A_4, Milagro's New γ-Hadron Separation Variable and Spectral Index Estimator

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Abstract

In this memo I introduce A_4, a new γ-Hadron separation variable and spectral index estimator in Milagro. With A_4, Quality factors as high as **3.0** are obtainanble. For hard A_4 cuts, signal to background ratios as high as **60%** are obtainable. For $\approx$ 1.54 years of data, a statistical significance of **10.55** on the Crab Nebula was obtained with A_4 when the weighting analysis was used, while the standard A_4 cut of $A_4 \geq 3.0$ gave a statistical significance of 8.02 for the same data set. I also show a new technique for measuring the spectral index of a γ-ray source in Milagro using A_4. This technique makes use of the energy dependence on A_4. The spectral index of the Crab obtained with this technique is $\alpha = \mathbf{-2.57 + (0.12 - 0.11)^{stat}}$. This value agrees with those measured by other experiments in the same energy range as Milagro.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

1 Data sets

In this analysis I used two different Monte Carlo simulations, GEANT 3.0 simulations, which was used in section 2, and GEANT 4.0 simulations V1.2, which was used in sections 3, 4, and 5. The data set used come from sub run 5953_108, taken on Oct. 19, 2004 at 06:28 UT. The number of events in this data sample is 355,475. In both cases of the Monte Carlo, the right experiment configurations were simulated, this includes the dead PMTs, the PMTs that didn't enter in the fit, the level of the water on top of the cover, and the air under the cover. In GEANT 3.0 simulations, only protons were simulated as background, while in GEANT 4.0 V1.2, helium and proton were simulated as background.

The Crab data set used in this analysis is the tped-reconstruction, recently produced by Curtis[4]. In this reconstruction, the airshower layer and the outriggers were used in the angular reconstruction. This data set includes ~1.54 years of data collected between Sept. 2003 and May 2005.

2 A_4

A_4 is a modified version of the variable I introduced in section 2.2 of my February 2005 memo[1]. A_4 is defined as:

$$A_4 = \frac{(fTop + fOut) \times nFit}{mxPE} \quad (1)$$

where

- $fTop$ is the fraction of the air shower layer PMTs hit in an event.[1]
- $fOut$ is the fraction of the outriggers hit in an event. [2]
- $nFit$ is the number of PMTs that entered in the angle fit.
- $mxPE$ is the number of PEs in the muon layer PMT with the highest hit.

The reason for using the fraction of the airshower layer and outriggers hit and not the actual numbers of the tubes hit is the fact that I want to

[1] $fTop = \frac{nTop}{450}$, where nTop is the number of air shower PMTs hit in an event.
[2] $fOut = \frac{nOut}{175}$, where nOut is the number of outriggers hit in an event.

give a higher weight for the outriggers in my variable. This is done for many reasons. One of these reasons is that events with cores on the pond seem to be more hadron like, while events with cores off the pond seems to be more γ-ray like. Also, events with large number of Outriggers hit have better angular and core resolutions. The use of mxPE instead of cxPE is due to the fact that cxPE was not calculated in most of the online reconstructed data, so instead mxPE was used. Also, cxPE was originally designed for HAWC so that γ-ray events with cores on the pond are not mistakenly removed when applying the γ-hadron cut. For HAWC this should work fine since it should have a core finder with better core resolution than Milagro.

The first part in the numerator of A_4 carries information about the size of the shower, while nFit carries information about how well the shower was reconstructed. mxPE carries information about the clumpiness in the Muon layer that is due to the penetrating Muons and hadrons which are mostly presented in hadronic air showers.

Figure 1 shows A_4 distributions for gamma Monte Carlo, proton Monte Carlo, and data. Figure 2 shows the efficiencies for retaining data, proton Monte Carlo, and gamma Monte Carlo as a function of A_4. In both figures we see a clear difference between the Monte Carlo gamma ray showers and the proton showers, while there is a good agreement between the data and the proton Monte Carlo. Figure 3 shows the Q-factor as a function of the minimum value of A_4 required to retain an event. Requiring events to have $A_4 \geq 3.0$ and $nFit \geq 40$ rejects 97.3% of the simulated proton induced air showers that trigger Milagro and 98.4% of the data (for this data sample), and retains 35% of the gamma-ray induced air showers. This results in a predicted Q-factor of 2.2 comparing Monte Carlo proton events to Monte Carlo gamma-ray events, and 2.7 comparing the data to Monte Carlo gamma-ray events.

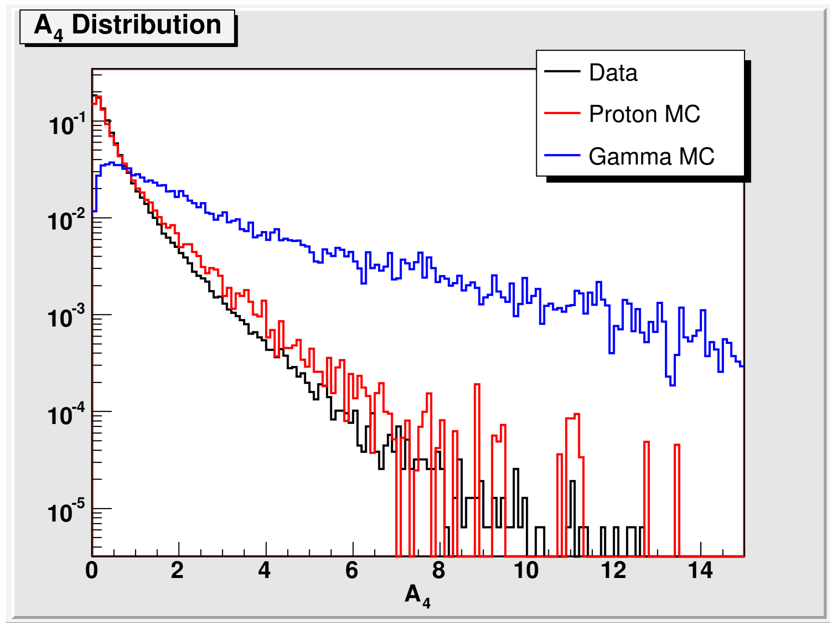

Figure 1: Distribution of A_4 for gamma Monte Carlo, proton Monte Carlo, and data.

