Sky maps, Geomagnetic field and the Moon shadow

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Abstract

In this short note we discuss the requirements that should be met when generating sky maps. We also suggest a better approach to generating the sky maps, if one is interested in studying geomagnetic effects. Some "standard" mappings are analyzed as to their adequacy for concrete tasks.

1 Sky maps

In counting-type astrophysical experiments, the brightness of a particular point in the sky is characterized by the number of events observed from that point during the exposure time. Such experiments measure the density of events on the surface of the Celestial sphere. Therefore, the procedure used to generate sky maps (projections of a sphere onto a plane surface), should conserve the density of events. To meet this requirement one must use an equal area projection of a sphere onto a plane. This requirement, however, is not enough to uniquely fix the projection, so there are several projections available which are used in solving different tasks. The projection which is most often used by Milagro is the *Sinusoidal Equal Area Projection* defined as:

$$\begin{cases} x = \alpha \cos \delta \\ y = \delta \end{cases}$$

where α is the Right Ascension and δ is Declination.

The area of a surface element on a sphere is described by

$$d\Omega = d\alpha\, d(\cos(\pi/2 - \delta)) = \cos \delta d\alpha d\delta$$

It is very easy to check that the Jacobian of this mapping conserves the area element:

$$J(\alpha, \delta \to x, y) = \begin{vmatrix} \partial x/\partial \alpha & \partial x/\partial \delta \\ \partial y/\partial \alpha & \partial y/\partial \delta \end{vmatrix} = \begin{vmatrix} \cos \delta & -\alpha \sin \delta \\ 0 & 1 \end{vmatrix} = \cos \delta \quad \Rightarrow$$

$$dx\, dy = d\Omega = \cos \delta\, d\alpha\, d\delta$$

It is crucial to understand that any area preserving mapping might distort the distances and directions on the map. That is why different area preserving mappings should be used for different tasks. For example, the sinusoidal projection has minimal distortions near the equator and that is why is very convenient for galactic plane studies. When the same mapping is used for an object far from the equator, the distortions become significant. For example, for the Crab nebula the contours of constant Right Ascension are titled by $\sim 26.5^\circ$ from the vertical in the DEC vs RAcos(DEC) plot.

2 Sky map relative to an object

When considering a problem of generating a sky map relative to some preselected point in the sky (an astronomical object or point of interest), one must be even more careful. In addition to the previously discussed constraints, one must make sure that all events with identical orientation with respect to the selected point are mapped into a unique location. Effectively, to solve such a problem, one must introduce a coordinate system attached to the preselected point. For example, the Equatorial Coordinate system (RA-DEC) is defined relative to the Northern Celestial pole.

3 Geophysical phenomena

If it is desired to study phenomena connected with the Earth's magnetic field, usually, the Geomagnetic Coordinate System[1] is the most convenient one. For an astronomer this means that the Geomagnetic Declination and Geomagnetic Right Ascension must be introduced. In this coordinate system, the Geomagnetic pole plays the role of the preselected point.

4 The Moon shadow

The shadow of the Moon is shifted by the magnetic field of the Earth. This shift is defined both by the direction of the magnetic field and by magnetic rigidity of the cosmic rays. To first approximation, the direction of the magnetic field of the Earth can be taken into account simply by considering the Moon shadow in the Geomagnetic Coordinate system. In this case, the shift of the particles due to the magnetic field will be always in one direction. This effect is important because the angle between the magnetic field and the Celestial meridian at the Moon changes as the Moon transverses the sky. This angle is histogrammed in figure 1. Neglecting this will lead to smearing and an average shift of the Moon shadow.

