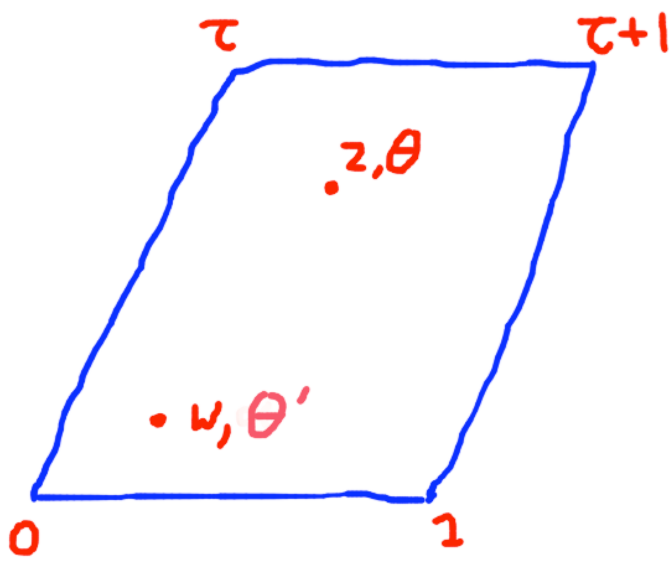

We have to integrate over the modular parameter τ of the torus, but this does not play any important role; we lose nothing essential if we keep it fixed. The really interesting parameters are only the positions $z|\theta$ and $w|\theta'$ at which the vertex operators are inserted.

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Because of the translation symmetry of the torus, we can shift w to zero, so the remaining moduli are $z|\theta, \theta'$. These are the important moduli that we have to integrate over. (With an even spin structure, the torus does not have any fermionic symmetries that could be used to eliminate θ or θ'.)

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The vertex operator for the scalar field in a massless chiral multiplet in the $\mathbf{26}^1$ of $SO(26) \times U(1)$ is

$$V^a(\widetilde{z}; z|\theta) = \lambda^a \lambda^{\widetilde{i}} b_{\widetilde{i} i}(X) D_\theta X^i \exp(ik \cdot X)$$

where λ^a are left-moving worldsheet fermions in the $\mathbf{26}^1$, $\lambda^{\widetilde{i}}$ are left-moving worldsheet fermions that carry an $SU(3)$ index (here $SU(3)$ is the holonomy group of the Calabi-Yau manifold K), and $X(\widetilde{z}; z|\theta)$ describes the motion of the string in spacetime; the wavefunction $b_{\widetilde{i} i}(X)$ is a function on K but $\exp(ik \cdot X)$ is a function on $\mathbb{R}^4$.

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If we want to integrate first over the θ's, we can do this by simply replacing each vertex operator by

$$W^a(\tilde{z};z) = \int \mathrm{d}\theta \, V^a(\tilde{z};z|\theta) = \lambda^a \tilde{\lambda^i} b_{\tilde{i}i}(X) \left(\partial_z X^i + \psi^i k \cdot \psi\right) \exp(ik{\cdot}X)$$

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We now want to calculate (and integrate over z) a two-point function

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where $\widehat{W}^a$ is a similar vertex operator with $i \leftrightarrow \bar{i}$ which describes a conjugate massless scalar. If we drop the right-moving fermions, we get a nonzero contribution by contracting $\langle\partial X^i \partial X^{\bar{i}}\rangle$, but this contribution, since it does not involve the RNS fermions at all, vanishes when we sum over spin structures by the same cancellation that causes the one-loop cosmological constant to vanish.

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$$k^2 \int \mathrm{d}^2 z \, \frac{1}{(\widetilde{z} z)^{1+k^2/2}}$$

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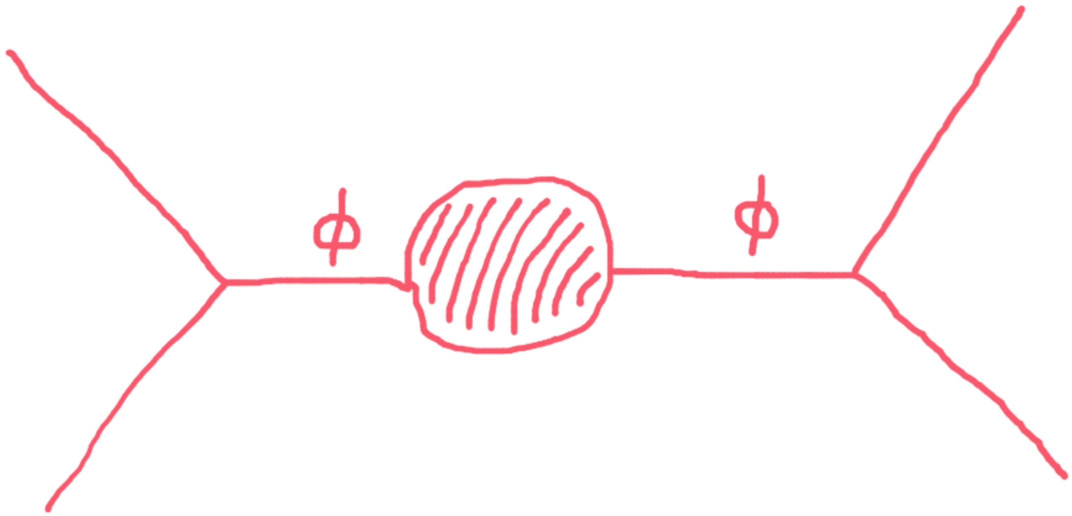