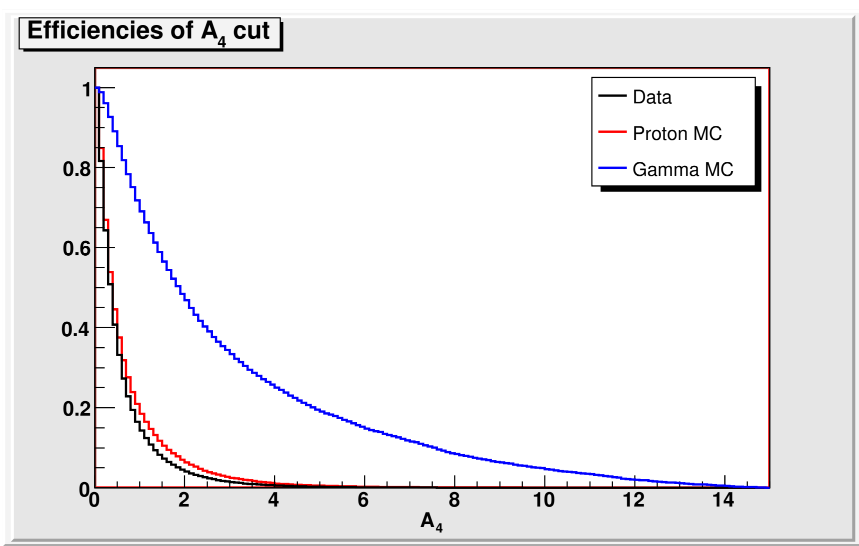

Figure 2: Efficiencies for retaining data, proton Monte Carlo, and gamma Monte Carlo as a function of A_4.

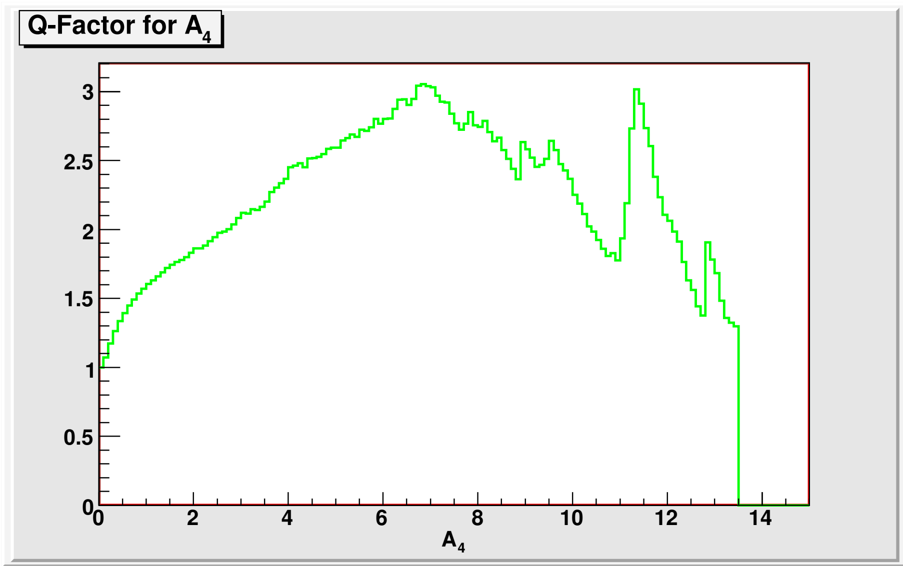

Figure 3: Q-factor as a function of the minimum value of A_4 required to retain an event.

3 Tests of A_4 on the Crab Nebula

Figure 4 shows a map of the statistical significance around the Crab Nebula with the $A_4 \geq 3.0$ and $nFit \geq 40$ cuts applied. In this map the Crab Nebula is seen at 8.02 σ. Figure 5 shows a map of the statistical significance around the same region for a harder A_4 cut of 12 and $nFit \geq 40$. The Crab Nebula is seen at 5.58 σ. Although there was a $\approx 30\%$ loss in statistical significance of the Crab Nebula when the harder A_4 cut was applied, the main advantage of applying the hard A_4 cut is the higher S/B ratio (60.0%) achieved with this cut compared to that with the soft A_4 cut (3.4%).

Figure 6 shows the distribution of the statistical significances in the location of the Crab Nebula for different A_4 cuts. As seen from this figure, the highest significance of the Crab is achieved when the $A_4 \geq 3.0$ cut is applied. Figure 7 shows the distribution of Excesses in the location of the Crab as a function of different A_4 cuts.

