A Very Short summary of The Spectral Index Estimation Analysis

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1 Spectral Index analysis

During the last Milagro collaboration meeting in Irvine I presented results from my analysis on the spectral Index Estimation through the use of the variable A_4 (http://www.ps.uci.edu/ allen/AousAbdoIrvine.pdf). In this analysis the excess from the Crab is binned differentially in A_4 from $A_4 \geq 1.0$ to $A_4 \geq 12$ in steps of 1.0. This distribution of the differential excess from the Crab was then compared to similar distributions for the Gamma Monte Carlo. The Gamma MC distributions were binned in A_4 in the same way. There were 11 Gamma MC distributions against which the Crab excess was compared, these distributions correspond to different spectral indexes ranging from -3.0 to -2.0 in steps of 0.1. The χ^2 for each case was calculated and then a distribution of the χ^2's as a function of the different α's was shown. The minimum of the χ^2 vs α distribution corresponds to the best fitted alpha and so to the spectral index of the Crab. This was found to occur at an α of -2.62 which corresponds to χ^2/ndf of 1.43. All of the above was done assuming no high energy cutoff in the spectrum.

Gus suggested in late December to try to determine if the Crab has a high energy cutoff by probing the space of α and E_{cut} for the minimum χ^2. In order to do this I had to calculate and simulate the new energy weight for such cases. I created a new re wighted 990 Gamma MC samples:

- 10 different alpha $[-2.9, -2.0]$ and for each of these
- 99 different E_{cut} $[1, 99]$ TeV

The result of the above is shown in figure 2. In order to test This method, and the code, I had to input a fake signal that I have some idea about its α and E_{cut}. Simply having one of my 990 Gamma MC subsets as my input is not accurate since this sample has an infinite amount of statistics compared to the actual Crab signal. To over come this obstacle I tried to Poisson fluctuate the signal by doing the following:

- For the selected Gamma MC A_4 distribution take the i'th bin content N_i to be a mean μ_i for a poisson randomly generated number
- The error assigned to this Poisson number is simply the square root of the number

and now I have a "fake" data set with a bin content equal to the Poisson RGN and the error bar is the square root of that PRGN. After talking to Jim last Thursday, he mentioned that the error bars will still be very small compared to those on the actual Crab data, and he was right. the fractional error bars on this "fake" data set were one order of magnitude less than those for the Crab. To over come this I talked to Andy on Friday and he suggested the following:

- We have to include the fluctuation in the background from the Crab in the creation of our "fake" data set . To do this the A_4 distribution for this fake data set is generated according to the following formula:

$$N_F^i(N_{MC}^i, \delta_{Crab}^i) = P(\mu = N_{MC}^i) + G(\mu = 0, \sigma^2 = \delta_{Crab}^i) \tag{1}$$

- and the error on each bin is given by:

$$\delta N_F^i(N_{MC}^i, \delta_{Crab}^i) = \sqrt{(\delta_{Crab}^i)^2 + (\delta_{MC}^i)^2} \tag{2}$$

Where:
N_F^i: Number of excess events in the "fake" data set in the ith A_4 bin
N_{MC}^i: Number of excess events in the reweighted Gamma MC data set in the ith A_4 bin [1].
δ_{Crab}^i: the error on the ith A_4 bin from the Crab excess, in this case:
$\delta_{Crab}^i = \sqrt{B^i}$: where B^i is the background count in the ith A_4 bin from the Crab.
$P(\mu = N_{MC}^i)$: a Poisson random number generated with a mean equal to N_{MC}^i
$G(\mu = 0, \sigma^2 = \delta_{Crab}^i)$: a Gaussian random number generated with a mean equal to 0., and a variance equal to δ_{Crab}^i

[1] The Gamma MC sample has been reweighted to have the same significance as the excess from the Crab

The first term in equation 1 takes care of poisson fluctuating the Excess in the reweighted Gamma MC sample, while the second term in the same equation includes the effect of the Background fluctuation seen in the Crab sample. Equation 2 assigns the error to be equal to the quadrature sum of the individual errors on the Crab and the Gamma MC samples. Since the errors on the Crab sample are much bigger than those in the Gamma MC sample, the error on the "fake" data set is dominated by the error from the Crab sample.

In total, 10 "fake" data sets were generated. For all of these data sets the input spectral index was taken to be equal to -2.6 and the cutoff energies were (3,7,9,11,21,31,41,51,61,71) TeV. The analysis code was run on each case 500 times, each time the minimum χ^2 as a function of α and E_{cut}, was reported. Two dimensinal distributions of the best fits as a function of α and E_{cut} were generated after words. Also one dimensional distributions of the best fits as a function of α and as a function of E_{cut} were generated. The reults of these runs are shown in figures 1 through 10.

In each of these figures, the upper left plot shows the distribution of the best χ^2 fits for the 500 runs as a function of α and E_{cut}. The upper right plot shows the distribution of the best χ^2 fits for the 500 runs as a function of α. The lower left plot showes the distribution of the best χ^2 fits for the 500 runs as a function of E_{cut}, while the lower right hand plot shows the $\chi^2 - \chi^2_{Min}$ for one of the 500 runs as a function of α and E_{cut}. Table 1 lists some numbers from these plots.

Input E_{cut} TeV	Mean α	Mean E_{cut} TeV
3.00	-2.50	3.00
7.00	-2.32	5.48
9.00	-2.60	8.99
11.00	-2.59	10.92
21.00	-2.58	20.13
31.00	-2.59	32.55
41.00	-2.59	42.29
51.00	-2.59	53.63
61.00	-2.59	58.06
71.00	-2.58	62.13

Table 1: Values of the Means of α and E_{cut} for different cutoff energies in the input spectrum for $\alpha = -2.6$.

Figure 11 shows the distribution of the Means of α as a function of the cutoff energy in the input spectrum. From this plot we see that we can determine the spectral index for cutoff energies greater then 9 TeV, below this value the detemination of the spectral index through this method is not accurate. Figure 12 shows the distribution of the means of the fit energy cutoffs as a function of the cutoff energy in the input spectrum. We see, from this plot that we can determine the cutoff energy in the spectrum up to 40 TeV, above this value this method can't correctly predict the cutoff energy in the input spectrum, one reason for this could be that higher values of A_4 should be considered, i.e. extra bins in A_4 up to 20 or 25 may be necessary to quantify the energies above 40 TeV. This can be seen in figure 13

