PROMPT AND DELAYED EMISSION FROM GAMMA RAY BURSTS

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Gamma ray bursts are cosmological spots of short, intense and narrowly beamed γ emission, whose observed isotropic luminosities, L_{iso}, are in the range of $10^{51} - 10^{52}$ erg/s. The GRB variability time scale δt is correlated with the width of the spectrum peaks and is up to 10^4 times shorter than the duration T of the GRB, which ranges between 0.01 to more than 100 seconds.[4]

The huge energy released by GRBs is almost certainly produced by ultra-relativistic flows, whose bulk Lorentz factors Γ_b can vary between 100 and 1000, with some indications that values bigger than 500 are more probable. [5] The widely accepted fireball internal-external shock model for GRBs envisages that high energy protons and electrons are accelerated in shocks by interaction with magnetic fields. The process of Fermi acceleration leads to electron and proton power-law spectra of index between 2 and 3. Electrons and protons cool through synchrotron emission. Since the electron cooling time is shorter than the proton cooling time, as a first approximation, the GRB prompt spectrum is believed to be dominated by electron synchrotron emission only. The observed prompt spectrum arising from electron synchrotron emission is a Band function, proportional to E^{-1} below the synchrotron peak energy, E_{peak}, and to $E^{-\alpha}$ above the peak energy, with α usually bigger than 2. Below the synchrotron self absorption energy, E_{ssa}, low energy photons are absorbed by fireball electrons in a magnetic field by synchrotron self-absorption. Ghirlanda et al. [6] derived the following relation between the peak energy and the isotropic equivalent energy $E_{\gamma,iso}$

$$E_{peak} = 258 \quad (\frac{E_{\gamma,iso}}{1.2 \times 10^{53}})^{0.40 \pm 0.05} \quad , \tag{1}$$

where $E_{\gamma,iso} = T\,L_{iso}$.

According to some authors TeV photons are created during the prompt phase, either through proton synchrotron or inverse Compton. TeV photons are mostly absorbed through pair production with cosmic background radiation (CBR), but some indications of TeV gamma rays from GRBs at low redshift were provided by experiments like Milagro [7], Hegra and Tibet. For values of the Band spectrum slope $\alpha < 2$, TeV prompt energy photons are taken into account in the prompt spectrum without investigating the details of the way they are created.

synchrotron-peak

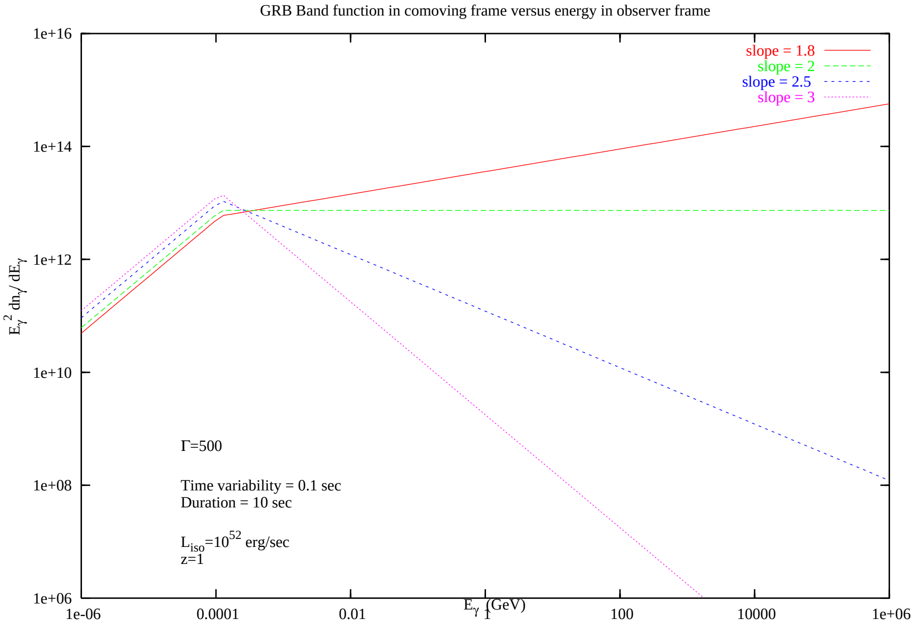

Figure 1: *Band spectrum*

GRB emission is narrowly beamed. According to the uniform jet model for GRBs the distribution of energy inside the jet is uniform and drops outside the opening angle θ. The product $\theta^2 L_{iso}$ seems to be constant. The competing model, called structured-jet model, proposes a universal non uniform distribution within the jet, which drops as the inverse square of angle from the jet axis. However, the structured-jet model predicts that $\theta^2 L_{iso}$ is constant, at least for an energy distribution $\propto \theta^{-2}$. Frail et al [8] found that when their emissions are corrected for the beaming, all GRBs release the same amount of energy, about 1.3×10^{51} erg. The average beaming factor ranges in the literature from around 75 [9] to 500 [8].