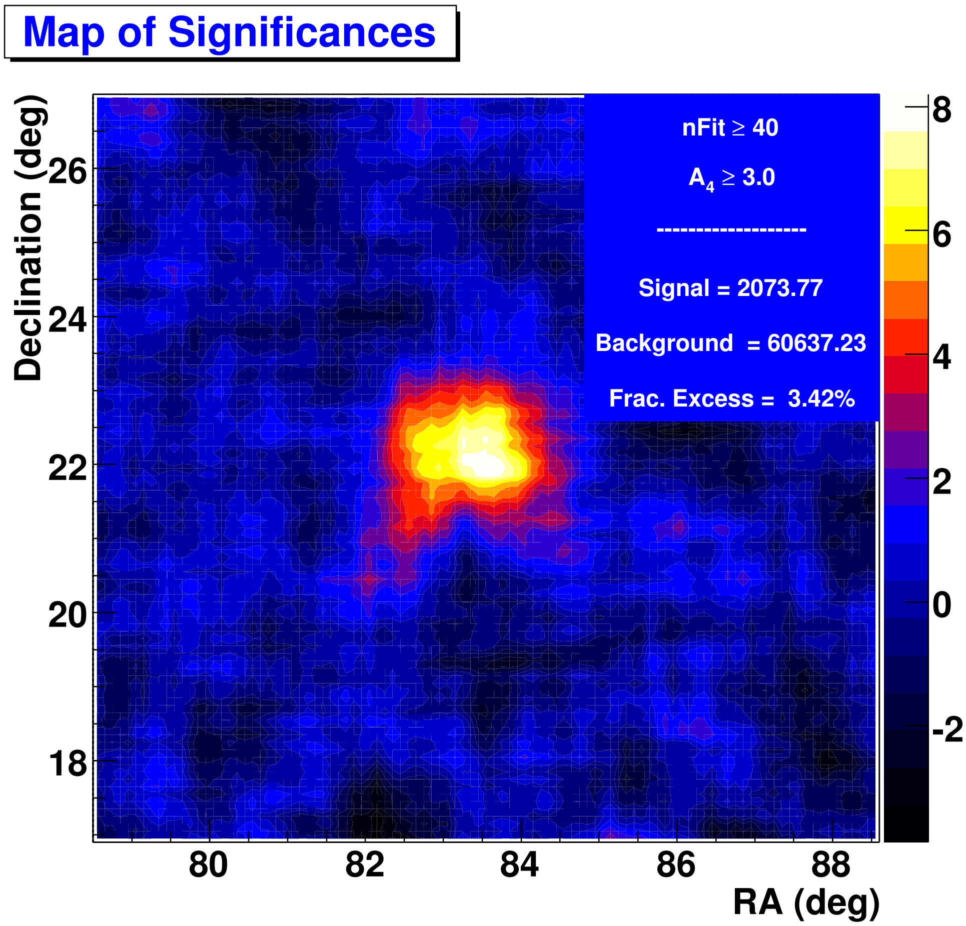

Figure 4: Map of the statistical significance around the Crab Nebula with the $A_4 \geq 3.0$ and $nFit \geq 40$ cuts applied.

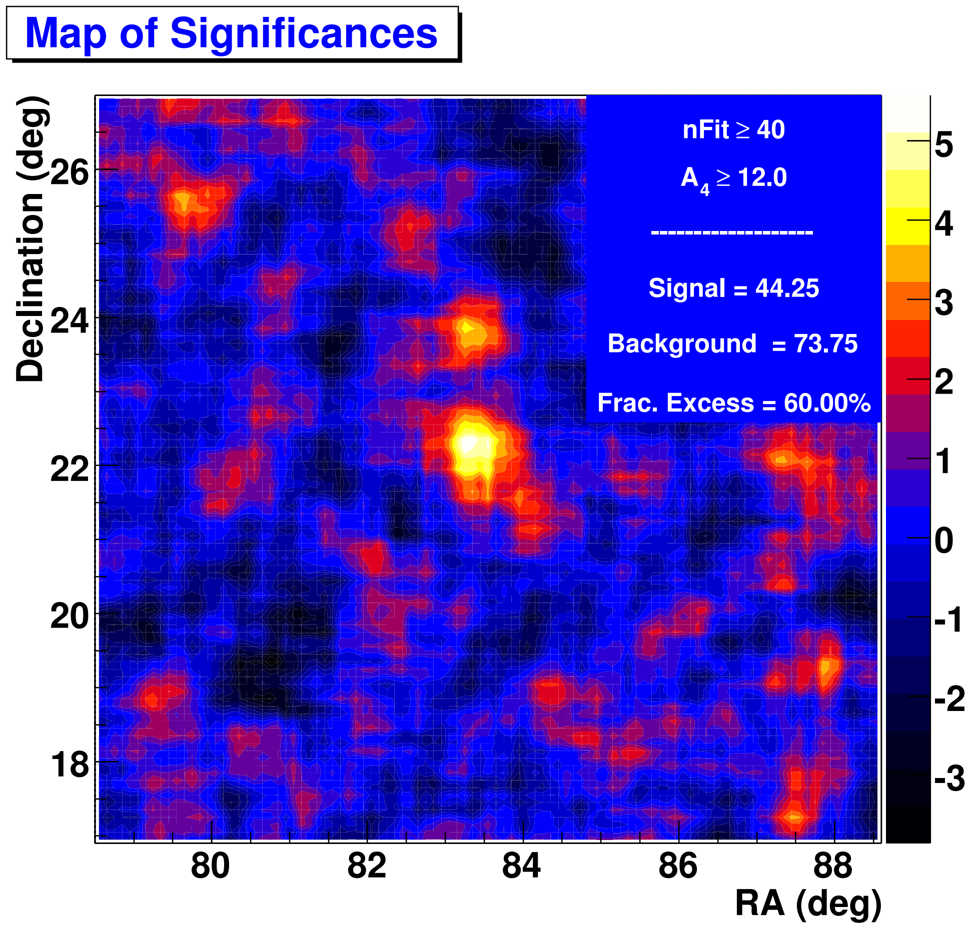

Figure 5: Map of the statistical significance around the Crab Nebula with the $A_4 \geq 12.0$ and $nFit \geq 40$ cuts applied.