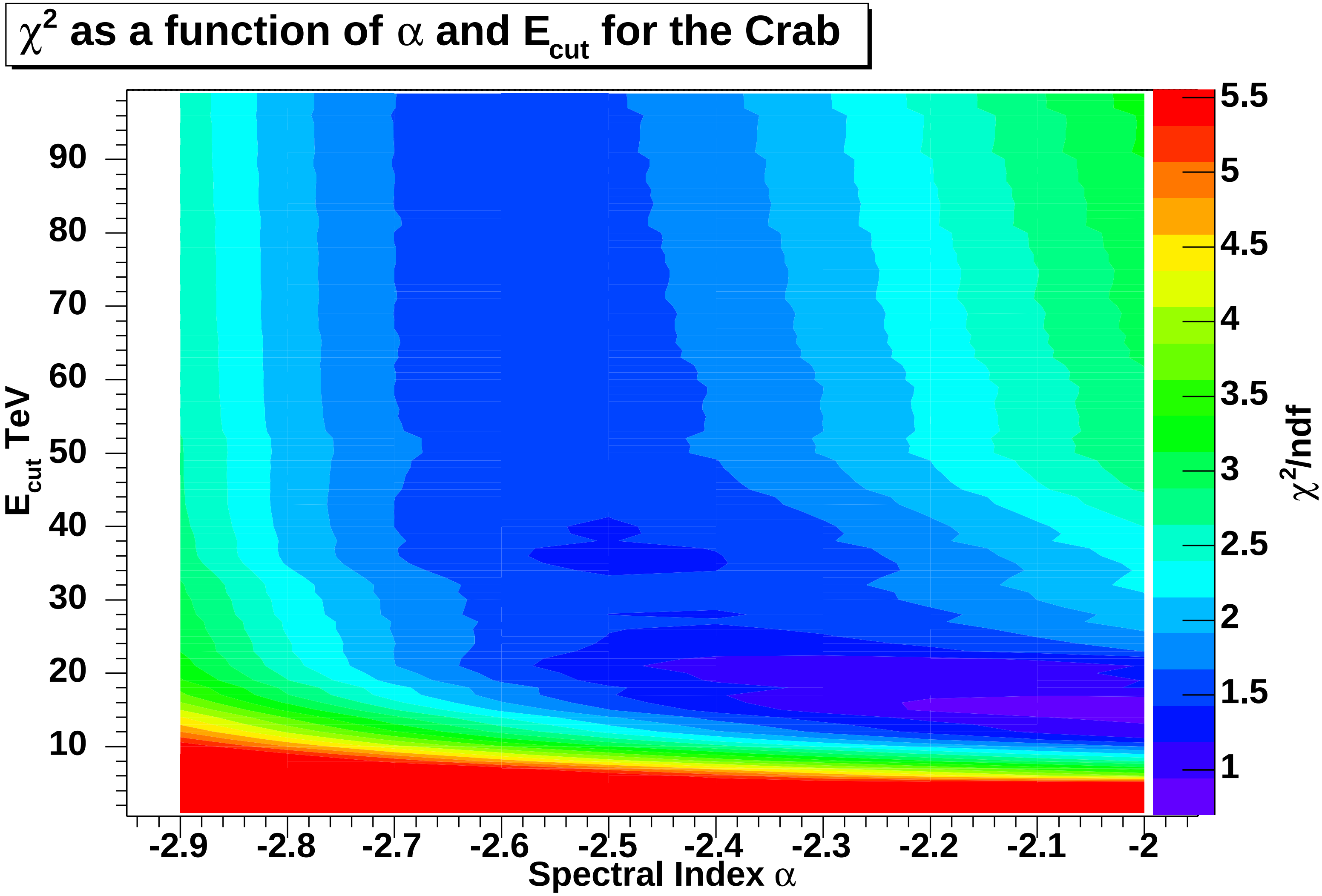

Table 2: χ^2/ndf as a function of α and E_{cut} for the Crab.