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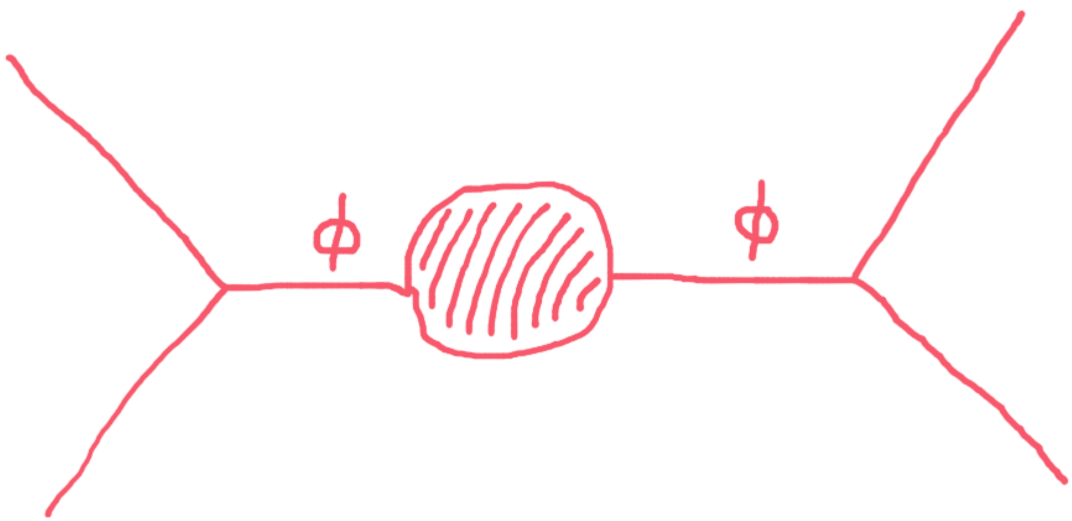

No ad hoc regularization is required.

They also showed that the term in

$$k^2 \int \mathrm{d}^2 z \frac{1}{|\widetilde{z} z|^{1+k^2/2}}$$

that survives for $k \to 0$ is the one-point function, on a torus, of what they called the vertex operator for the D-auxiliary field:

$$V_D = \lambda^i \lambda^{\bar{i}} g_{i\bar{i}} \psi^j \psi^{\bar{j}} g_{j\bar{j}}.$$

This interpretation was useful and V_D will appear in some of our later formulas, though unfortunately I don't have much to say about how general is its role as an auxiliary field vertex operator.

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$$V^a(\widetilde{z}z|\theta) = \lambda^a \lambda^i b_{i\bar{i}}(X) D_\theta X^i.$$

Since we have set $k = 0$, we are not going to get a nonzero result from k^2/k^2.

In fact, if we calculate the two-point function of V^a, we get a result that is independent of the θ's, roughly

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This comes from the OPE expansion

$$\mathcal{V}^a(\widetilde{z}; z|\theta)\widehat{V}^b(0;0|\theta') \sim \frac{\delta^{ab}V_D}{\widetilde{z}} + \dots$$

Now, we still have to integrate over $\widetilde{z}$ and $z|\theta, \theta'$. But how can we get a nonzero result from the integral

$$\int \mathrm{d}\widetilde{z}\,\mathrm{d}z\,\mathrm{d}\theta\,\mathrm{d}\theta' \frac{\langle V_D \rangle}{\widetilde{z}}$$

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$$I = \int \mathrm{d}\widetilde{z}\,\mathrm{d}z\,\mathrm{d}\theta\,\mathrm{d}\theta'\,\frac{\langle V_D\rangle}{\widetilde{z}}$$

converges but not absolutely, because of the singularity at $\widetilde{z} = 0$; to define the integral, we need to supply an infrared regulator, for instance by explaining the order in which we are going to perform the integrations near $z = \widetilde{z} = 0$.

