Application of the Kalman Filter in GLAST

Bill Atwood, Jose A. Hernando, Robert Johnson

University of California, Santa Cruz

GLAST Tracking tasks

Parameter Estimation from the tracker:

Gamma direction

Energy

Process:

Selection of the best reconstruction

Track Pattern Recognition

Track Fitting

LSQ Fit *VS* Kalman Filter

LSQ: A Track estimator

Cov Matrix with non-diagonal errors

Pattern - Cone around a guess ray

Kalman: A track Follower, a iterative method

Consider Error plane by plane

Helps Pattern Recognition

Application of the Kalman Filter in GLAST

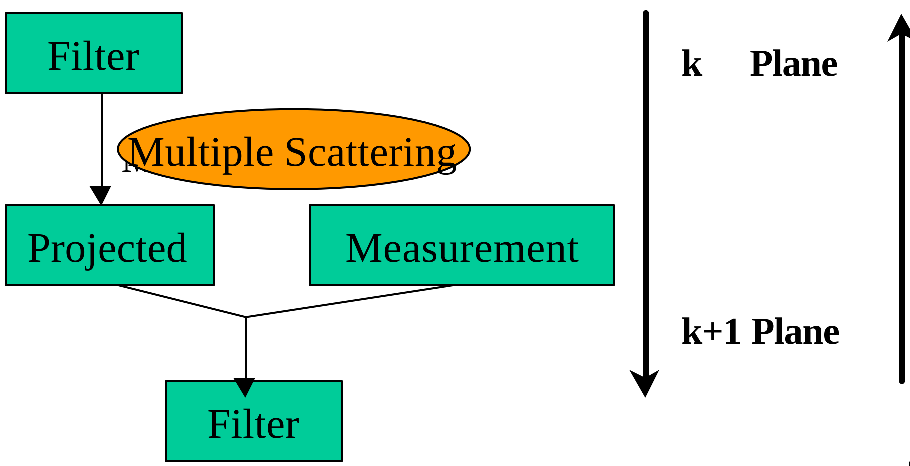
The Kalman Filter Steps

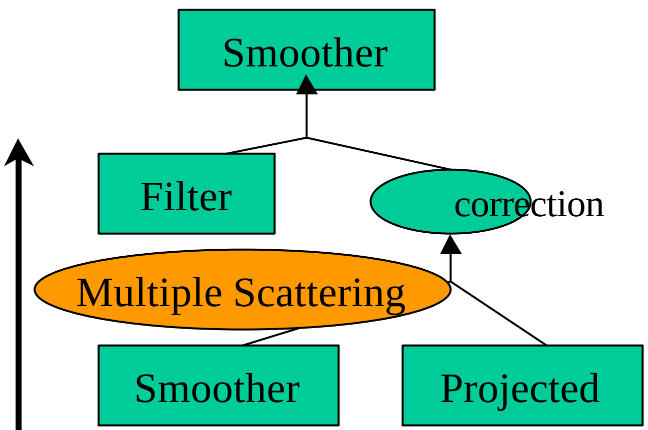
Kalman Filter:

Based in the minimun squared method

Iterative procedure !

A robust mathematical method => $c^{2}\ \vec{p}_k, \mathbf{C}_k$





Filter:

Extrapolate to the next plane

Weight extrapolated and measured hit

Smoother:

Correct previous hit with the posterior information

Kalman Filter in GLAST

Application of the Kalman Filter in GLAST:

- Theoretical calculation of the track parameters resolution.
- Proper treatement of the Fitting process.
- Simple Handling of the Pattern Recognition.
- Simple extrapolation to other subdetectors.

Application of the Kalman Filter in GLAST

Kalman Filter Technique

1.- History: Kalman (61), Billoir (84), Fruhwirth (87)