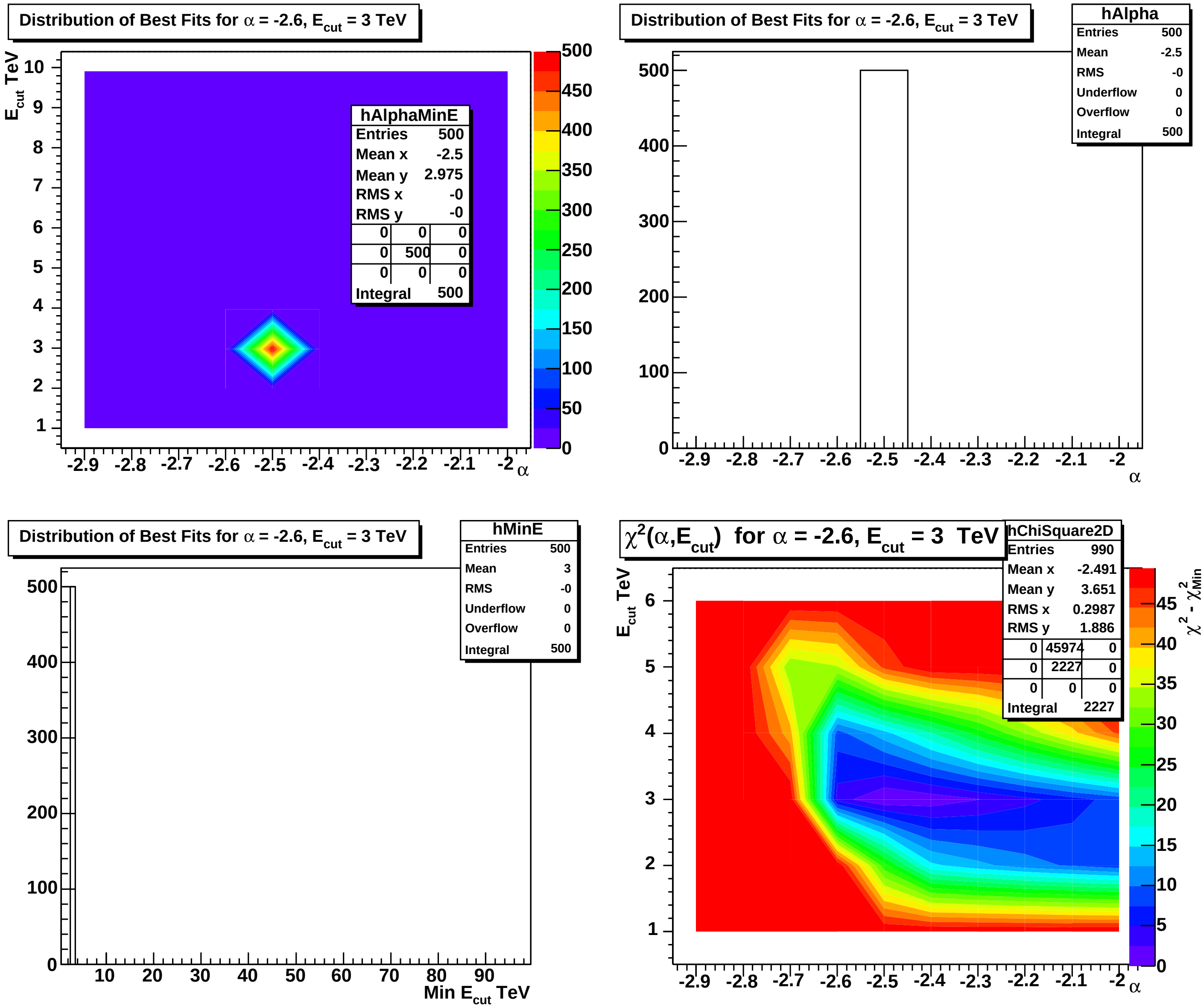

Figure 1: Best Fits for an input spectrum of α = -2.6 and E_{cut} = 3 TeV

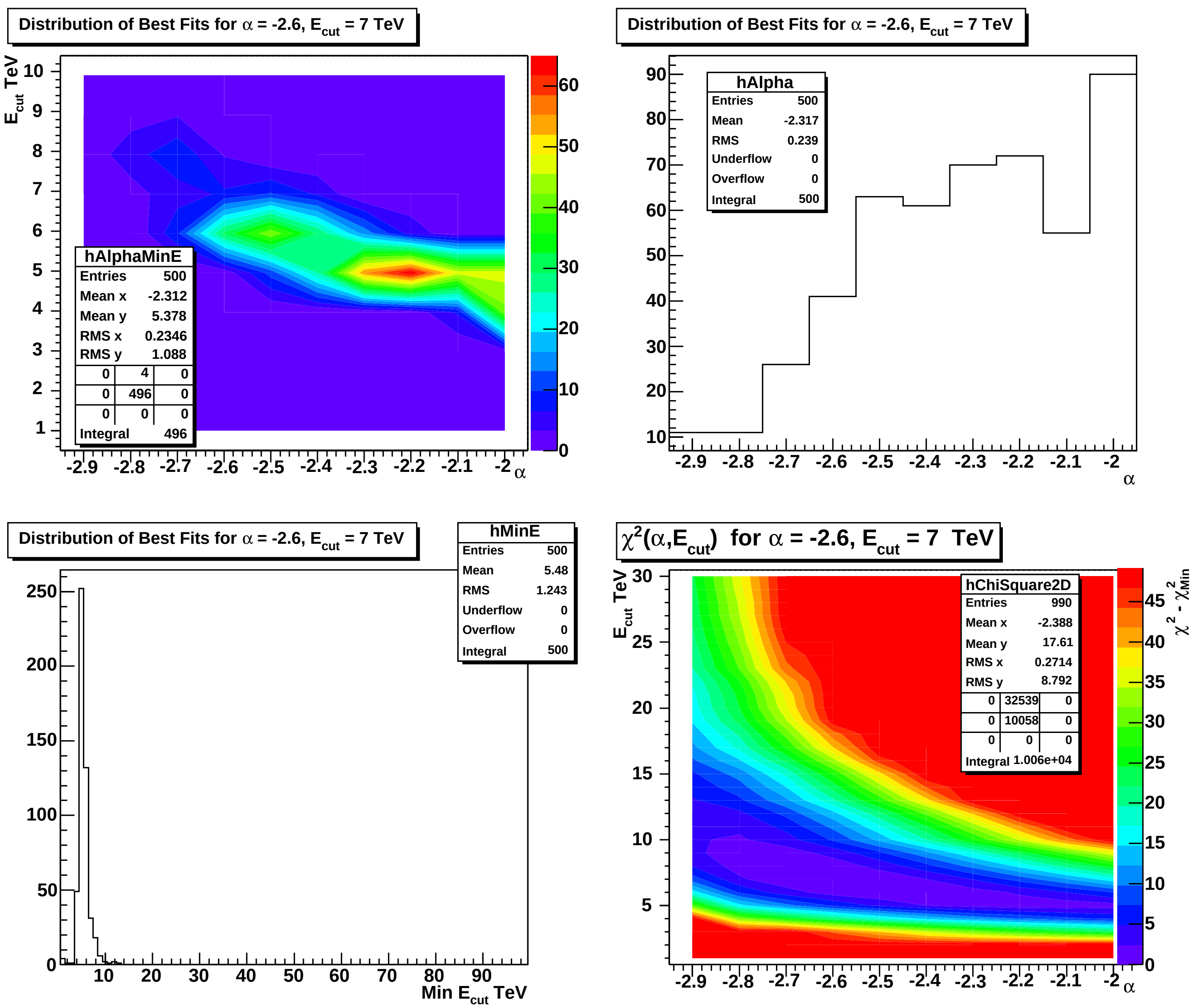

Figure 2: Best Fits for an input spectrum of α = -2.6 and E_{cut} = 7 TeV

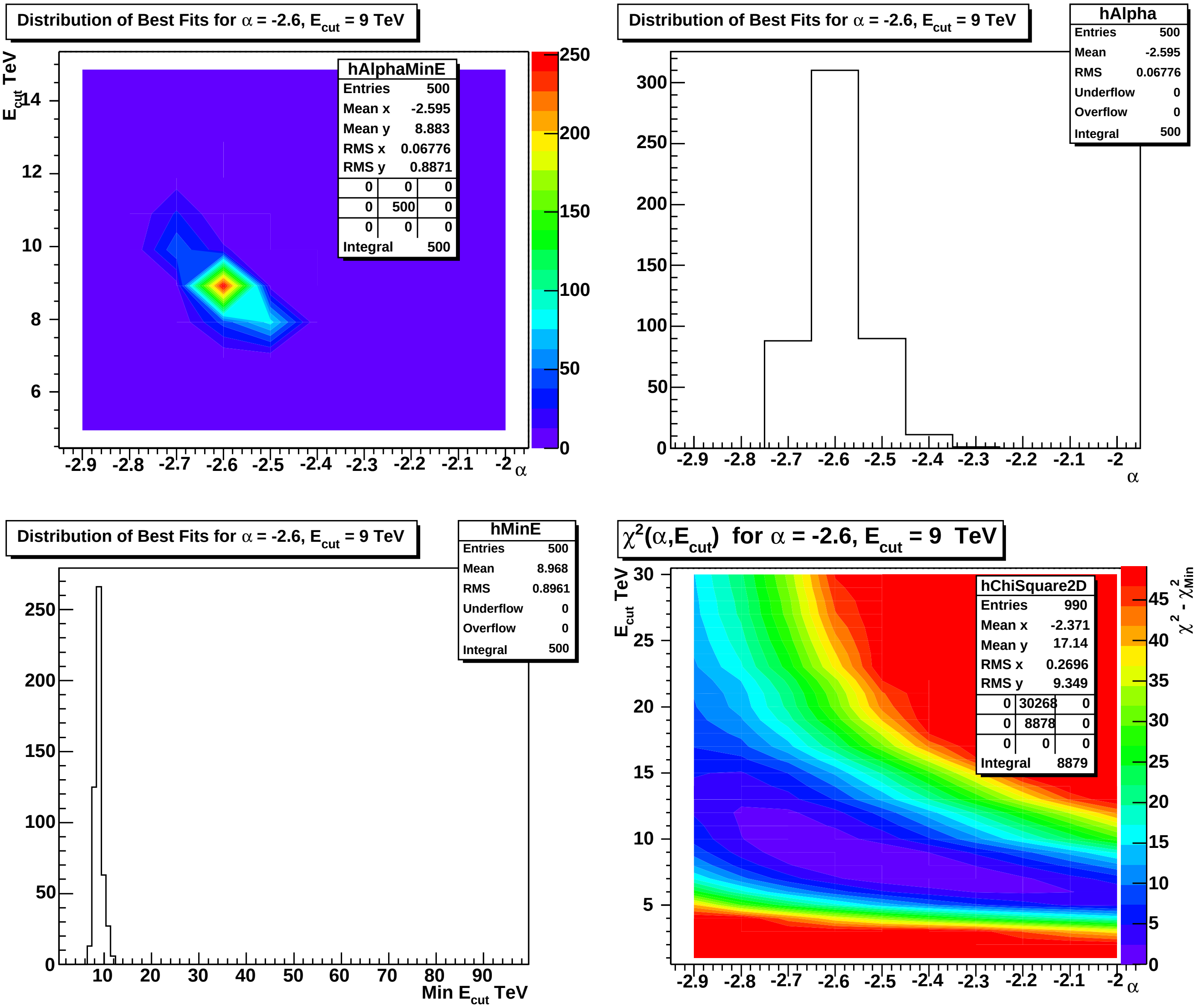