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$$I \to I + \langle V_D \rangle \int \mathrm{d}\widetilde{z}\,\mathrm{d}z\,\mathrm{d}\theta\,\mathrm{d}\theta' \frac{\partial}{\partial z}\left(\frac{\theta\theta'}{\widetilde{z}}\right)$$

and because of

$$\partial_z \frac{1}{\widetilde{z}} = 2\pi i \delta^2(\widetilde{z}; z),$$

this particular infinitesimal change of variables does not leave the integral I invariant.

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$$I \to I + \langle V_D \rangle \int \mathrm{d}\widetilde{z}\, \mathrm{d}z\, \mathrm{d}\theta\, \mathrm{d}\theta' \frac{\partial}{\partial z}\left(\frac{\theta\theta'}{\widetilde{z}}\right)$$

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this particular infinitesimal change of variables does not leave the integral I invariant. Moreover, now we have a factor $\theta\theta'$ so the Berezin integral gives a nonzero result. So the answer we get depends on what variable we use near $z = 0$; we get the wrong answer if we use z, and the right answer if we use $z - \theta\theta'$.

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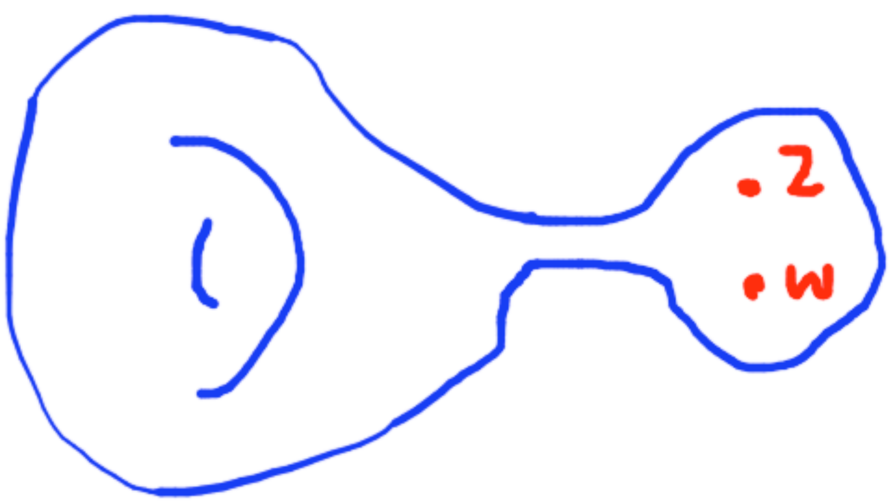
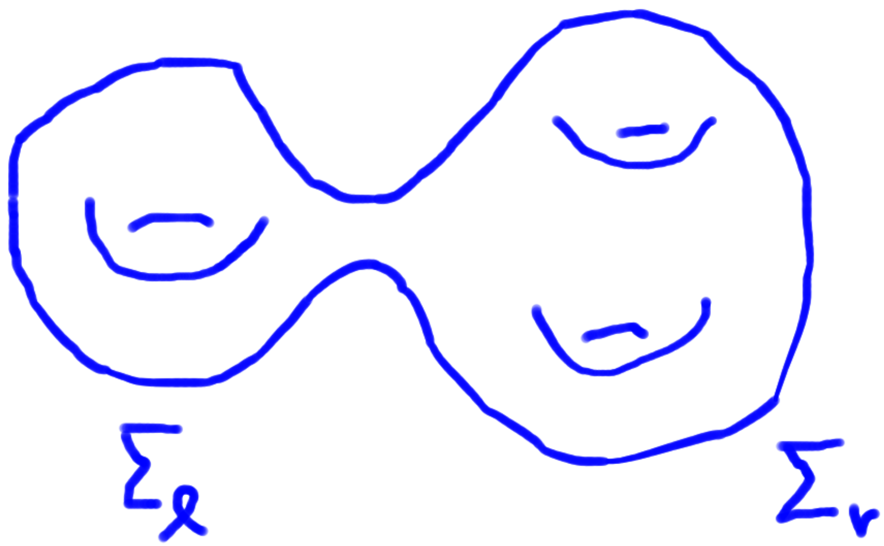


This is a special case of a more general process in which a string worldsheet Σ splits into a pair of worldsheets Σ_ℓ and Σ_r joined by a long tube:



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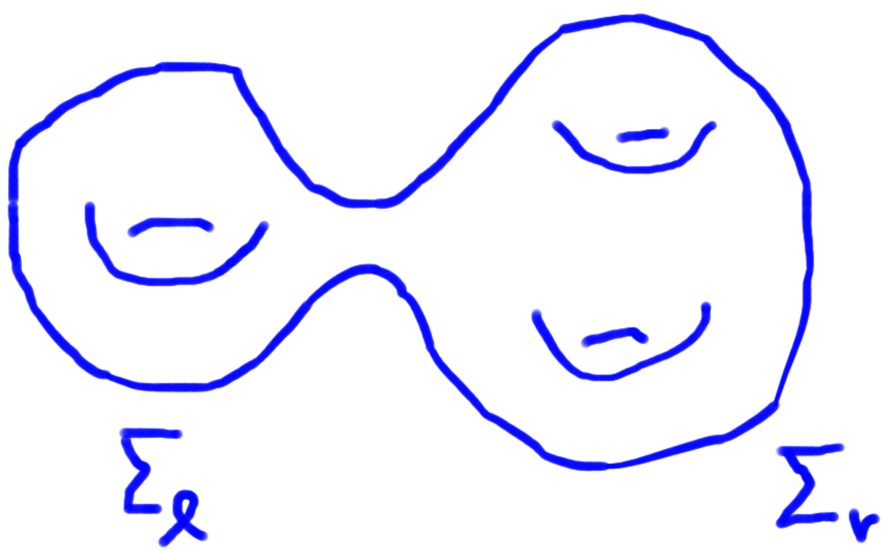

What we have been studying is the special case in which Σ_r is a genus 0 surface with two punctures z and w.

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where q is the gluing parameter – the tube joining the two branches has length $\log 1/|q|$. There is a supersymmetric version of this; one glues a branch parametrized by $x|\theta$ to one parametrized by $y|\psi$ by

$$xy = \varepsilon^2, \quad y\theta = \varepsilon\psi, \quad x\psi = -\varepsilon\theta.$$

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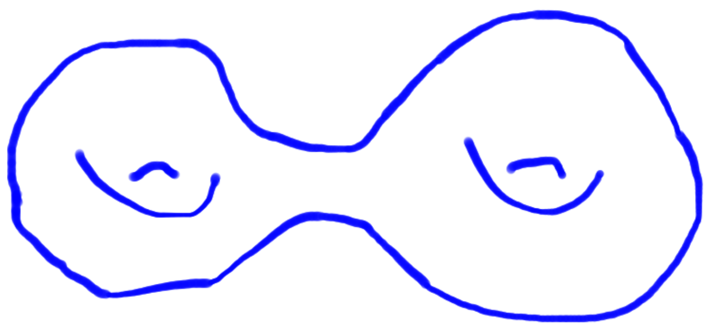
Now we turn to the second topic, which is the two-loop cosmological constant. First I want to say a little about the celebrated calculation made by D'Hoker and Phong for superstring theory in $\mathbb{R}^{10}$. A genus two super Riemann surface



has three even moduli m_1, m_2, m_3 and two odd ones η_1 and η_2. However, as in the previous case where the freedom $z \to z - \theta\theta'$ was important, we run into the fact that the even moduli can be redefined by functions of the odd ones, e.g. $m_1 \to m_1 + f(m_1, m_2, m_3)\eta_1\eta_2$. Unless one has a nice definition of the variables, one runs into unmanageably complicated calculations.

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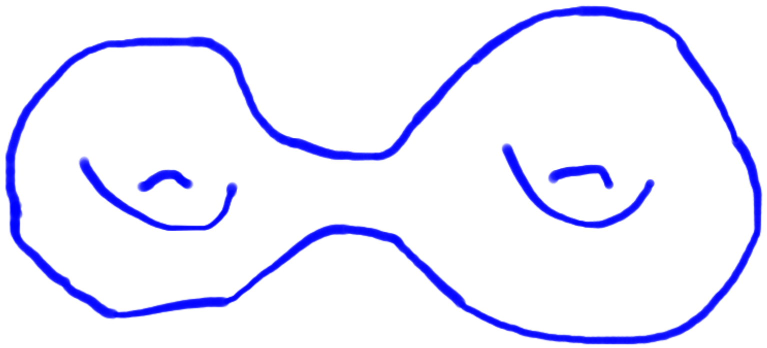
The procedure of D'Hoker and Phong was simply to integrate out the odd variables keeping fixed m_1, m_2, m_3. Technically, it was a hard calculation, but because m_1, m_2, and m_3 were globally-defined, they had a sound framework for the calculation, and were able to carry it out successfully.

However, from what I have told you, there is something one might worry about. A genus two surface can split into two components



and when it does, we may get the wrong answer if we integrate out the odd moduli keeping the wrong bosonic variables fixed. Are m_1, m_2, m_3 the correct variables that should be kept fixed when we integrate out the odd variables in the limit that the worldsheet is degenerating?

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The answer to this question is actually "no, but it doesn't matter for most supersymmetric models," such as the ones actually studied by D'Hoker and Phong. However, precisely in the case of the $SO(32)$ heterotic string on a Calabi-Yau manifold, to get the right answer, we need to correct their procedure near the degeneration limit, in a way that is just analogous to $z \to z - \theta\theta'$ in our genus 1 discussion.

