Angle Fitter 3++

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Abstract

This memo describes a proposed modification of the present angle fitter with new important features for Milagro.

1 Introduction.

In this memo we would like to propose a modification of the current plane fitter with the following features:

1. The least square fit is linear; no iterations are necessary; the fit never fails. Therefore the computations are faster.
2. In some circumstances it allows the consideration of non-flat arrays and the experimental determination of the speed of propagation of the showers in air.
3. In other circumstances we need to insist on flat arrays, which can be in several tiers, and supply the propagation speed as an external parameter.

The general method opens the possibility of including the muon/hadron layer and outriggers, in addition to the shower layer, in the angular reconstruction.

2 Nonplanar shower detectors in uniform medium.

Examples: Outrigger Array or Cygnus (after the fact).

The shower front is defined by a normal vector $\vec{n} = (a, b, c)$ $||\vec{n}|| \neq 1$ (note that we do NOT demand that $\vec{n}$ is a unit vector, so that a, b and c are independent). With a set of identical detectors at different heights in air, the times registered by the detectors are (after time-pedestal subtraction has given us a common clock for the event):

$$T_i = \frac{ax_i + by_i + cz_i}{v_0\sqrt{a^2+b^2+c^2}} + T_0 \tag{1}$$

where (x_i, y_i, z_i) are coordinates of the PMTs in the outriggers, v_0 is the speed of shower front in air and T_0 is a common time offset for the whole event.

Introducing notation:

$$A = \frac{1}{v_0}\frac{a}{\sqrt{a^2+b^2+c^2}} \quad B = \frac{1}{v_0}\frac{b}{\sqrt{a^2+b^2+c^2}} \quad C = \frac{1}{v_0}\frac{c}{\sqrt{a^2+b^2+c^2}} \tag{2}$$

we see that T_i's are linear functions of parameters A, B, C, T_0.

$$T_i = Ax_i + By_i + Cz_i + T_0 \tag{3}$$

and thus, the least square fit will lead to a set of four linear equations for four unknowns A, B, C, T_0:

$$\left\{ \begin{array}{lcl} \sum_i (T_i - Ax_i - By_i - Cz_i - T_0)\, x_i & = & 0 \\ \sum_i (T_i - Ax_i - By_i - Cz_i - T_0)\, y_i & = & 0 \\ \sum_i (T_i - Ax_i - By_i - Cz_i - T_0)\, z_i & = & 0 \\ \sum_i (T_i - Ax_i - By_i - Cz_i - T_0) & = & 0 \end{array} \right. \tag{4}$$

After the system is solved, the shower direction is reconstructed as follows:

$$\left\{ \begin{array}{lcl} A & = & \frac{1}{v_0}\sin\theta\cos\phi \\ B & = & \frac{1}{v_0}\sin\theta\sin\phi \\ C & = & \frac{1}{v_0}\cos\theta \end{array} \right. \Bigg| \Rightarrow \left\{ \begin{array}{lcl} \phi & = & atan2(B, A) \\ \theta & = & \arccos\frac{C}{\sqrt{A^2+B^2+C^2}} \end{array} \right. \tag{5}$$

As one can see, the fit is linear (never fails!) and there was no assumption that the array is flat. The fitted speed of the shower front in air, for that event, is gotten from the fitted parameters A,B,C by

$$v_0 = \frac{1}{\sqrt{A^2 + B^2 + C^2}} \tag{6}$$

This will be a sensitive measurement of the speed if the height differences between the detectors are significant. The average over many events will be a good measure of the effective shower propagation speed in air. If for any given event the speed v_0 is very different from the average one, the fit might be considered as a failure due to physical arguments. The neccessary condition for this method to work is that the shower propagates the extra distance to a deeper detector at the **same** speed as above the detectors.

3 Flat array in air.

Example: a fictitious EAS array on the Great Salt Lake.

However, if we do have a flat array, then all z_i are the same and we are free to choose $z_i \equiv 0 \ \forall i$. The C disappears from the equations:

$$\left\{ \begin{array}{lcl} \sum_i (T_i - Ax_i - By_i - T_0)\, x_i & = & 0 \\ \sum_i (T_i - Ax_i - By_i - T_0)\, y_i & = & 0 \\ \sum_i (T_i - Ax_i - By_i - T_0) & = & 0 \end{array} \right. \tag{7}$$

$$\left\{ \begin{array}{lcl} A & = & \frac{1}{v_0} \sin\theta \cos\phi \\ B & = & \frac{1}{v_0} \sin\theta \sin\phi \end{array} \right| \Rightarrow \phi = atan2(B, A) \tag{8}$$

In this case the speed v_0 is an externally supplied parameter in the fit. One has to assert its value in order to find θ:

$$\theta = \arcsin(v_0 \sqrt{A^2 + B^2}) \tag{9}$$

4 The Case of Two Media.

Example: Milagro pond Detector.