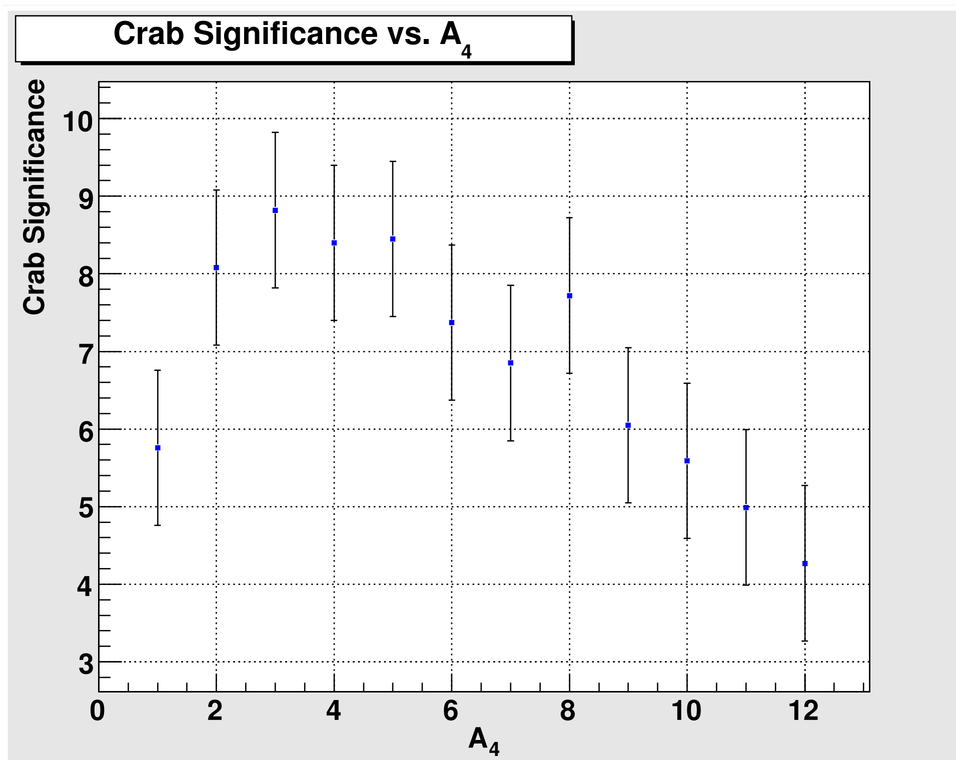

Figure 6: Distribution of statistical significances in the location of the Crab Nebula as a function of different A_4 cuts.

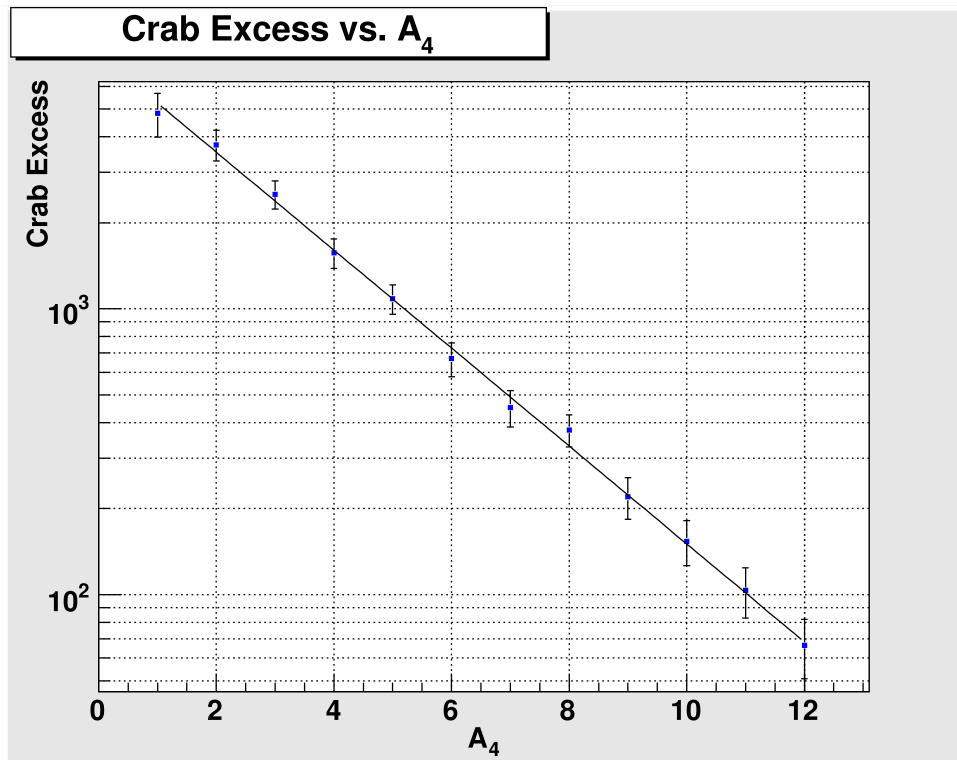

Figure 7: Distribution of Excesses in the location of the Crab Nebula as a function of different A_4 cuts.

4 γ-Hadron Relative Weighting

In [2] Andy introduced the weighting analysis of the Crab data. Here, I show a similar weighting analysis of the Crab data using A_4.