2.- Based in the minimum squared method

Minimizes the residuals

3.- Iterative procedure addition plane by plane

a.- simplifies the inclusion of the MS

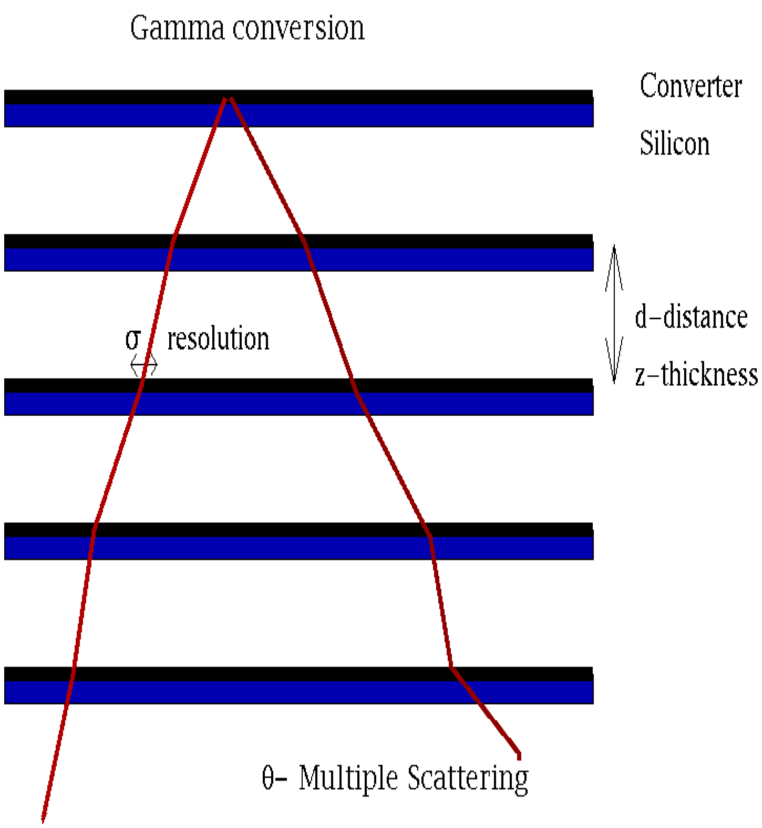
b.- helps the patter recognition

4.- A robust mathematical method

Track parameter resolution => estimation of the PSF

Application of the Kalman Filter in GLAST

GLAST tracking parameters



1.- Track Model: Straight line

2.- Periodic system

- d — distance between planes
- s — spatial resolution
- z/X0 — converter thickness

3.- Random error = Multiple Scattering

$$J_0 \cong \frac{0.015GeV}{p}\sqrt{z / X_0}$$ rms MS angle

- E — Energy
- ? — Incident angle

Application of the Kalman Filter in GLAST

Kalman Filter for a Straight line

The track parameters and covariance matrices

$$\vec{p}_k = \begin{pmatrix} x_k \\ b = \tan J \end{pmatrix} \qquad \mathbf{C}_k = \begin{pmatrix} f_x^2 s^2 & f_{xb} \frac{s^2}{d} \\ f_{xb} \frac{s^2}{d} & f_b^2 \left(\frac{s}{d}\right)^2 \end{pmatrix}$$

The Filter step

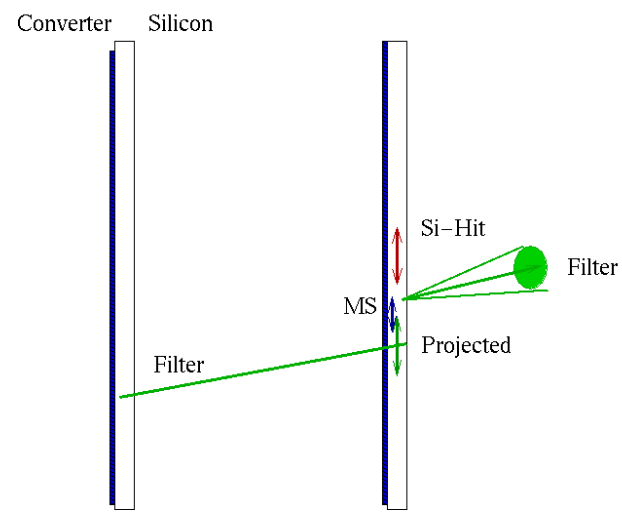
$$\vec{p}_{k+1}^{extrap} = \mathbf{F}\vec{p}_k; \quad \rightarrow \mathbf{F} = \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}; \; x_{k+1} = x_k + \tan J \; d$$

$$\mathbf{C}_{k+1}^{extrapol} = \mathbf{F}\mathbf{C}_k\mathbf{F}^T + \mathbf{Q} \quad \rightarrow \mathbf{Q} = \begin{pmatrix} J_0^2 z^2/3 & J_0^2 z/2 \\ J_0^2 z/2 & J_0^2 \end{pmatrix}$$

***

$$\mathbf{C}_{k+1} = \left[\mathbf{C}_{k+1}^{extrapol,-1} + \mathbf{G}\right]^{-1} \quad \rightarrow \mathbf{G} = \begin{pmatrix} 1/s^2 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\vec{p}_{k+1} = \mathbf{C}_{k+1}\left[\mathbf{C}_{k+1}^{extrapol,-1} p_{k+1}^{extrapol} + \mathbf{G}\vec{m}\right] \rightarrow \vec{m} = \begin{pmatrix} x_{strip} \\ 0 \end{pmatrix}$$