Figure 3: Best Fits for an input spectrum of α = -2.6 and E_{cut} = 9 TeV

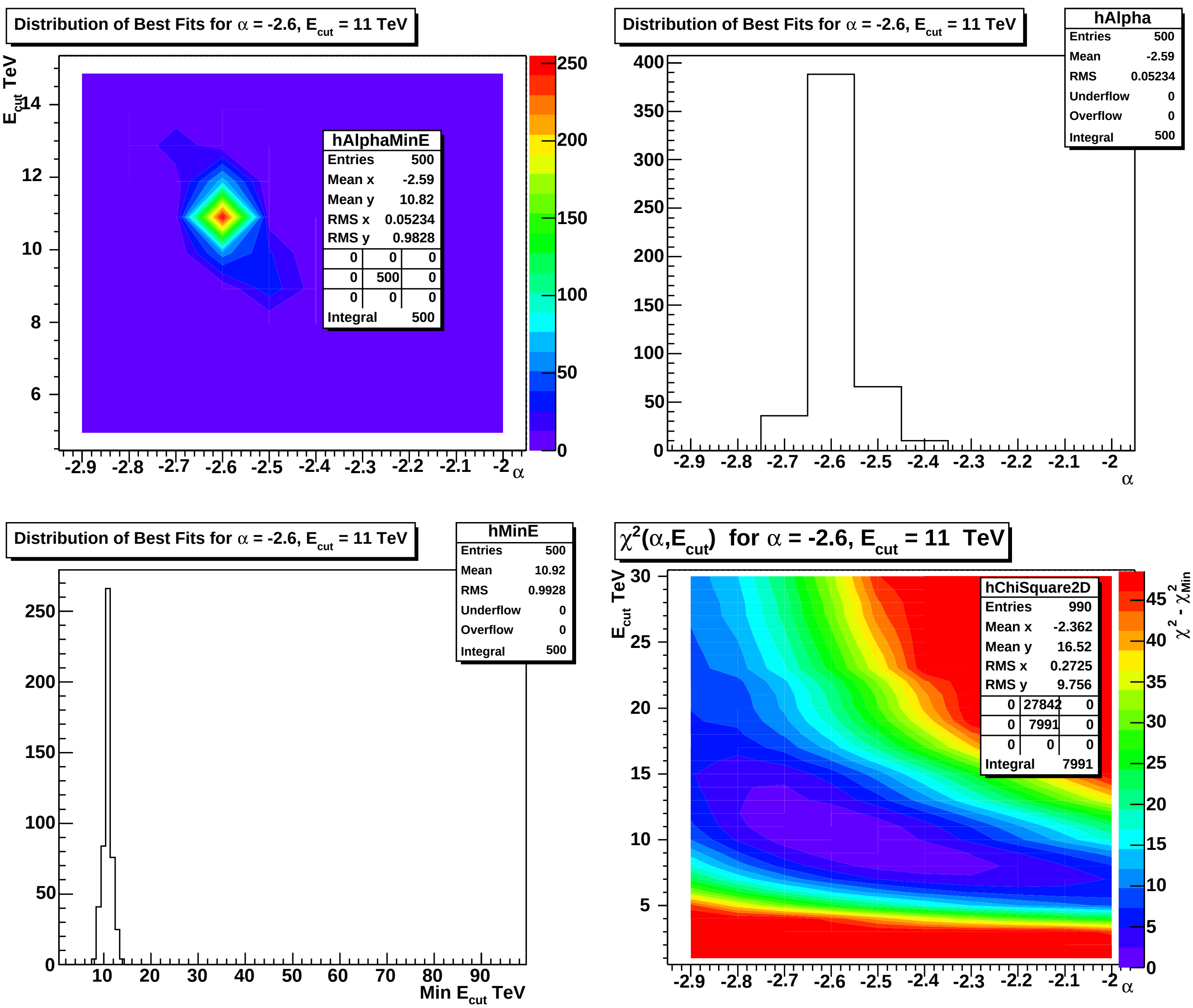

Figure 4: Best Fits for an input spectrum of α = -2.6 and E_{cut} = 11 TeV

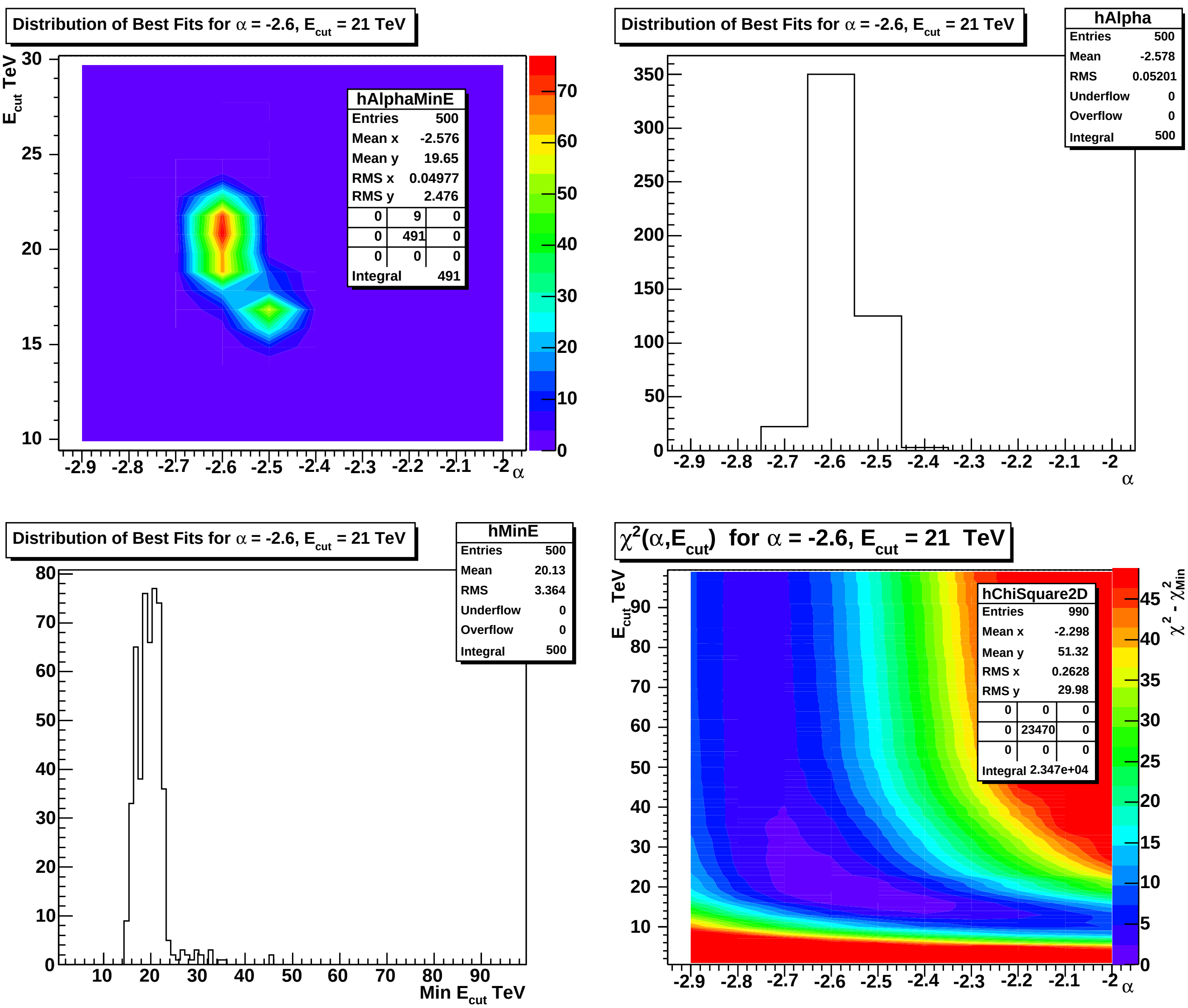