Photon-photon absorption through pair production inside GRBs controls the range in energy and the amount of radiation emitted. In Fig (2) we plot the optical depths corresponding to the different processes taking place inside GRBs, $\gamma\gamma$ absorption, electron Compton scattering and $e\,\gamma \to e^{\pm}$. The GRB isotropic luminosity is $L_{iso} = 10^{52}\,erg/s$ and the time variability is $\delta t = 0.01s$. $\gamma\gamma$ pair production within the GRB attenuates the prompt spectrum, while electron Compton scattering and $e\,\gamma \to e^{\pm}$ have optical depths less than 1 for every set of values of the isotropic luminosity, the Lorentz factor and the time variability. The optical depth for γ-ray absorption through pair production is

$$\tau_{grb} = \frac{r_{sh}}{\Gamma_b} \int \sigma_{\gamma\gamma} \frac{dN_\gamma}{dE_\gamma} \, dE_\gamma \tag{2}$$

where the GRB shell radius is $r_{sh} = 2\, c\, \delta t\, \Gamma_b$, $\frac{dN_\gamma}{dE_\gamma}$ is the GRB synchrotron spectrum and the cross section σ_{pair}

$$\sigma_{pair} = \frac{3}{16}\, \sigma_{th} (1-\beta^2)\, (2\, \beta\, (\beta^2 - 2) + (3 - \beta^4\, log((1+\beta)/(1-\beta))) \tag{3}$$

with

$$\beta = \sqrt{1 - \frac{(m\, c^2)^2}{E_\gamma\, \epsilon_{cmb}}}\, . \tag{4}$$

Prompt photons are absorbed above the cutoff energy E_{cutoff} [12]

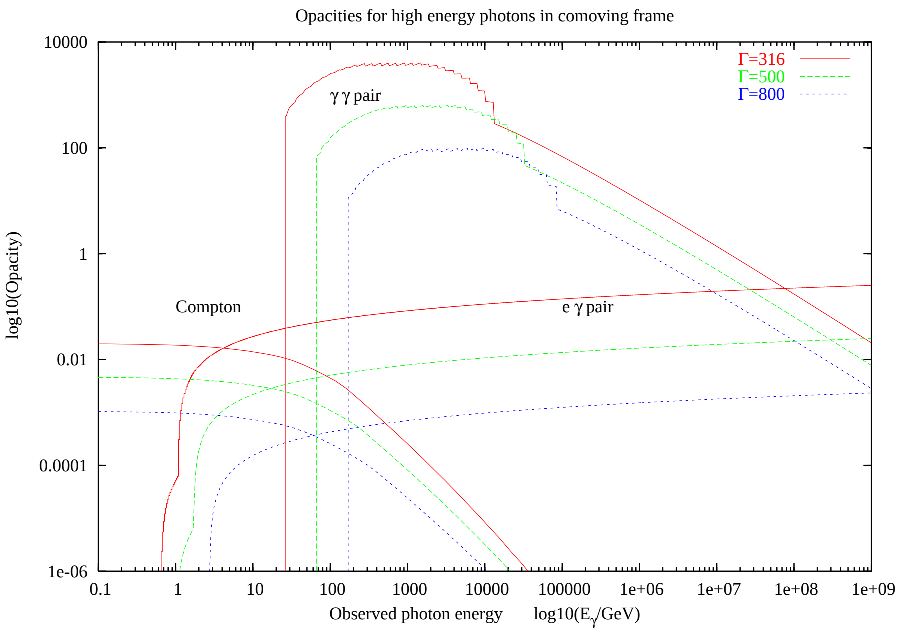

Figure 2: *Optical depths in the GRB fireball.*

$$E_{cutoff} = \frac{m_e^2\, c^4 \Gamma_b{}^2}{2\, E_{peak}}\, , \tag{5}$$

corresponding to the photon threshold energy for pair production with lower energy photons. The prompt photons re-appear again above the so called thinning energy, the photon energy corresponding to optical depth $\tau_{\gamma\gamma} = 1$