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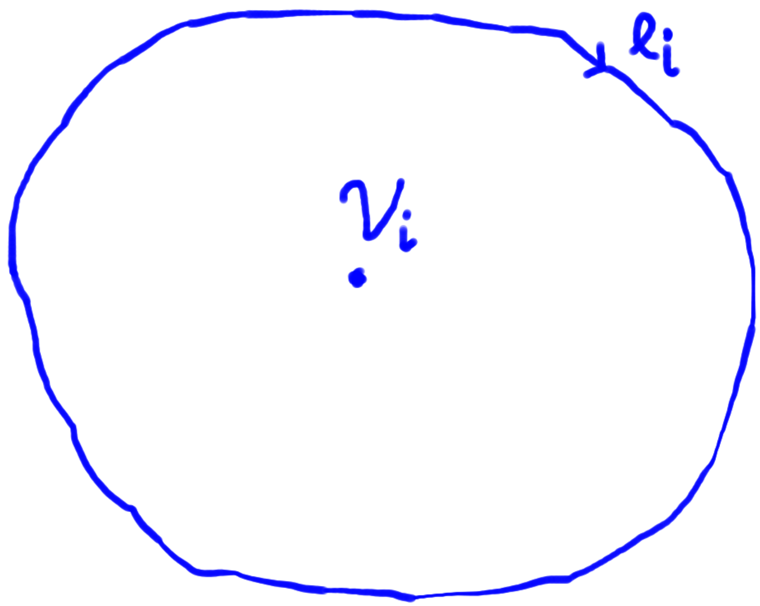
Once one has a worldsheet conservation law, one can derive a Ward identity saying that the expectation value of a product of vertex operators is zero unless the conservation law is satisfied. In the usual way, one considers a correlation function

$$\frac{\partial}{\partial \sigma^\mu} \langle T(J^\mu \mathcal{V}_1 \dots \mathcal{V}_n \rangle) \rangle$$

where σ^μ are the worldsheet coordinates (for example, for the heterotic string, the σ^μ are $\widetilde{z}, z, \theta$).

If the $\mathcal{V}_i$ have definite charge in the sense that

$$\oint_{\ell_i} J_\mu \mathrm{d}\sigma^\mu \cdot \mathcal{V}_i = q_i \mathcal{V}_i,$$



then

$$0 = \int_\Sigma \frac{\partial}{\partial \sigma^\mu} \langle T(J^\mu \mathcal{V}_1 \dots \mathcal{V}_n)\rangle = \sum_i q_i \langle \mathcal{V}_1 \dots \mathcal{V}_n \rangle,$$

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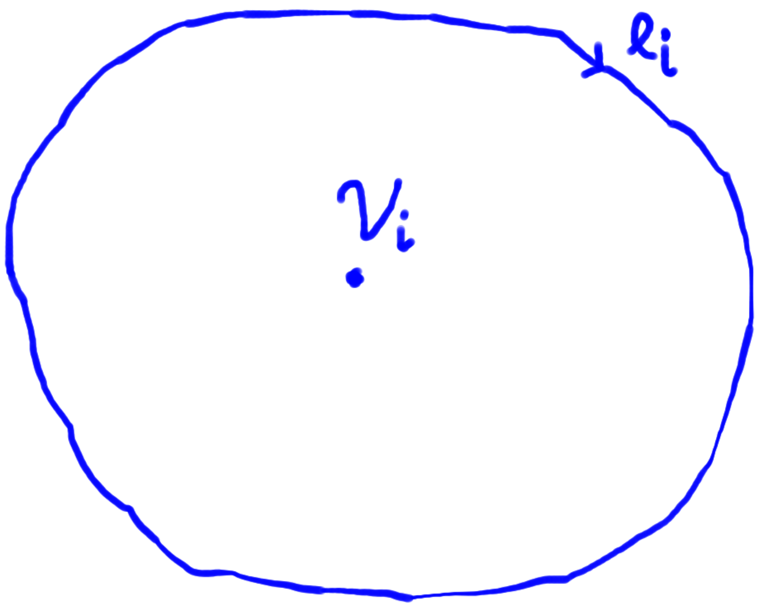

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so $\langle \mathcal{V}_1 \ldots \mathcal{V}_n \rangle = 0$ unless

$$\sum_i q_i = 0.$$

That is our conservation law, and since we proved it without any integration over moduli, we can take it to the bank: the contribution to a scattering amplitude with $\sum_i q_i \neq 0$ vanishes even before any integration over moduli, and so such an amplitude is certainly zero.