Thus, the requirements on maps around the Moon can be summarized as:

1. The mapping should be area preserving.

2. The orientation of events relative to the Moon should be conserved.

3. The sky map should have constant orientation with respect to geomagnetic field.

[1] Geomagnetic Coordinate System is defined relative to the geocentric dipole model poles

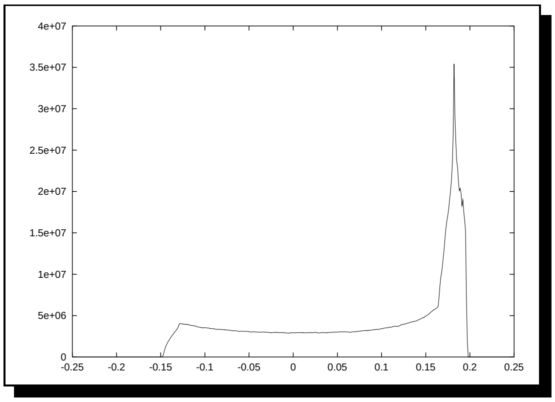


Figure 1: Histogram of the angle between Magnetic and Celestial meridians at the Moon, radians

4.1 How we do it now

The mapping which is frequently used by Milagro[2] is:

$$\begin{cases} x = (\alpha - \alpha_0)\cos\delta_0 \\ y = \delta - \delta_0 \end{cases}$$

where (α_0, δ_0) are coordinates of the Moon.

This is not an area preserving mapping because the Jacobian

$$J = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 \neq \cos\delta$$

This mapping does not preserve the orientation of the events, relative to the Moon either. Figure 2 shows how points with fixed coordinates relative to the Moon get smeared as the Moon transverses the sky.

One might be tempted to fix the present mapping by taking:

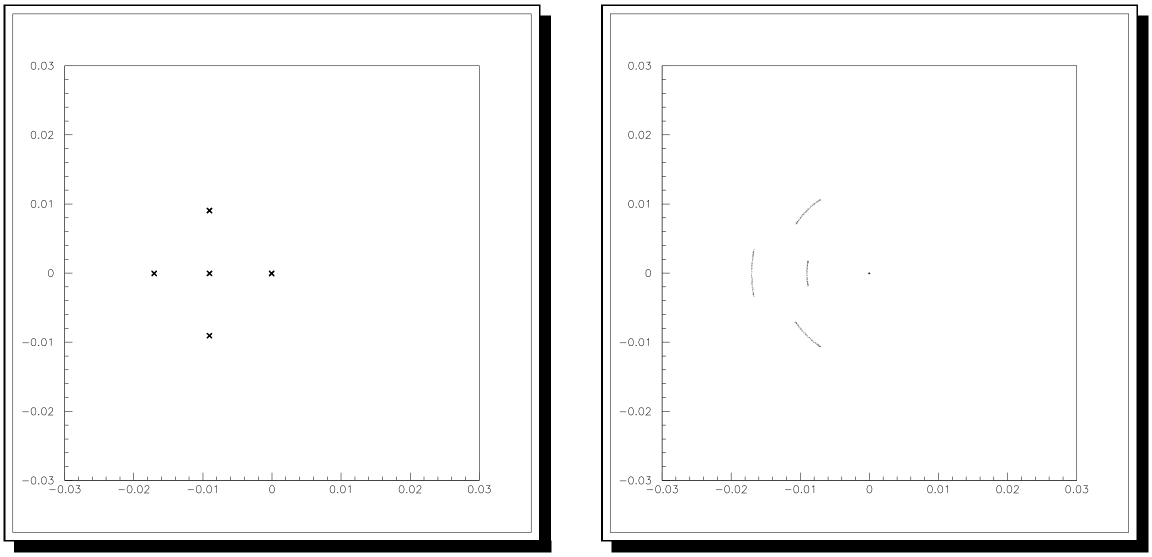
$$\begin{cases} x = \alpha\cos\delta - \alpha_0\cos\delta_0 \\ y = \delta - \delta_0 \end{cases}$$

but even though this is an area preserving map attached to the Northern Celestial pole with the Moon at the origin, it still does not preserve the relative position of events with time.

4.2 How we should do it

Usually, when generating projections of the sky maps around an object which does not move on the sky, the Northern Celestial pole is chosen as the preselected point and the sinusoidal equal area projection is used for its simplicity. Of course, one should be aware of the distortions this mapping will cause in the neighborhood of the selected object.