$$E_{thinning} = \frac{3\Lambda L_{iso}\sigma_{TH} m_e^2}{64\pi\Gamma^2\delta t\, {E_{\gamma\, ssa}}^2} \tag{6}$$

where $\Lambda = log[2\,(2E_{\gamma\, ssa}E_\gamma)^{1/2}]/(m_e\,\Gamma_b)$. For high enough bulk Lorentz factors and for short enough time variabilities, GRBs are optically thin at all energies. In Fig (3), we plot the predicted optical depths for GRB 010921, which has time variability of 2 seconds, duration of 24.6 seconds and fluence of the order of $1.02\,10^{-5}\,\mathrm{erg/cm^2}$ in the 30-400 KeV range. The isotropic luminosity is obtained by multiplying the fluence by $4\,\pi {D_L}^2$, where D_L is the luminosity distance, and dividing it by the GRB duration T. High energy

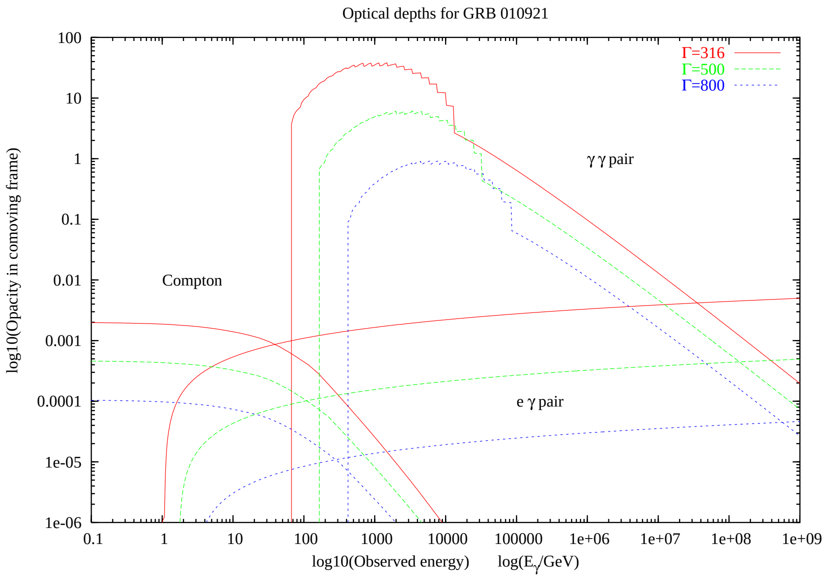

Figure 3: *Optical depths for GRB 010921*

gamma rays are attenuated also when travelling to us because they form pairs in collisions with low energy photons from the meta-galactic radiation field. Following Kneiske et al. [13], the optical depth of gamma ray depending on the redshift z and the gamma ray energy E_γ is

$$\tau_{bkg_{\gamma\gamma}}(E_\gamma, z) = z^{1.33}\,(E_\gamma/E_0)^{3/2} \tag{7}$$

where $E_0 = 90$ GeV. The parameterisation in Eq.(7) is valid up to a redshift of 1. For higher redshifts the attenuation of gamma ray does not depend on the redshift and we parametrize the optical depth as done by [14] (Fig(4)).

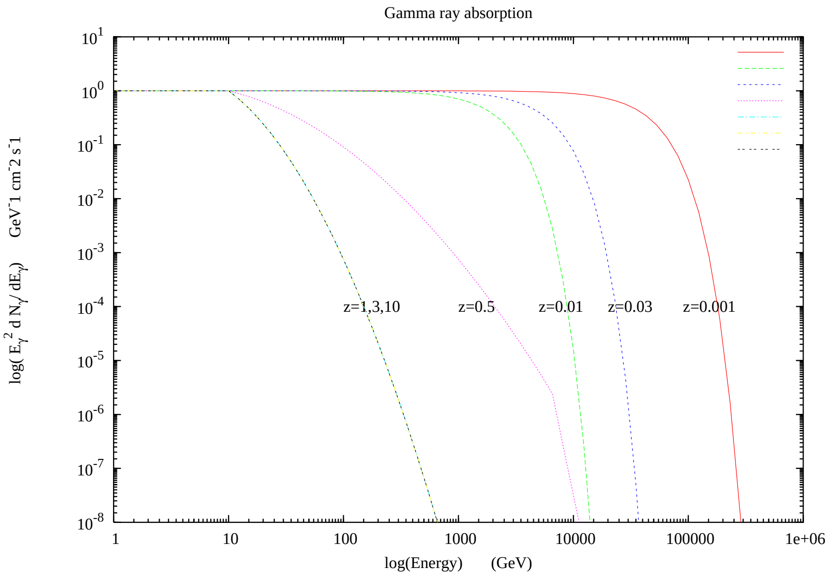

Figure 4: *Attenuation through $\gamma\gamma$ absorption at different redshifts.*