The data set is sliced in 12 bins in A_4. In each of these bins, events with A_4 value greater than or equal to the lower end of the bin and smaller than the upper end of the bin are kept in that bin. i.e. for the ith A_4 bin, only events that satisfy the criteria

$$B_i^{min} \leq A_4 < B_i^{max} \tag{2}$$

are kept in that bin (with the exception of the last bin, for which the upper end is ∞), B_i^{min} and B_i^{max} being the bin's lower and upper limits, respectively.

Table 1 lists the set of cuts applied for each slice along with the number of gamma Monte Carlo events expected in that slice $\langle S_i \rangle$, the number of measured background events in the same slice $\langle B_i \rangle$, and the weight assigned for that slice ω_i. The weight assigned for the ith slice is equal to[3]:

$$\omega_i = \frac{\langle S_i \rangle}{\langle B_i \rangle} \tag{3}$$

All weights have been normalized to that of the first slice. Figure 8 shows the map of statistical significance around the Crab with the A_4 weighted analysis applied. The significance at the location of the Crab is 10.55 σ. This is an increase by 32% over the significance achieved with the standard A_4 cut ($A_4 \geq 3.0$).

Slice Number	Cuts	N_γ^{Exp}	N_B^{Meas}(x 10^6)	Weight
1	$1 \leq$ A4 < 2	1262.5	409.314	1.00
2	$2 \leq$ A4 < 3	700.3	126.831	1.79
3	$3 \leq$ A4 < 4	410.1	50.166	2.65
4	$4 \leq$ A4 < 5	254.1	23.601	3.49
5	$5 \leq$ A4 < 6	182.4	13.055	4.53
6	$6 \leq$ A4 < 7	162.2	8.404	6.26
7	$7 \leq$ A4 < 8	161.4	6.186	8.46
8	$8 \leq$ A4 < 9	49.5	1.335	12.03
9	$9 \leq$ A4 < 10	32.2	0.743	14.03
10	$10 \leq$ A4 < 11	27.9	0.479	18.90
11	$11 \leq$ A4 < 12	7.4	0.275	26.93
12	$12 \leq$ A4 $< \infty$	5.1	0.159	32.01

Table 1: List of the set of cuts applied for each slice along with the number of gamma Monte Carlo events expected in that slice, the number of measured background events in the same slice, and the weight assigned for each slice. All weights have been normalized to that of the first slice.

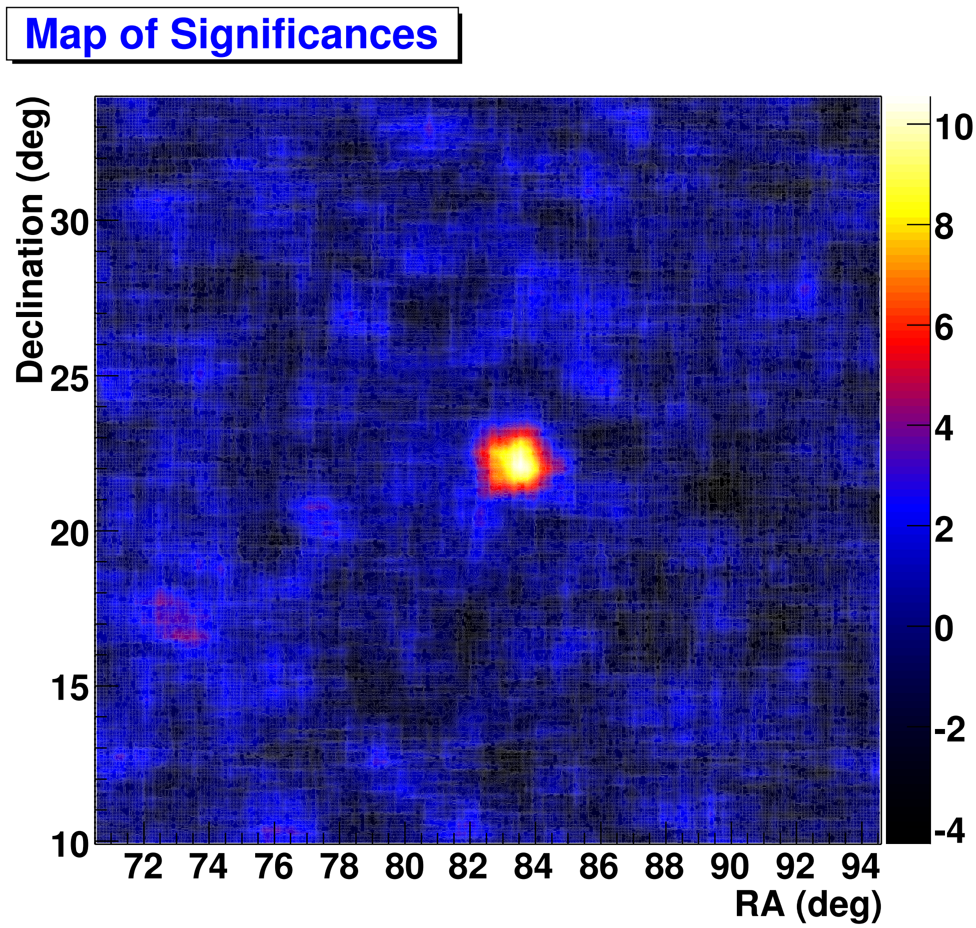

Figure 8: Map of the statistical significance around the Crab Nebula with the new γ-Hadron weighting analysis method applied.