Life is more difficult in the presence of two media with different propagation speeds. Trying to directly apply the method described in section 2 for the PMT's at varying heights inside the pond we immediately run into the following problem. The times, registered by the PMT's at different depths are affected differently by the complicated signal propagation in the detector, where the signal speed is certainly not the shower speed in air. In fact we have a messy multi-component shower mix in the pond (gammas, eventually converting, electrons, Cerenkov photons and perhaps muons) propagating at different speeds and drastically different directions: the effective signal speed is likely to be angle and depth dependent! The conditions for the method of section 2 are not met.

The above inconsistency does not exist for a planar, horizontal array under water. If the depth of water above the PMT's is uniform we can expect the in water propagation time Δ from the surface to the PMT's to be independent of their location in the array. Under this assumption (which we'll call HPS for horizontal position symmetry), the time Δ adds the same delay to each PMT and can be thought of as part of the detector transit time; it just adds a constant to T_0 in that event, while the time differences come from the shower arrival on the pond surface. Therefore, we are back to case of the flat array of Section 3, and can fit the angle using equations [(7),(8), (9)]. With the HPS assumption the propagation speed v_0 must be the one in air.

In practice, the HPS assumption is not expected to be exact, because of phenomena happening inside the water, but systematic deviations from it must go to zero as the shower angle approaches the zenith. To first order, the corrections may be lumped into a small change in effective speed to $v_0^{'}$, whatever their origin [1],[2],[3]. This effective speed is likely dependent on the depth of the flat layer.

MILAGRO contains three flat arrays: the 450 PMT shower layer (L_1), the 209 PMT muon bottom tier (L_2) and the 64 PMT muon mezzanine (L_3).

We propose to treat each layer independently, applying the flat version of the linear fitter [(7),(8), (9)], allowing different effective speeds $v_{0L_i}^{'}$ for each layer. Thus, we will obtain three statistically independent angle determinations θ, ϕ for each event.

Three separate direction determinations allow several studies, systematics checks, and some systematic corrections. Comparison of T-chi distributions gives the depth dependence of relative resolution. "Del_layer" can be an alternative to del_EO as a resolution measure. The ratios $v_{0L_i}^{'}/v_{0L_j}^{'}$ are obtained experimentally from a sinθ vs sinθ' plot as discussed in [2]. One of the velocities, say v'_{0L_1}, still has to be provided.

Once systematic problems and relative resolutions are understood, one can get a single direction determination for each event from a weighted average of the the three or four (if outriggers included) angle determinations. Detailed studies using real data and Monte Carlo can shed light on how to do it best. Equivalently one could sum the χ^2 from all the layers, to get a single χ^2 fit. We do not advocate such a combined fit for early MILAGRO running, but give an example of it, for completeness, in Appendix A.

5 Conclusion.

We propose to use linear χ^2 fit method as the angle fitter for MILAGRO, using the "flat" version of Section 3 for three angle determinations from the pond and the "hilly" version of Section 2 for an angle determination from the outriggers.

6 References

1. Stefan Westerhoff's Monte Carlo studies show such an effect.

2. L. Fleysher, R.Fleysher "How are Dec and RA affected by ..."

3. Allen Mincer's milagro@milagro e-mail note of 12/24/98 discusses the language, applicability, and criteria for applying such a correction.

7 APPENDIX A.

If we wanted to combine all 3 pond layers into a single χ^2-fit:

$$\chi^2_{total} = \sum_i \chi^2_{L_i}$$

we would get a set of equations analogous to (7):

$$\sum_{i\in L_1,L_2,L_3} (T_i - \eta_L(Ax_i + By_i))\, x_i - T_{01}\sum_{i\in L_1} x_i - T_{02}\sum_{i\in L_2} x_i - T_{03}\sum_{i\in L_3} x_i = 0$$

$$\sum_{i\in L_1,L_2,L_3} (T_i - \eta_L(Ax_i + By_i))\, y_i - T_{01} \sum_{i\in L_1} y_i - T_{02} \sum_{i\in L_2} y_i - T_{03} \sum_{i\in L_3} y_i = 0 \quad (10)$$

$$\sum_{i\in L_1} (T_i - \eta_1(Ax_i + By_i) - T_{01}) = 0$$

$$\sum_{i\in L_2} (T_i - \eta_2(Ax_i + By_i) - T_{02}) = 0$$

$$\sum_{i\in L_3} (T_i - \eta_3(Ax_i + By_i) - T_{03}) = 0$$

where $\eta_L = \frac{v_0}{v_{0L}}$ are relative effective speeds for different layers.

This set of five linear equations in five unknowns $A, B, T_{01}, T_{02}, T_{03}$ can be easily solved. To extract the shower direction one has to use equations (8) and (9), where v_0 is the externally imposed effective speed. Adding weights in front of the χ^2 terms is a straightforward modification (not shown) for a weighted average.