Inside and outside the GRB, the synchrotron Band spectrum is only attenuated by $\gamma\gamma$ reactions and the spectrum which leaves the source and is observed at a luminosity distance $D_L(z)$ is

$$\frac{dN_{\gamma_{prompt}}}{dE_\gamma\, dt\, dA}(E_\gamma, z, L_{iso}) = \frac{(\alpha-1)\, L_{iso}}{4\pi D_L(z)^2 {E_{peak}}^2}\, (\frac{E_\gamma}{E_{peak}})^{-\alpha}\, e^{-\tau_{grb\,\gamma\gamma}(E_\gamma)}\, e^{-\tau_{bkg\,\gamma\gamma}(E_\gamma, z)} \tag{8}$$

where $\tau_{grb\,\gamma\gamma}$ and $\tau_{bkg\,\gamma\gamma}$ are the optical depths for pair production inside and outside the source, respectively. Outside the GRB, due to $\gamma\gamma$ interactions with cosmic infra-red background photons and CBR photons, most of the high energy photons produce high-energy $e^{\pm}$ pairs. We assume that each lepton of the pair shares half of the energy of the initial GRB photon, $\gamma_e = E_\gamma / 2\, m_e$. The electron-positron flux at a distance $D_L(z)$ is

$$\frac{dN_e}{dA\, d\gamma_e\, dt}(\gamma_e, z, L_{iso}) = \frac{(\alpha-1)\, L_{iso}\, {E_{peak}}^{(\alpha-2)}}{2\pi D_L^2(z)\, (2m_e)^{(\alpha-1)}\, {\gamma_e}^{\alpha}}\, e^{-\tau_{grb_{\gamma\gamma}}(E_\gamma)}\, [1 - e^{-\tau_{bkg_{\gamma\gamma}}(E_\gamma, z)}]\,. \tag{9}$$

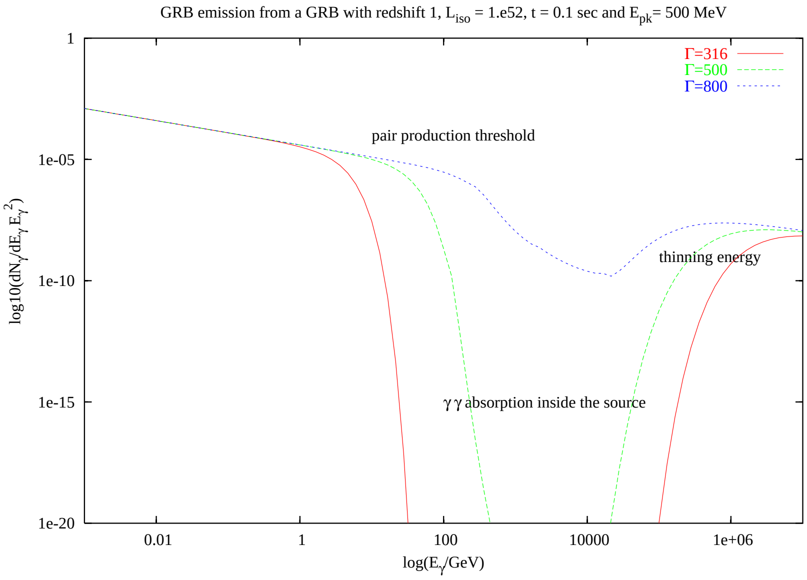

Figure 5: *Prompt spectrum attenuated by $\gamma\gamma$ absorption inside and outside the source.*

The pair flux will be enriched as long as the burst lasts. The time integrated electron-positron fluxes which interact with CMB photons are given by

$$\frac{dN_e}{dA\, d\gamma_e} = \int_0^T dt\, \frac{dN_e}{dA\, d\gamma_e\, dt}\,. \tag{10}$$