Application of the Kalman Filter in GLAST

Dimensionless general case

The periodicity of the system allows to consider a general dimensionless case

The covariance matrix

$$C_k = \begin{pmatrix} f_x^2 s^2 & f_{xb}\dfrac{s^2}{d} \\ f_{xb}\dfrac{s^2}{d} & f_b^2\left(\dfrac{s}{d}\right)^2 \end{pmatrix} \rightarrow C_k = \begin{pmatrix} f_x^2 & f_{xb} \\ f_{xb} & f_b^2 \end{pmatrix}$$

The improving factors $f_x \,;\, f_b$

The nominal resolutions $s \,; J_n = \dfrac{s}{d}$

The multiple Scattering covariance matrix

$$Q = \begin{pmatrix} J_0^2 z^2/3 & J_0^2 z/2 \\ J_0^2 z/2 & J_0^2 \end{pmatrix} \rightarrow Q = \begin{pmatrix} 1/3\, f_z^2 f_{ms}^2 & 1/2\, f_z\, f_{ms}^2 \\ 1/2\, f_z\, f_{ms}^2 & f_{ms}^2 \end{pmatrix} \rightarrow Q_{(f_z = 0)} = \begin{pmatrix} 0 & 0 \\ 0 & f_{ms}^2 \end{pmatrix};$$

The thickness factor $f_z = \dfrac{z}{d}$

The multiple scattering factor $f_{ms} = \dfrac{J_0}{J_n}$

Application of the Kalman Filter in GLAST

The Slope Improving factor at the Vertex Position

The slope improving factor at the Vertex:

1.- For a given N and f_{ms} *two variables!*

2.- Apply the Filter - Covariance matrices

3.- Apply the Smoother-Covariance matrices

4.- Extrapolate to the Vertex the Cov matrix

Title:
kalmannote.talk.eps
Creator:
HIGZ Version 1.22/09
Preview:
This EPS picture was not saved
with a preview included in it.
Comment:
This EPS picture will print to a
PostScript printer, but not to
other types of printers.

Dependence with the incident angle

Increasing of the f_{ms}

$$f_{ms} \rightarrow \frac{f_{ms}}{\cos^{5/2} J}$$

(the dependence with f can be included)

Application of the Kalman Filter in GLAST

Number of relevant planes

Conclusions:

1.- The MS tends to dominate the resolution very quickly

2.- The number of relevant planes

$$\mathbf{C}_k = \mathbf{C}_{k+1}$$

Title:
kalmannote.ftetanplanes.eps
Creator:
HIGZ Version 1.22/09
Preview:
This EPS picture was not saved
with a preview included in it.
Comment:
This EPS picture will print to a
PostScript printer, but not to
other types of printers.

Application of the Kalman Filter in GLAST

An Estimation of the PSF

An estimation of the PSF

$$\text{PSF} \cong \sqrt{\left(\frac{E_+}{E_g}\right)^2 s_+^2 + \left(\frac{E_-}{E_g}\right)^2 s_-^2}$$

$$E_+ = 0.75 E_g$$

Conclusions:

1.- The PSF is better than the MC *(factor 0.7-08)*

2.- Little dependence with the angle

3.- Asymptotic limits (low and high energies)

4.- In the intermediate region the KF should improve the present algorithm

5.- Understand the differences

Second effects (tails,pattern) or KF

Title:
kalmannote.enesixteen.eps
Creator:
HIGZ Version 1.22/09
Preview:
This EPS picture was not saved
with a preview included in it.
Comment:
This EPS picture will print to a
PostScript printer, but not to
other types of printers.

Application of the Kalman Filter in GLAST

An estimation of the PSF

Dependence with z/X0 and s

E = 0.1, 1., 10., 100 GeV

Title:
kalmannote.dosseventeenb.eps
Creator:
HIGZ Version 1.22/09
Preview:
This EPS picture was not saved
with a preview included in it.
Comment:
This EPS picture will print to a
PostScript printer, but not to
other types of printers.

Application of the Kalman Filter in GLAST

Conclusions

1.- The Kalman Filter allows to compute the Track Parameters resolutions

2.- In a periodic system, the *improving factor* can be computed in a dimensionless general case:

It depends on two parameters : N -planes

$f_{ms} = \frac{J_0}{J_n}$; $J_n = \frac{s}{d}$ *the ratio MS/resolution*

3.- A PSF can be estimated using this method (Burnett,Jones,Hernando)

It is an approximation: physics -> non Gaussian tails, dE/dx

detector-> pattern recognition, eff, etc

Application of the Kalman Filter in GLAST

In a near Future

1.- A KalFit class in the GLAST simulation program

2.- Understand the second order effects

3.- Can the Kalman Filter help the energy resolution?

4.- Application of the Kalman Filter to the Patter Recognition Problem