Figure 5: Best Fits for an input spectrum of α = -2.6 and E_{cut} = 21 TeV

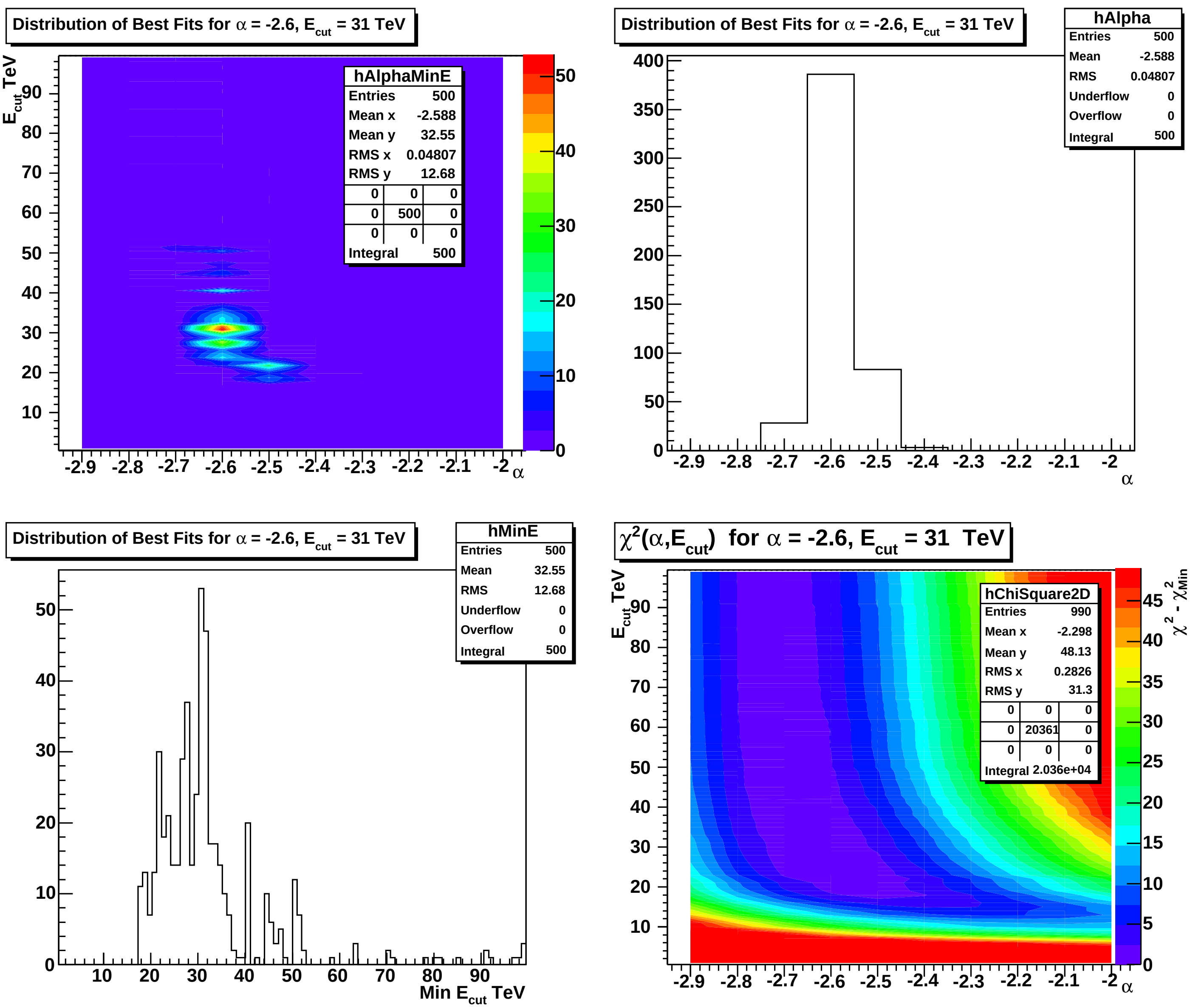

Figure 6: Best Fits for an input spectrum of α = -2.6 and E_{cut} = 31 TeV

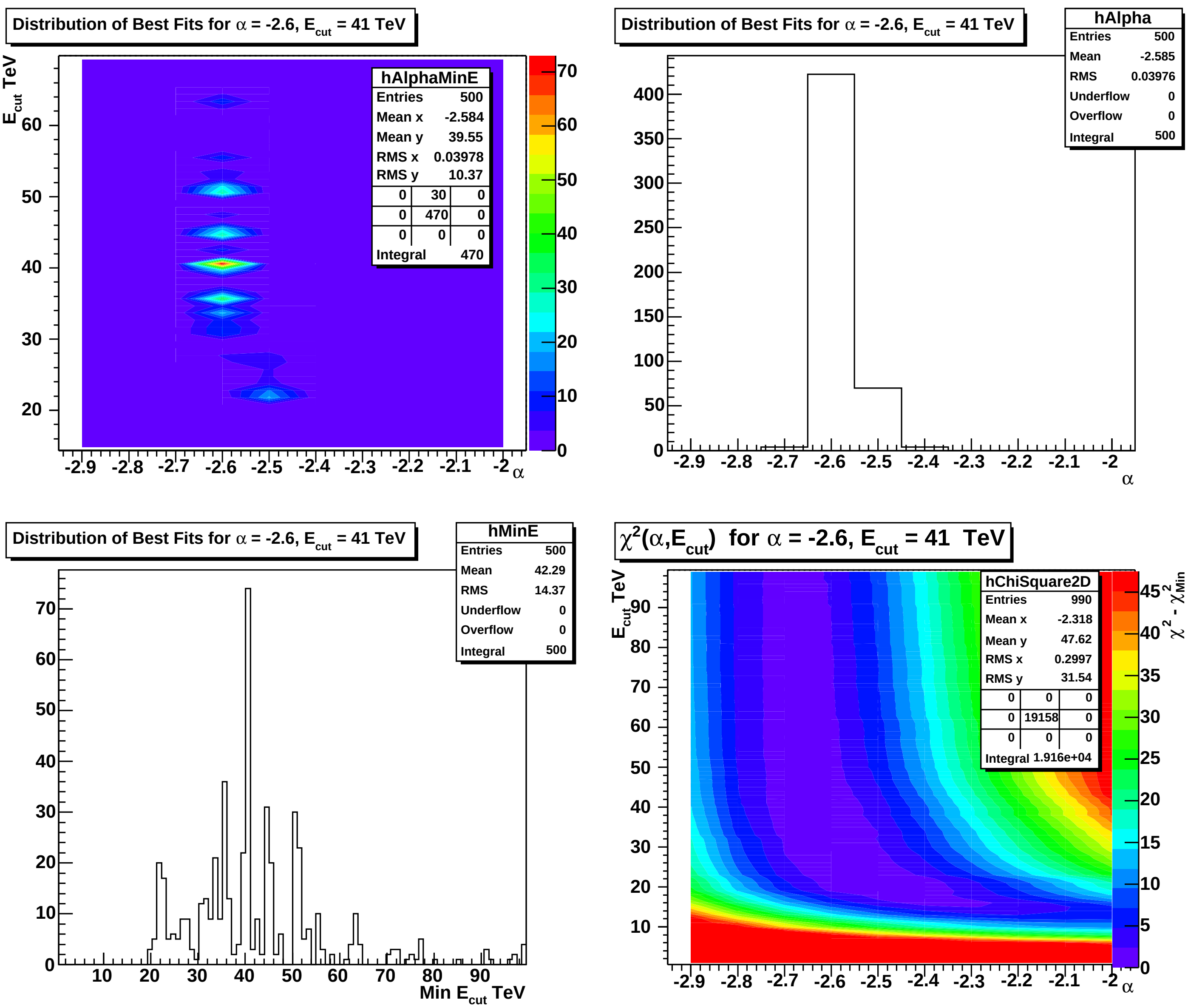

Figure 7: Best Fits for an input spectrum of α = -2.6 and E_{cut} = 41 TeV