The pairs will inverse Compton scatter off CMB photons and produce secondary photons, which will in turn interact with IR photons and generate other pairs. Multiple inverse Compton scatterings will happen until the energy of the secondary photons is no longer enough to trigger a pair production with the IR photons. When the energy of the scattered photons is unsufficient to produce a subsequent pair, the photons will travel to the detector without undergoing further absorption. When simulating the series of inverse Compton scatterings we assume that all interactions happen very close to the source, at the redshift of the source itself. The number of secondary photons per unit volume per photon energy interval created in the vicinity of a GRB through one inverse Compton scattering of the pairs off cosmic of the background radiation photons is [15]

$$q_{IC}(E_\gamma, z, L_{iso}) = \frac{d\,N_\gamma}{d\,E_\gamma\,dV} = \int d\gamma_e \int d\epsilon_{cmb} \frac{d\sigma_{IC}}{dE_\gamma}(E_\gamma, \gamma_e\,\epsilon_{cmb})\,u_{cmb}(\epsilon_{cmb}, z)\,\frac{dN_e}{dA\,d\gamma_e} \tag{11}$$

where $\frac{d\sigma_{IC}(E_\gamma, E_e, \epsilon_{cmb})}{dE_\gamma}$ can be either the differential Thomson cross section (for low energy pairs) or the differential Klein-Nishina formula (for high energy pairs)[17]. $u_{cmb}(\epsilon_{cmb}, z)$ is the density of CBR photons per unit volume per photon energy interval at redshift z [16]. Photons produced through inverse Compton of the highest energetic pairs do not appear in the inverse Compton fluxes since they are absorbed by the IR photons. The secondary source

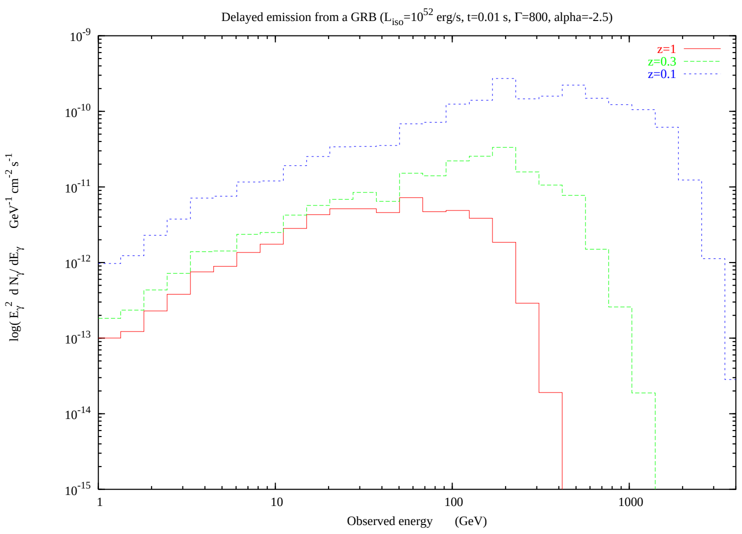

Figure 6: *Redshift dependence of delayed emission from a GRB with bulk Lorentz factor of 800 and* $\alpha = -2.5$.

function in Eq.(11), iterated over multiple scatterings, is integrated over the volume, divided by the unit area of detector and weighted over the number of GRBs per unit time and per unit volume to obtain the total counting rate

$$Flux_{delayed} = \frac{dN_{\gamma_{delayed}}}{dA\,dE_\gamma\,dt\,d\Omega} = \int dV\,\frac{1}{4\,\pi D_L^2(z)}\,q_{IC}(E_\gamma, z, L_{iso})\,n_{GRB}(z) \tag{12}$$

where $n_{GRB}(z)$ is the observed GRB redshift distribution [9, 10]

$$n_{GRB}(z) = \frac{R_{GRB}(z)}{1+z}\,\frac{dV}{dz}\,\int d\,log\,(L_{iso})\,\Phi_0(L_{iso})\,. \tag{13}$$

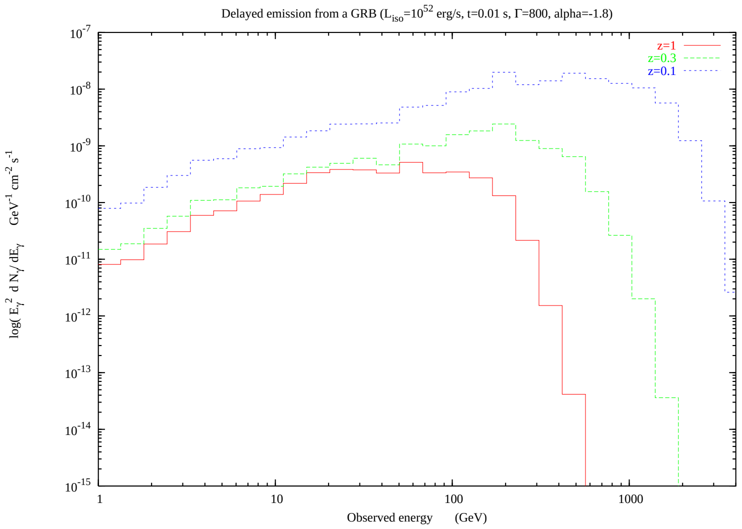

Figure 7: *Redshift dependence of delayed emission from a GRB with bulk Lorentz factor of 800 and* $\alpha = -1.8$.