5 Spectral Index Analysis

In this section I introduce a new technique for measuring the spectral index of a γ-ray source in Milagro using A_4.

5.1 Energy Dependence on A_4

In order to be able to measure the spectral index of a γ-ray source one needs to have a variable that is well correlated with energy, A_4 is such a variable. Figure 9 shows the relation between the energy and A_4 for gamma Monte Carlo. As can be seen in this figure, there is a very good correlation between the energy of a γ-ray event and the A_4 value of that event in the energy range 2-20 TeV.

5.2 Spectral Index Determination Technique

In order to determine the spectral index of a γ-ray source, the following steps were done:

- Eleven different gamma Monte Carlo sets were created. These data sets were simulated with different spectral indeces ranging from -2.0 to -3.0 in increment of 0.1.
- Excess from the data were binned in A_4, differentially.
- Gamma MC sets were binned in A_4, differentially.
- The different gamma MC differential distributions were fit to the differential excess from the data.
- χ^2 for each of these eleven fits were calcualted.
- A plot of these χ^2 values as a function of spectral index was generated.
- Minimum of this plot corresponds to spectral index of the source.

5.2.1 Crab Nebula Spectral Index Estimation

The Crab Nebula serves as a standard candle in γ-ray astronomy and a new technique is best tested on this steady source. In addition to this, the fact that this source has been well studied by many experiments in the same energy range as Milagro helps test the new technique by cross checking the results of this new technique with those of the other experiments.

Figure 10 shows the distribution of differential excess from the Crab Nebula as a function of A_4. The last bin in this figure contains all excess events with $A_4 \geq 12.0$, this is the reason why this bin has more events than the two previous bins. Figure 11 shows the A_4 differential distributions of four gamma Monte Carlo sets with spectral indeces -2.1, -2.3, -2.6, and -2.9. For comparison, the differential excess from the Crab (figure 10) is shown on each of the plots.

Figure 12 shows the distribution of the χ^2 values of the fits of the different gamma Monte Carlo stes to the Crab data as a function of the spectral index α. From this plot we see that the minimum χ^2 corresponds to a spectral index of:

$$\alpha_{crab} = -2.57 + (0.12 - 0.11)_{stat}$$

The statistical errors were obtained by fitting the distribution in figure 12 to a quadratic function and then going up one unit in χ^2 from the minimum. As can be seen from this figure, the distribution is not symmetric around the minimum, this is why the statistical errors are not equal. In total there are 12 degrees of freedom (ndf). After the subtraction of one degree of freedom for the minimization of χ^2 with respect to α and another one for the minimization of χ^2 with respect to the scaling factor[3], one ends up with 10 degrees of freedom. χ^2 at the minimum is equal to 13.9 this corresponds to chance probability of $\approx 18\%$

The value of α for the Crab obtained in this analysis is in good agreement with results form other expermiments. Table 2 lists Measurements of the Crab spectral index by other experiments in the same energy range as Milagro.

[3]The scaling factor of the i'th gamma MC distribution, S_i is defined as:

$$S_i = \frac{A_c}{A_i}\delta_i$$

where A_c and A_i are the areas under Crab differential excess and the i'th gamma MC differential distribution, respectively, and δ_i is a correction applied to S_i to minimize χ^2 of the fit of the i'th gamma MC distribution to the Crab data.

Tibet	$-2.62 \pm 0.17_{stat}$
HEGRA	$-2.59 \pm 0.03_{stat} \pm 0.05_{sys}$
Whipple	$-2.49 \pm 0.06_{stat} \pm 0.04_{sys}$

Table 2: Measurements of the Crab spectral index by other experiments in the same energy range as Milagro

5.2.2 Cosmic Rays Spectral Index Estimation

The same technique was applied to measure the spectral index of cosmic rays. To do this, eleven different proton and helium Monte Carlo sets with different spectral indeces were created. These data sets were simulated with different spectral indeces ranging from -2.0 to -3.0 in increment of 0.1. The data used to measure the spectral index is the Crab off source data. The analysis proceeds in the same steps as in the one previous section.

The result for the measurement of the cosmic rays spectral index is:

$$\alpha_{cr} = -2.786 + (0.088 - 0.108)_{stat}$$

This is in good agreement with BESS measurements in the energy range 30-540 GeV:

$$\alpha_{proton} = -2.732 \pm 0.011_{stat} \pm 0.019_{sys}$$

$$\alpha_{helium} = -2.699 \pm 0.04_{stat} \pm 0.044_{sys}$$

χ^2 at the minimum is equal to 7.54 this corresponds to chance probability of $\approx 67\%$