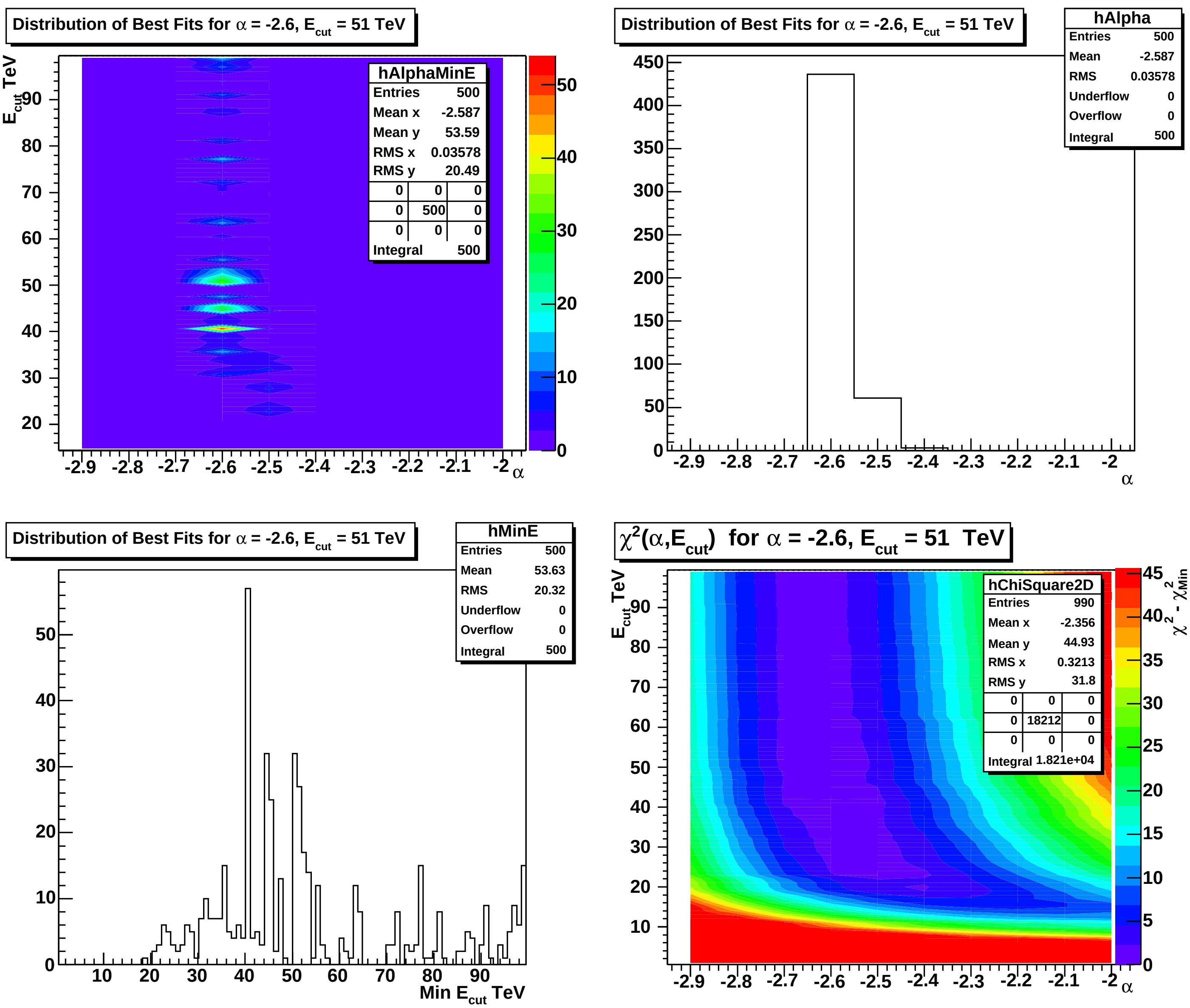

Figure 8: Best Fits for an input spectrum of α = -2.6 and E_{cut} = 51 TeV

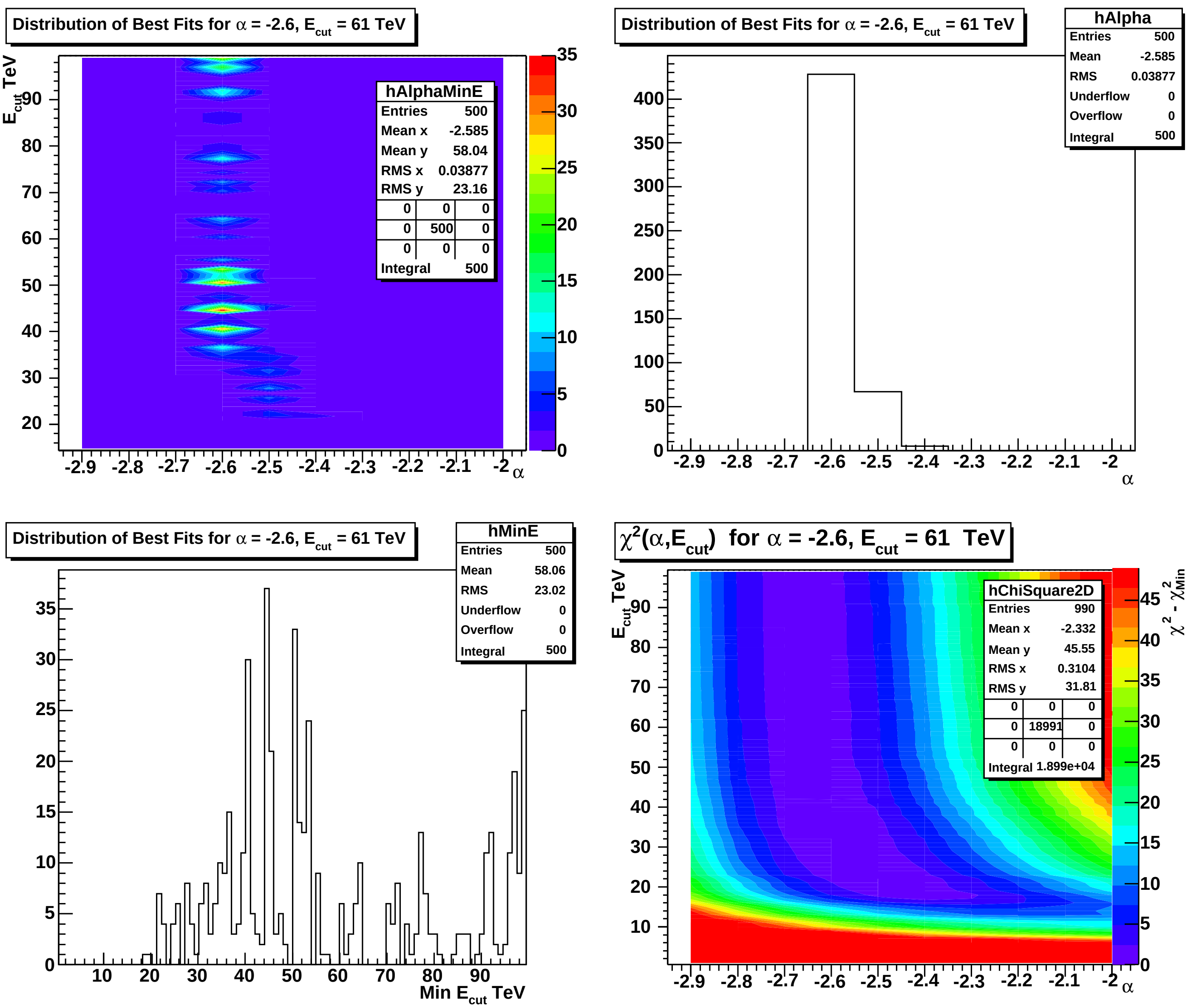

Figure 9: Best Fits for an input spectrum of α = -2.6 and E_{cut} = 61 TeV