In Eq.(13) $\Phi_0(L_{iso})$ is the luminosity function [9]. The local luminosity function $\Phi_0(L_{iso})$ is defined as the comoving space density of GRBs in the interval $log(L_{iso})$ and $log(L_{iso}) + d\,log(L_{iso})$. Following [11], we derive the luminosity function $\Phi_0(L_{iso})$ by assuming a broken power law with lower and upper limit, $L_{lower} = \frac{L_{iso}{}^*}{30}$ and $L_{upper} = 10\,L_{iso}{}^*$, respectively, where $L_{iso}{}^* = 4.4\,10^{51}$,

$$\Phi_0(L_{iso}) = \begin{cases} c\,\left(\frac{L_{iso}}{L^*_{iso}}\right)^{-0.1} & \text{for } L_{lower} < L_{iso}{}^* < L_{upper} \\ \\ c\,\left(\frac{L_{iso}}{L_{iso}{}^*}\right)^{-2} & \text{for } L_{lower} < L_{iso}{}^* < L_{upper} \end{cases} \tag{14}$$

The normalization constant c is obtained by imposing the integral of $\Phi_0(L_{iso})$ to be unity. The integration over L_{iso} is performed over the interval indicated in [9].

In Eq.(13) the redshift distribution of GRBs $R_{GRB}(z)$ is

$$R_{GRB}(z) = \frac{23\,e^{3.4z}\,\rho_{GRB}\,F(z, \Omega_m, \Omega_\Lambda)}{22 + e^{3.4z}} \tag{15}$$

where

$$F(z, \Omega_m, \Omega_\Lambda) = \frac{\sqrt{\Omega_m\,(1+z)^3 + \Omega_k\,(1+z)^2 + \Omega_\Lambda}}{(1+z)^{3/2}} \tag{16}$$

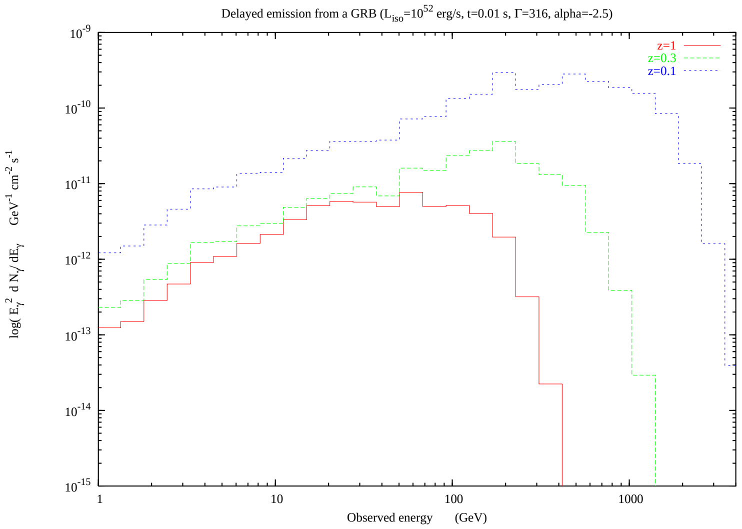

Figure 8: *Redshift dependence of delayed emission from a GRB with bulk Lorentz factor of 316 and* $\alpha = -2.5$.

and $\rho_{GRB} = 0.44 \, Gpc^{-3} \, yr^{-1}$ is the observed rate of GRB per co-moving volume [9]

$$dV = \frac{d_h \, (1+z)^2 d_a{}^2 \, d\Omega dz}{E(z)} \, . \tag{17}$$

The function $E(z)$ is defined as

$$E(z) = \sqrt{\Omega_m \, (1+z)^3 + \Omega_k \, (1+z)^2 + \Omega_\Lambda} \tag{18}$$

where d_h is the Hubble distance and d_a is the angular diameter distance. The luminosity distance is defined as $D_L(z) = (1+z)^2 \, d_a$.