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If we cannot interpret $\mathcal{S}_\alpha$ as a conserved current on the worldsheet, how can we use it to derive a Ward identity and why is spacetime supersymmetry ever valid? The answer to this question begins with the fact that the correlation function

$$\langle \mathcal{S}_\alpha \cdot \mathcal{V}_1 \dots \mathcal{V}_n \rangle,$$

where the $\mathcal{V}_i$ are physical state vertex operators, is not a number, or a measure on $\mathfrak{M}_{g,n}$, the moduli space of super Riemann surfaces (of genus g with n punctures), that can be integrated to get a number, because the ghost number is wrong by 1. (That is because the antiholomorphic part of $\mathcal{S}_\alpha$ is 1, of ghost number 0 rather than 1.)

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$$0 = \int_{\mathfrak{M}_{g,n}} \mathrm{d}\langle \mathcal{S}_\alpha \cdot \mathcal{V}_1 \ldots \mathcal{V}_n \rangle = \int_{\partial \mathfrak{M}_{g,n}} \langle \mathcal{S}_\alpha \cdot \mathcal{V}_1 \ldots \mathcal{V}_n \rangle,$$

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where $\partial \mathfrak{M}_{g,n}$ is the "boundary" of the moduli space of super Riemann surface. Here we are using the supermanifold version of Stokes's theorem in order to integrate by parts. Unfortunately there isn't time today to explain such matters.

This "boundary" is the union of components $\mathcal{D}_i$ that represent different ways that the surface Σ can degenerate. So we get a Ward identity

$$0 = \sum_i \int_{\mathcal{D}_i} \langle \mathcal{S}_\alpha \cdot \mathcal{V}_1 \dots \mathcal{V}_n \rangle$$

and this is the identity that will under favorable conditions lead to spacetime supersymmetry.

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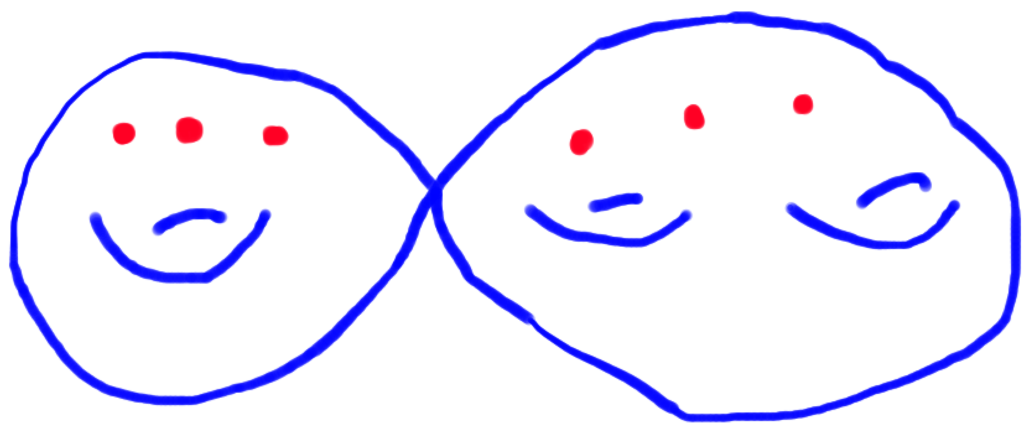

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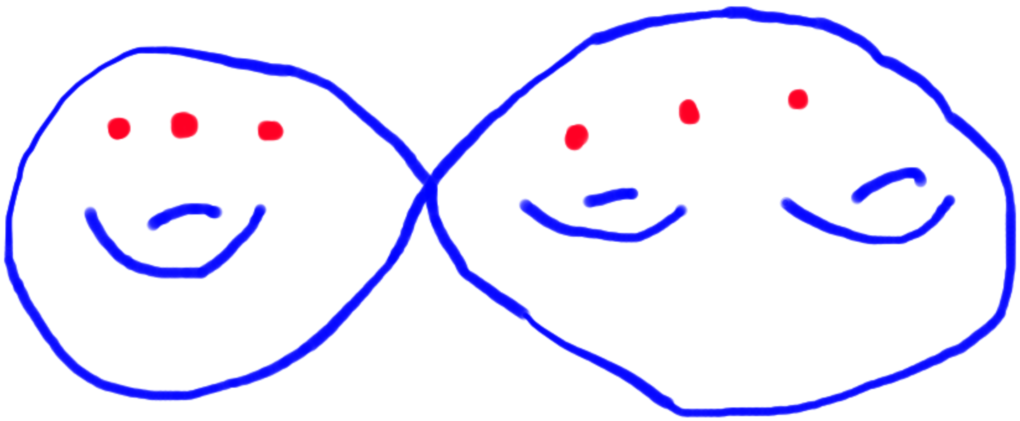