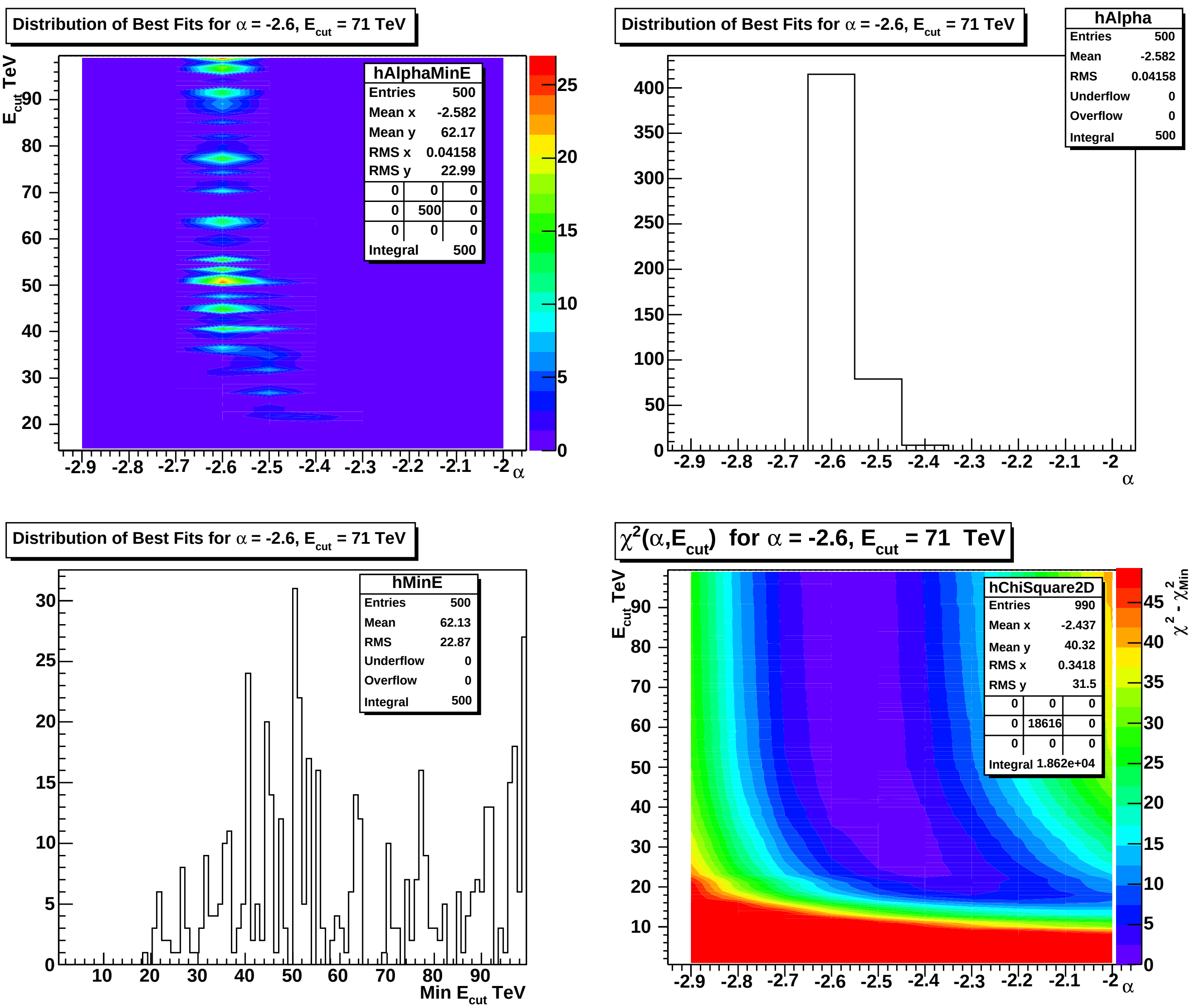

Figure 10: Best Fits for an input spectrum of α = -2.6 and E_{cut} = 71 TeV

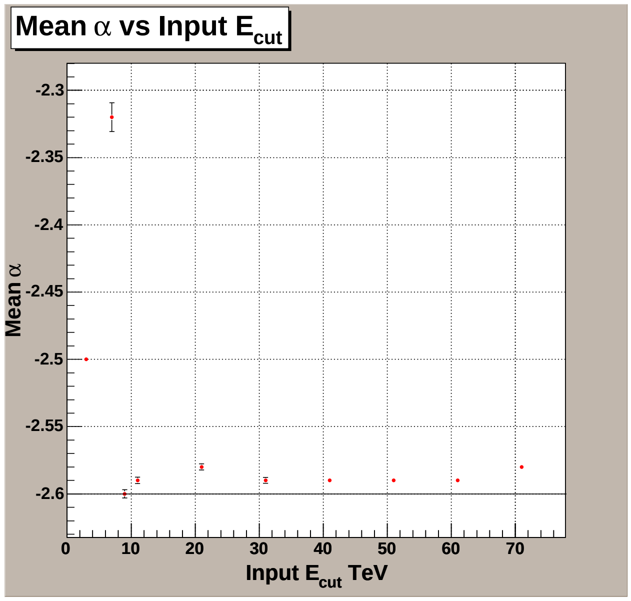

Figure 11: Distribution of the Means of α as a function of the cutoff energy in the input spectrum for $\alpha = -2.6$.