Thus the integral over dV in Eq.(12) can be transformed to an integral over the redshift z and the total delayed photon flux due to multiple inverse Compton scatterings off CMB photons becomes

$$Flux_{delayed} = \frac{dN_{\gamma_{delayed}}}{dA \, dE_\gamma \, dt \, d\Omega} = \int dz \, q_{IC}(E_\gamma, z, L_{iso}) \, n_{GRB}(z) \, \frac{d_h}{4 \, \pi \, (1+z)^2 \, E(z)} \, e^{-\tau_{bkg\,\gamma\gamma}(E_\gamma, z)} \tag{19}$$

where the integration over the redshift is performed from redshift 0.01, corresponding to about 40 Mpc, to redshift 10.

In Fig(6) and (7), we plot the expected delayed emissions from a GRB with $L_{iso} = 10^{52}$ erg/s and bulk Lorentz factor of 800 at different redshifts for $\alpha = -2.5$ and $\alpha = -1.8$, respectively, which correspond to different production mechanisms inside the GRB. In Fig (8) we plot the expected delayed emission from a GRB with bulk Lorentz factor of 316. Comparing Fig(6) and Fig (8) we conclude that the delayed emission does not depend on the bulk Lorentz factor.

The total flux from prompt GRB emission is given by the following integral

$$Flux_{prompt} = \frac{dN_{\gamma_{prompt}}}{dA\, dE_\gamma\, dt\, d\Omega} = \frac{1}{beaming} \int dz \frac{d\,N_{\gamma_{prompt}}}{d\,E_\gamma dt\, dA} n_{GRB}(z) \qquad (20)$$

where $\int_0^T dt \frac{d\,N_{\gamma_{prompt}}}{d\,E_\gamma dt\, dA}$ is the differential gamma ray prompt flux from each GRB in Eq.(7), measured in $GeV^{-1}cm^{-2}$, $n_{GRB}(z)$ is the redshift dependent number of GRBs per unit time and the average beaming factor $f_{beaming}$ is assumed to be 75. In Fig (9) and (10) we plot the contribution from the attenuated prompt emission from all GRBs at different redshifts. Contrary

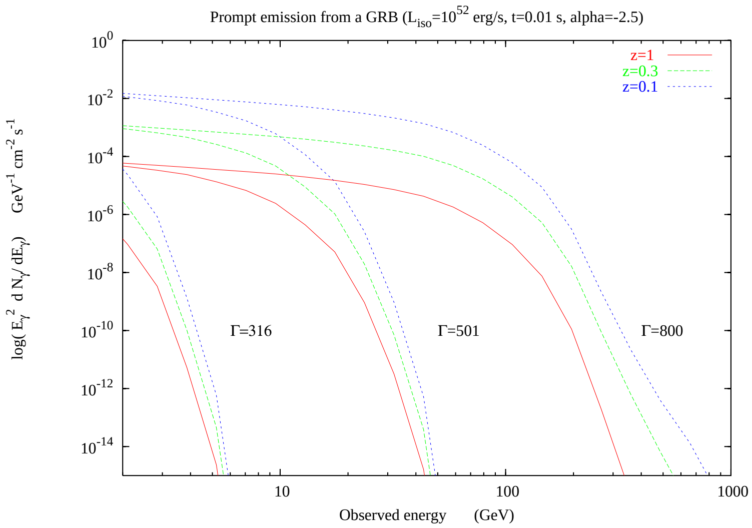

Figure 9: *Redshift dependence of prompt emission from a GRB with* $\alpha = -2.5$.

to the delayed emission, GRB prompt emission is strongly dependent on the bulk Lorentz factor.

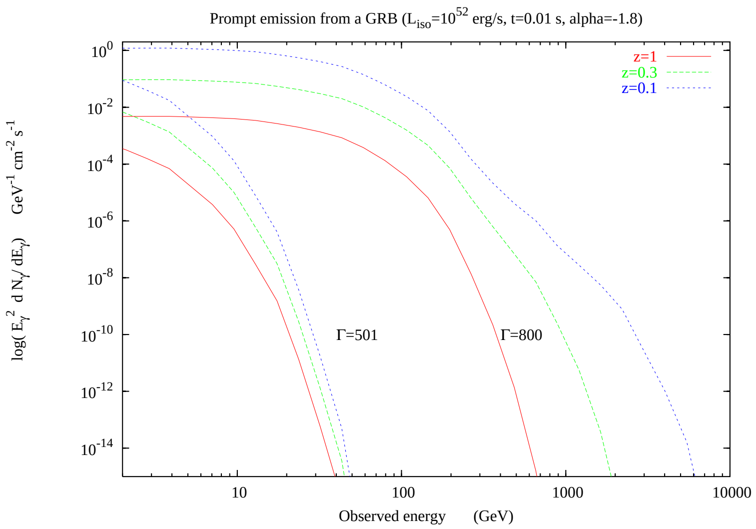

Figure 10: *Redshift dependence of prompt emission from a GRB with* $\alpha = -1.8$.