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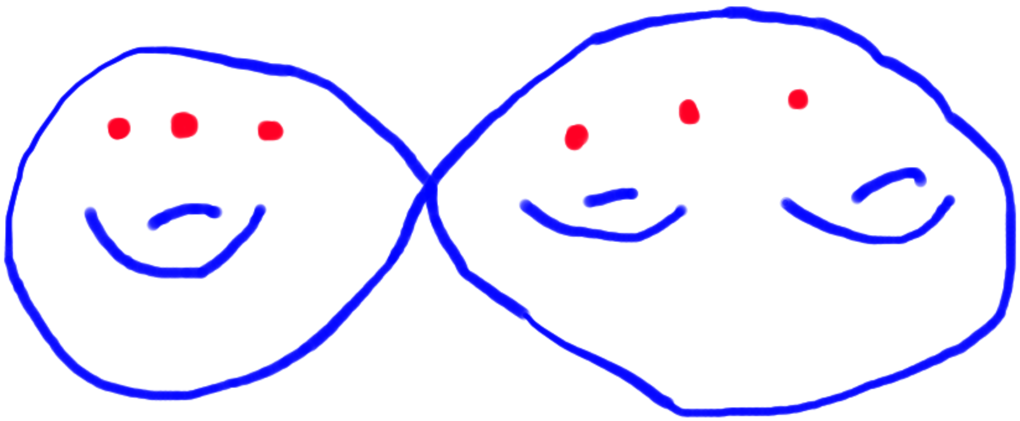

However, any component in which the momentum flowing between the two sides is generically not on-shell does not contribute to the Ward identity. That identity hence receives contributions from only rather special components.

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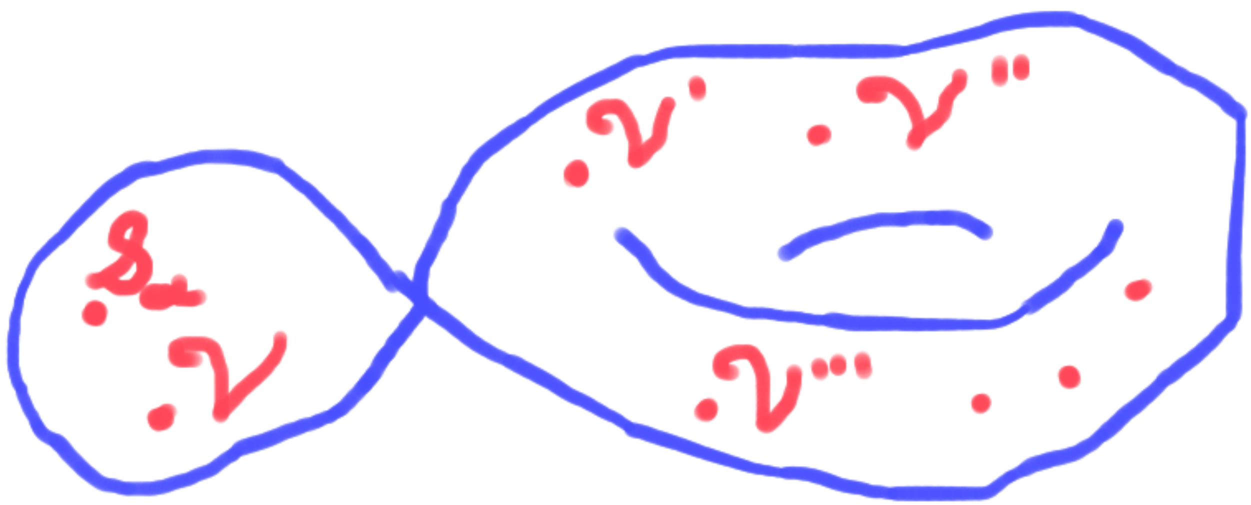

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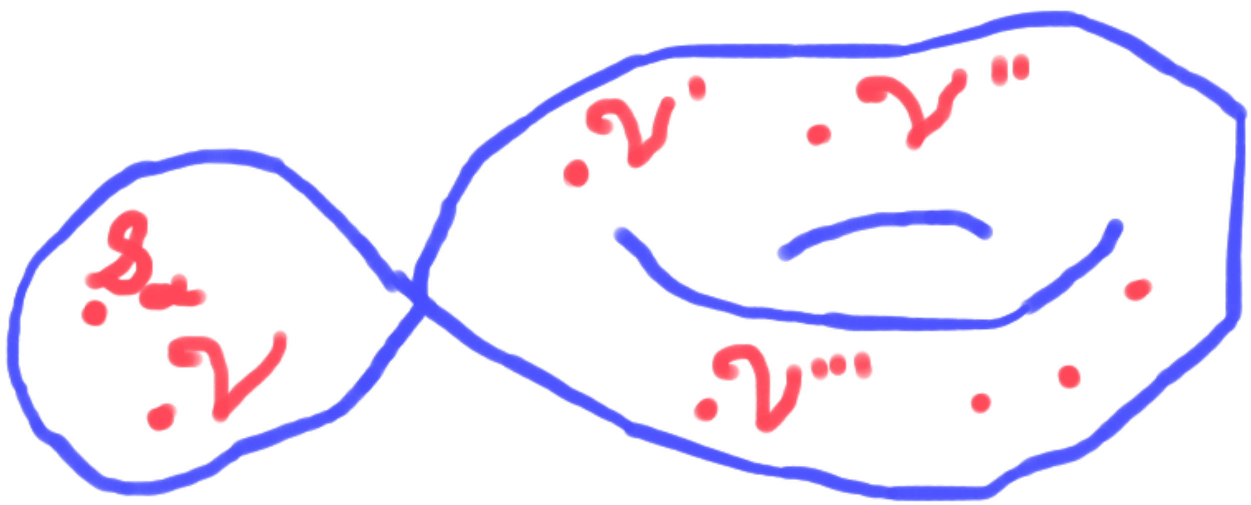