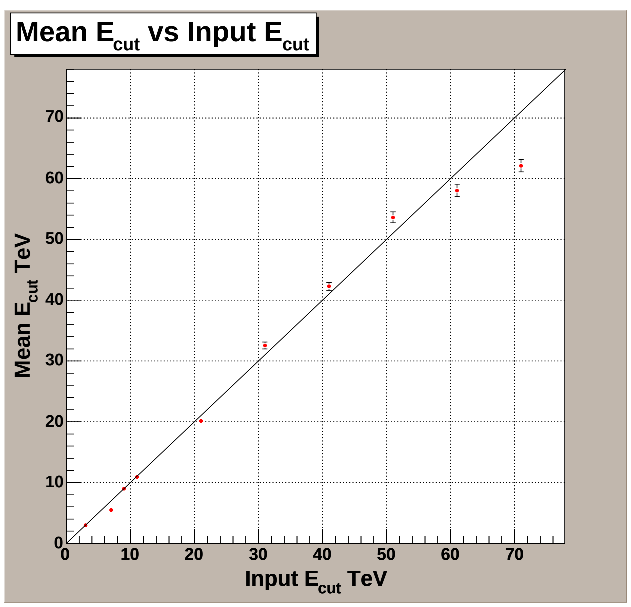

Figure 12: Distribution of the Means of E_{cut} as a function of the cutoff energy in the input spectrum for $\alpha = -2.6$.

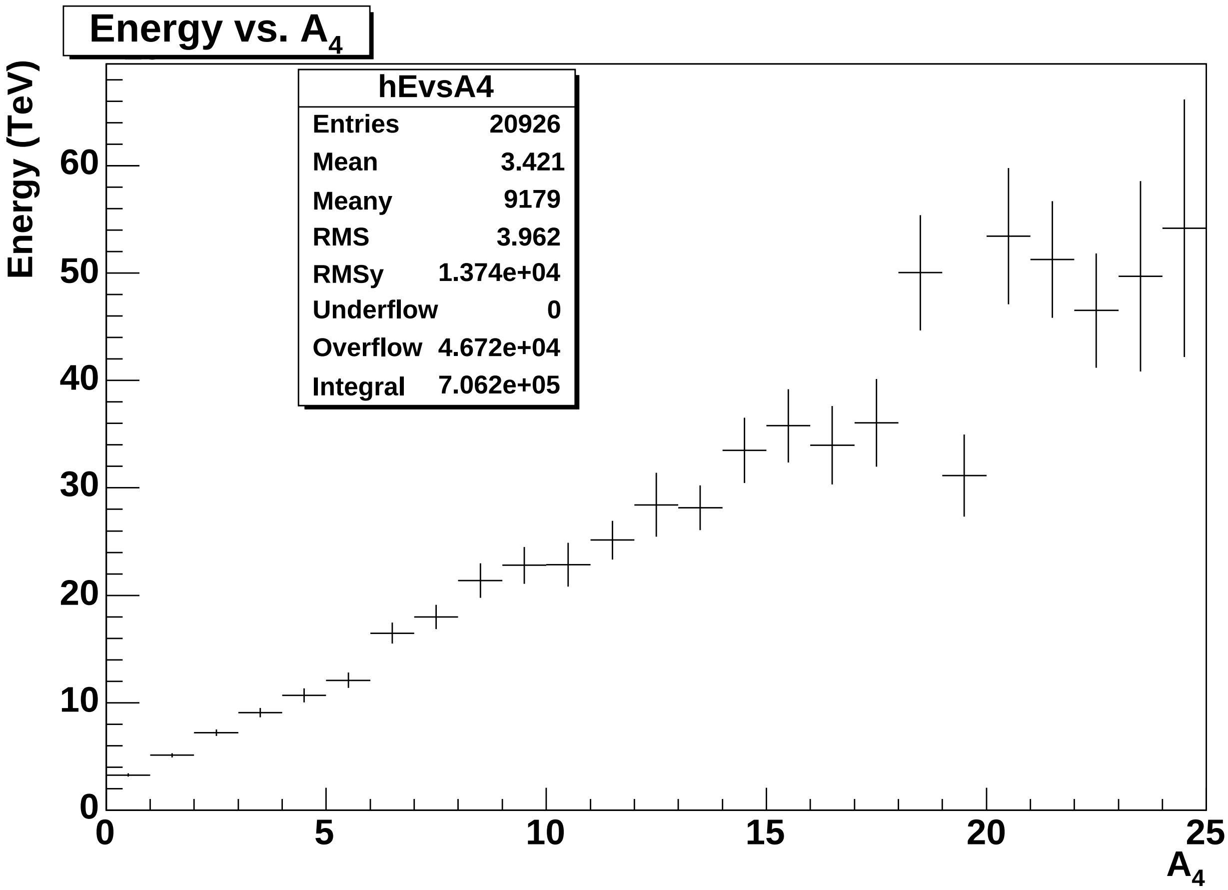

Figure 13: Energy vs A_4. Profile plot