[2]See of instance: R. Atkins et al., Nucl. Instr. and Meth. A 449 (2000) 478-499



(a) points with fixed coordinates $(-1.0^\circ, 0.0^\circ)$, $(-0.5^\circ, 0.5^\circ)$, $(-0.5^\circ, 0.0^\circ)$, $(-0.5^\circ, -0.5^\circ)$ and $(0.0^\circ, 0.0^\circ)$ relative to the Moon(0.0,0.0)

(b) smearing caused by the Moon's motion on the sky in 1 month

Figure 2: Smearing of events due to Moon's motion

If, however, the astronomical object of interest moves on the sky, this object must play the role of the preselected point. As was discussed earlier, the "standard" approach has drawbacks. One way to solve this problem is to introduce new coordinates on the Celestial sphere: an analog of latitude (χ) and longitude (ξ) which are measured with respect to the preselected point. The sinusoidal mapping has maximal distortions at the origin, so one is forced to look for a different mapping function.

The Azimuthal Equal Area Projection in the polar case, for instance, has the required properties.(figure 3)

$$\begin{cases} x = \sqrt{2(1-\cos\chi)}\,\sin\xi \\ y = \sqrt{2(1-\cos\chi)}\,\cos\xi \end{cases}$$

where χ is angle between an event and the Moon, and ξ is longitude of an event measured along the line circle $\chi = const$. The meridian of the zeroth longitude (the y-axis) is arbitrary and can be chosen to be oriented along the geomagnetic field.

Thus, this mapping not only satisfies the first two requirements and can be properly oriented with respect to the Earth's magnetic field, it has several other important features — minimal distortions at the Moon (the most interesting region) and conservation of the directions as seen from the origin.

The Moon shadow for 10 months of Milagro data is shown in figure 4 for the standard and azimuthal mapping with 2.1° smoothing binsize. Since the former includes smearing of about 0.5°, an analysis of optimal bin size should be performed for the proposed method.

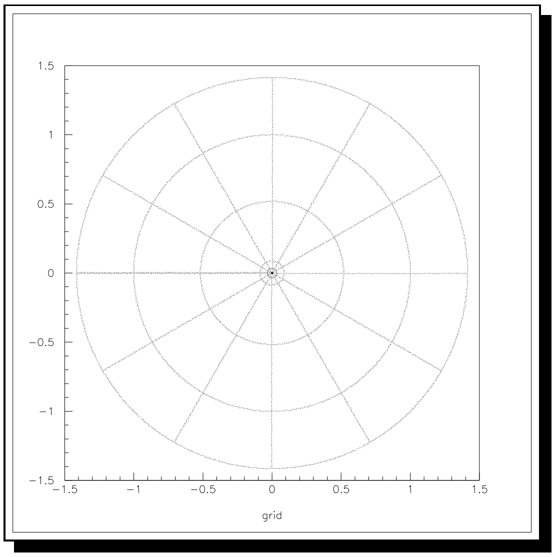
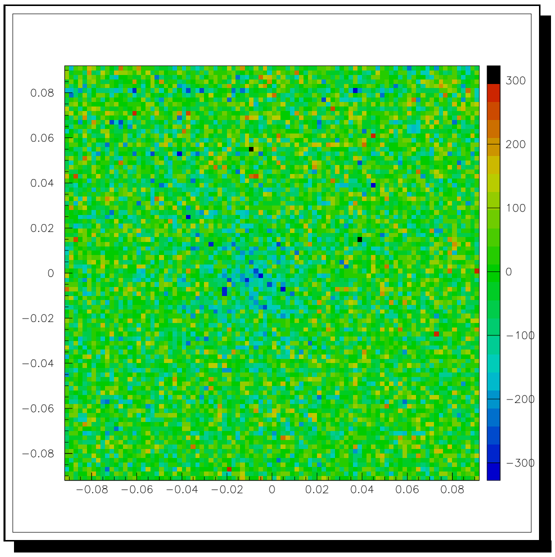