One branch of the worldsheet contains the supercurrent $\mathcal{S}_\alpha$ and precisely one other vertex operator $\mathcal{V}$.

The contribution of this type of component is an S-matrix element obtained by replacing the branch that contains the product $\mathcal{S}_\alpha \cdot \mathcal{V}$ by an effective operator that couples to the right hand side of the picture. This operator is linear in $\mathcal{S}_\alpha$ and $\mathcal{V}$, so we can call it $\{Q_\alpha, \mathcal{V}\}$, where this formula defines a linear operator Q_α acting on vertex operators.

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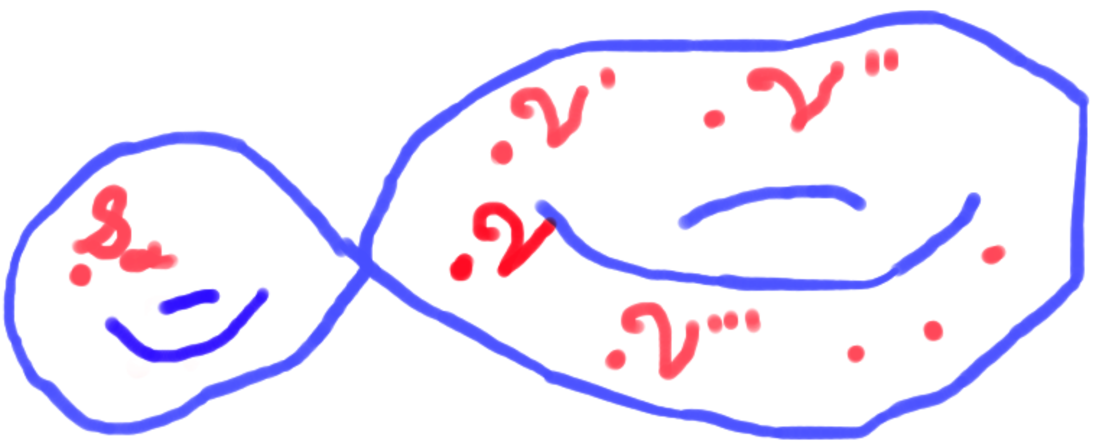

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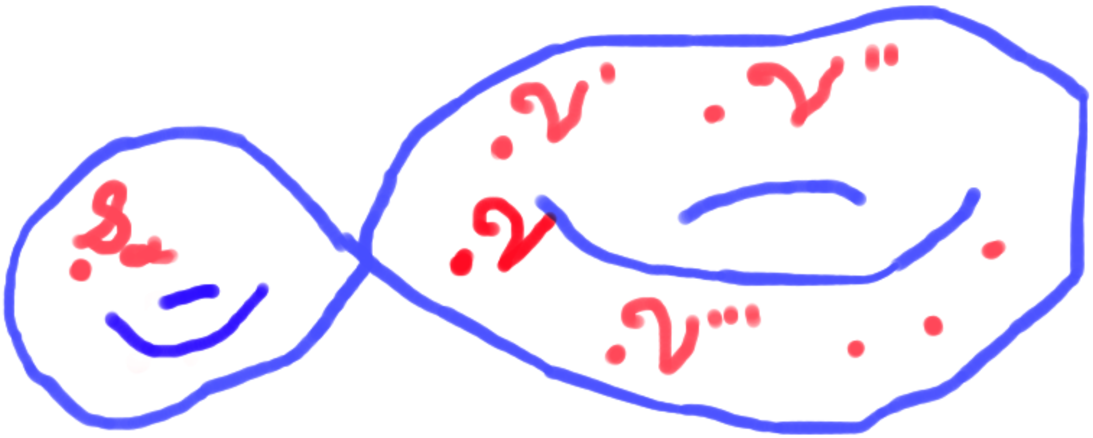

In field theory terms, this contribution involves the matrix element for the supercurrent to create a Goldstone fermion that then couples to $\mathcal{V}_1 \ldots \mathcal{V}_n$.

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