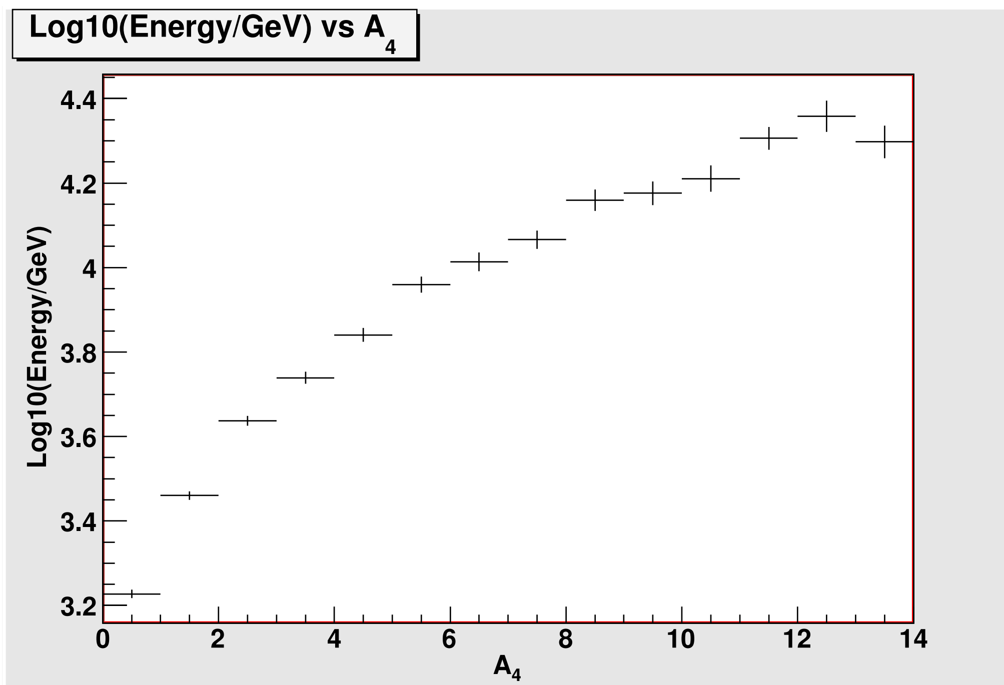

Figure 9: Energy of γ-ray events as a function of A_4

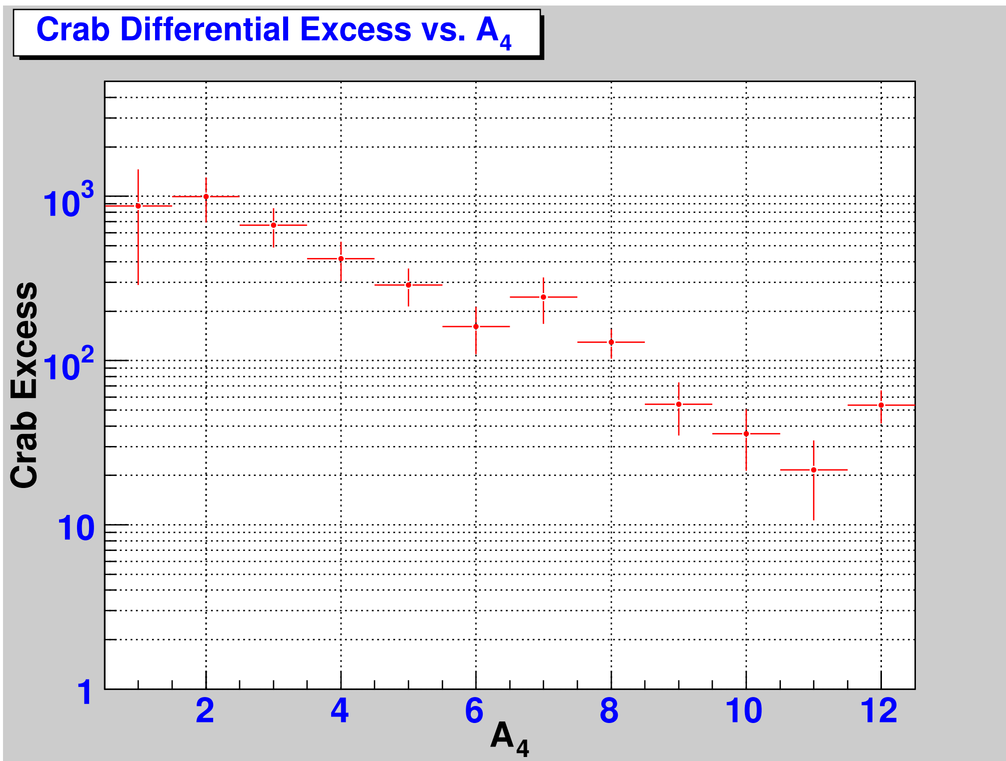

Figure 10: Differential excess from the Crab Nebula as a function of A_4

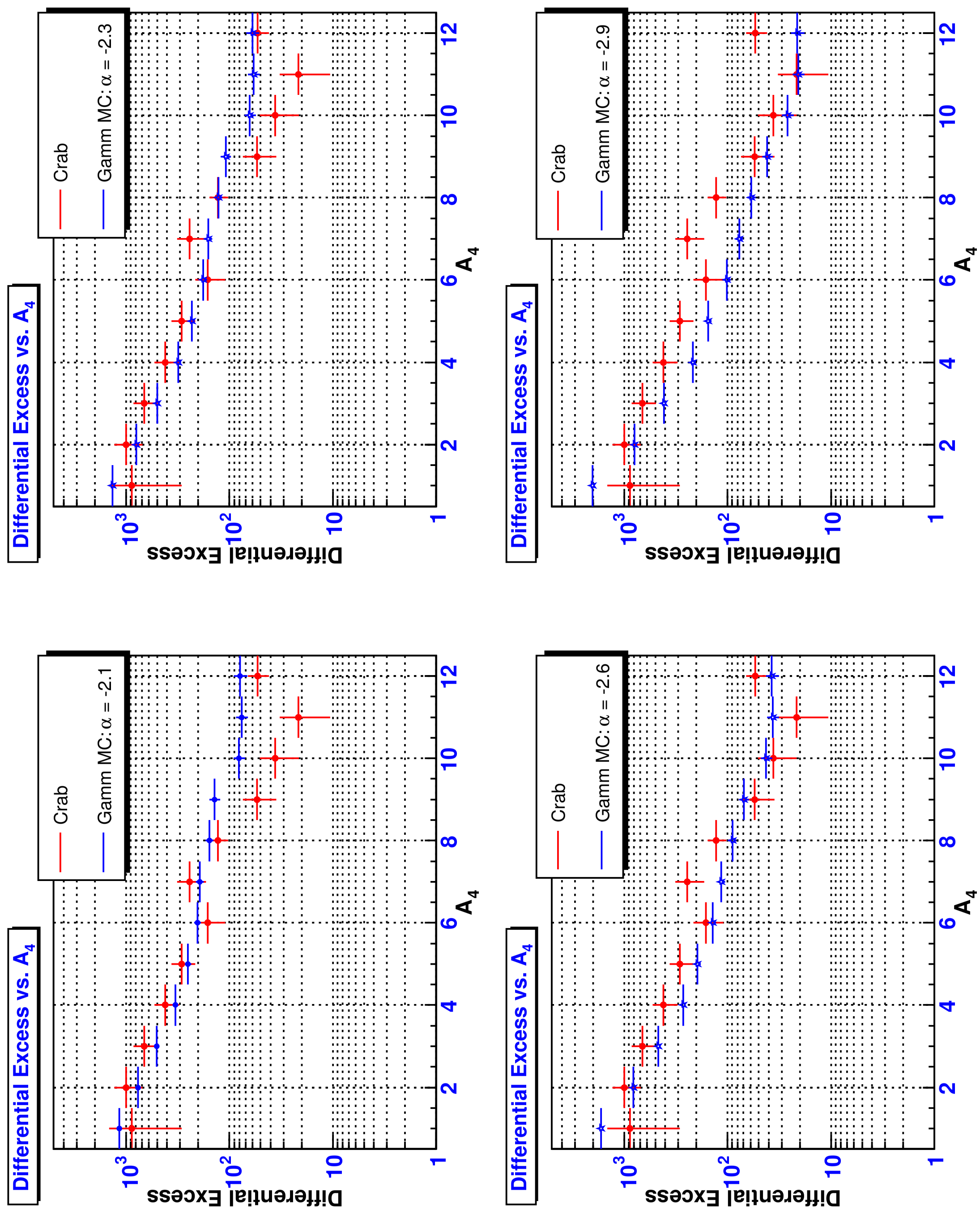

Figure 11: A_4 differential distribution of four gamma Monte Carlo sets with spectral indeces -2.1, -2.3, -2.6, and -2.9. For comparison, the differential excess from the Crab is shown on each of the plots.

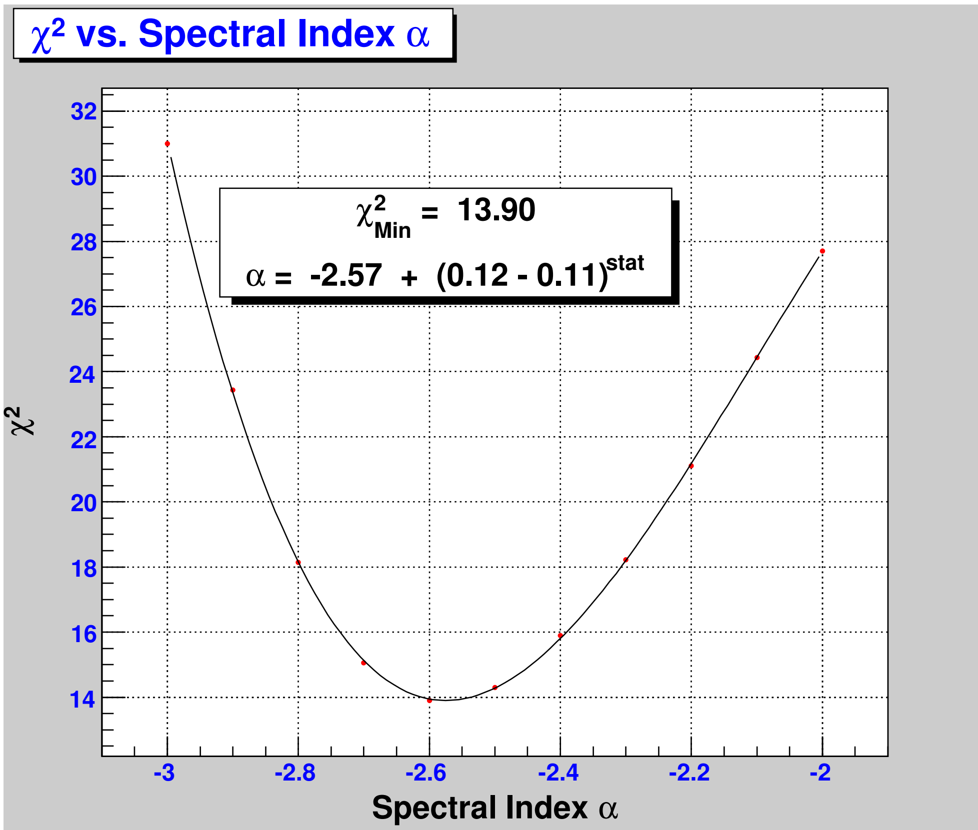

Figure 12: Distribution of the χ^2 values of the fits of the different gamma Monte Carlo sets to the Crab data as a function of the spectral index α.

References

[1] Abdo, A., New Gamma-Hadron Separation Technique and Variables in Milagro.
Milagro memo.

[2] Smith, A., A Weighting Analysis of Crab Data.
Milagro memo.

[3] Linnemann, J., Weighting for Bins of a Hadron Identification or Energy Variable.
Milagro memo.

[4] Lansdell, C., Timing Pedestal Shifting and the Crab.
Milagro memo.