Figure 3: Azimuthal Equal Area projection grid, polar case. Circles — $\chi = const$, straight lines — $\xi = const$

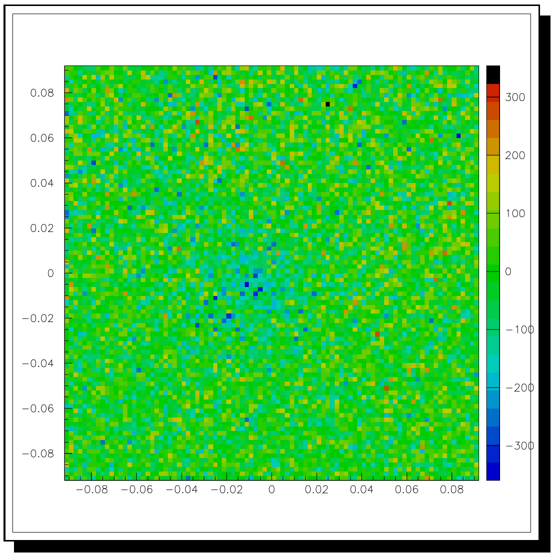
This must be done in any event because the Moon is not a point source for which the 2.1° binsize was obtained.

5 Conclusions

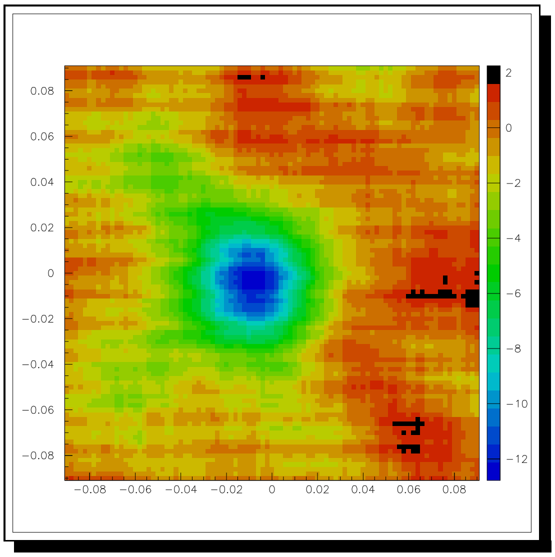
In this brief note we discussed the importance of the choice of the mapping function for sky map generation. Also, we have proposed a new approach for mapping of the Moon shadow. Results of the Moon shadow analysis are applicable to the Sun shadow problem too.



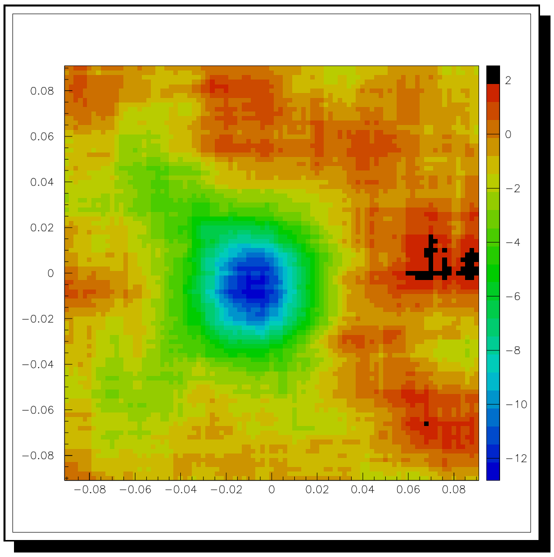
(a) “Standard” mapping $0.1° x 0.1°$ non-overlapping bins



(b) Azimuthal mapping $0.1° x 0.1°$ non-overlapping bins



(c) “Standard” mapping smoothed with $2.1°$ bin



(d) Azimuthal mapping smoothed with $2.1°$ bin

Figure 4: Milagro Moon