Finally, in Fig.(11) and (12) we plot the sum of prompt and delayed GRB emissions for Lorentz factors equal to 800, for a GRB duration of 10 seconds and for a time variability $\delta t = 0.01$ seconds for $\alpha = -2.5$ and $\alpha = -1.8$, respectively.

References

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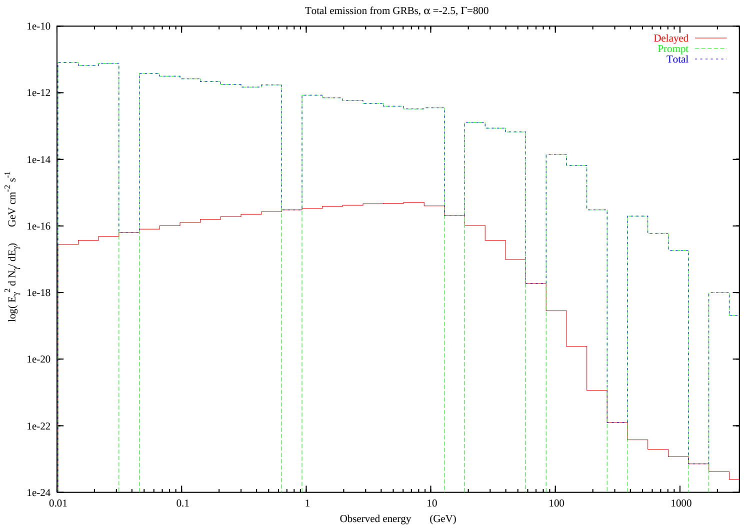

Figure 11: GRB Total Emission

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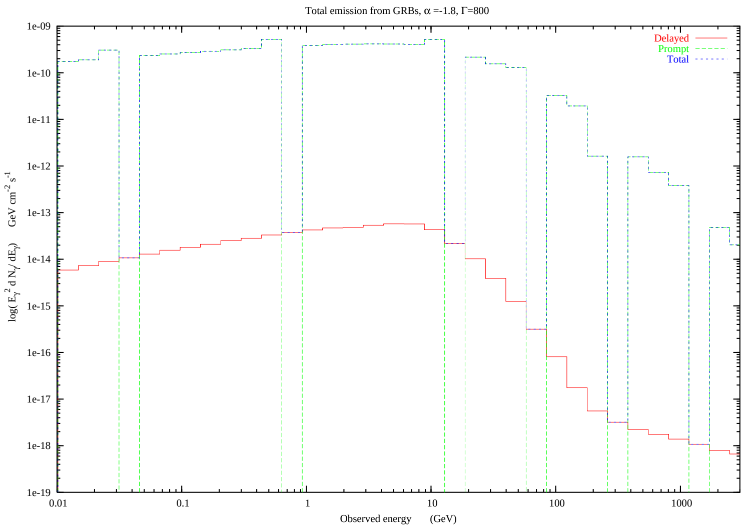

Figure 12: GRB Total Emission

1 Appendix

In the directory /HOME/disk03/people/casanova/memos/grb you can find some FORTRAN programs (sorry, not in C++) which calculate prompt and delayed emission from a single GRB or from all GRBs. The programs code the widely accepted fireball internal-external shock model for GRBs and use Primack or an alternative model for gamma gamma absorption. A generally accepted model for GRB redshift distribution and rate is also implemented.

The GRB opacity is coded in *grbOpacity.f* The prompt emission from a single GRB is coded in the program called *prompt.f*. The delayed emission from a single GRB is coded in the program called *delayed.f*. The prompt and delayed emission from all GRBs is coded in the short Monte-Carlo *promptDelayedAll.f*, in which multiple inverse Compton scatterings and pair production of photons travelling from the GRBs to the detector are coded.

For every program you will be asked to give your value for GRB parameters like the Lorentz Bulk factor, the time variability and the GRB duration. For other parameters like the Band photon spectrum index or the fraction of initial GRB energy to leptons I have already given a value but this can be easily modified.