Methods of Photo-Electron Calibration

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1 PE calibration using Occupancy method

Photo-electron (PE) calibration is based on the well-known occupancy method, which was one of the methods of PE calibration of Milagrito [1] and is also described in IMB calibration paper [2]. In this memo we describe the general idea of the method, and then starting from section 2 we discuss the innovations of the implementation and conclude with the study of the errors.

The data used for PE calibration is the same as for the timing calibration which is obtained with the laser light passing through the neutral density filter wheel oriented at different angles of rotation.

The ToT-to-PE conversion is split into two steps:

1. At low light levels, calibrate using occupancy method.
2. Extend to higher light levels using known attenuation properties of the filter wheel.

Low light level calibration

At low light levels, it is possible to measure PE as a function of ToT directly. Occupancy method is based on the following assumptions:

- number of photo-electrons produced at PMT's photocathode obeys Poisson distribution: $P(n, \lambda) = \frac{\lambda^n}{n!} e^{-\lambda}$
- constant light intensity source is used in calibration.[1]

[1] The effects of instability of laser light on the validity of the occupancy method is studied in [3].

The probability that **at least one** photo-electron was produced when the PMT's photocathode was illuminated is called occupancy (η) and is given by the following expression:

$$\eta = P(n > 0, \lambda) = 1 - P(n = 0, \lambda) = 1 - e^{-\lambda}$$

where λ — is the mean number of the photoelectrons observed. Occupancy can be easily measured[2] and the mean number of photoelectrons can be inferred:

$$\eta = \frac{\#\ of\ observed\ pulses}{total\ \#\ laser\ pulses}, \qquad \lambda = -\ln(1-\eta)$$

However, this method is applicable only at relatively low light levels ($\lambda \leq 2$), because at high light levels a small measurement error in η will lead to an exponentially big error in λ:

$$\Delta\lambda = \frac{1}{1-\eta}\Delta\eta = e^{\lambda}\Delta\eta$$

High light level calibration

At high light levels we assume the following:

- mean number of produced photo-electrons is proportional to the light intensity at the photocathode.
- constant light intensity source is used in calibration.

Under these assumptions and if the transmittance of the filter wheel T is known, then:

$$\lambda = a \cdot T$$

where a is some coefficient. This coefficient can be found by fitting a straight line to the λ vs T plot for low light levels when λ can be determined by the occupancy method. a varies from PMT to PMT.

$$\chi^2 = \sum_i (\lambda_i - aT_i)^2 \quad \Rightarrow \quad a = \frac{\sum_i \lambda_i T_i}{\sum_i T_i^2}$$

Thus, given the transmittance T, average number of photoelectrons λ can be determined at high light levels and the ToT-to-PE conversion can be extended to the region beyond $\lambda \geq 2$.

[2]See section 4 for some comments on this problem

2 Filter Wheel Calibration

As it was mentioned in section 1, it is important to know the transmission properties of the filter wheel to be used in the occupancy method. The method of filter wheel calibration is based on the same assumptions as the occupancy method in section 1 with the following supposition:

For any two sufficiently close transmittances of the filter wheel (T_1 and T_2) there exist a PMT in the pond for which the occupancy method can be applied on both light intensities.

Since for such a PMT the occupancy method is valid, the average number of photoelectrons (λ_1 and λ_2) can be determined and:

$$\lambda = a \cdot T \quad \Rightarrow \quad \frac{T_2}{T_1} = \frac{\lambda_2}{\lambda_1}$$

This line of arguments can be used to relate T_2 with T_3, T_3 with T_4 and so on, leading to the restoration of the levels of transmittance for the whole filter wheel. Without any doubt, one can always set $T_0 = 1$.

Figure 1 shows filter wheel calibration curves obtained with different sets of laser balls and PMT's. Agreement between them proves the stability of the method.

3 Morse Code

Historically, during data taking the laser ball number and filter wheel position are "transmitted" to the DAQ system and are encoded into the data stream as a special sequence of triggers. The sequence became known as "the Morse code". Each message consists of four digits (two for laser ball setting and two for filter position) separated from each other and from the main laser data by five delimiters. Because the laser can operate at 20 Hz at the most, the delimiters can be recognized as series of triggers at a high rate (more than 100 Hz). Since the laser ball numbers range from 1 to 30 and filter positions — from 0 to 35 (filter wheel is rotated in 10° quanta), the "message" was converted to the base-6 system before the transmission. To ensure high fidelity of the "channel", the following error-correcting algorithm is employed. Assuming low probability of loosing more than one trigger pulse per digit in a message, for each digit of value x $3(x+1)$ trigger pulses are sent to the DAQ at about 10 Hz. During data processing, the number of registered triggers

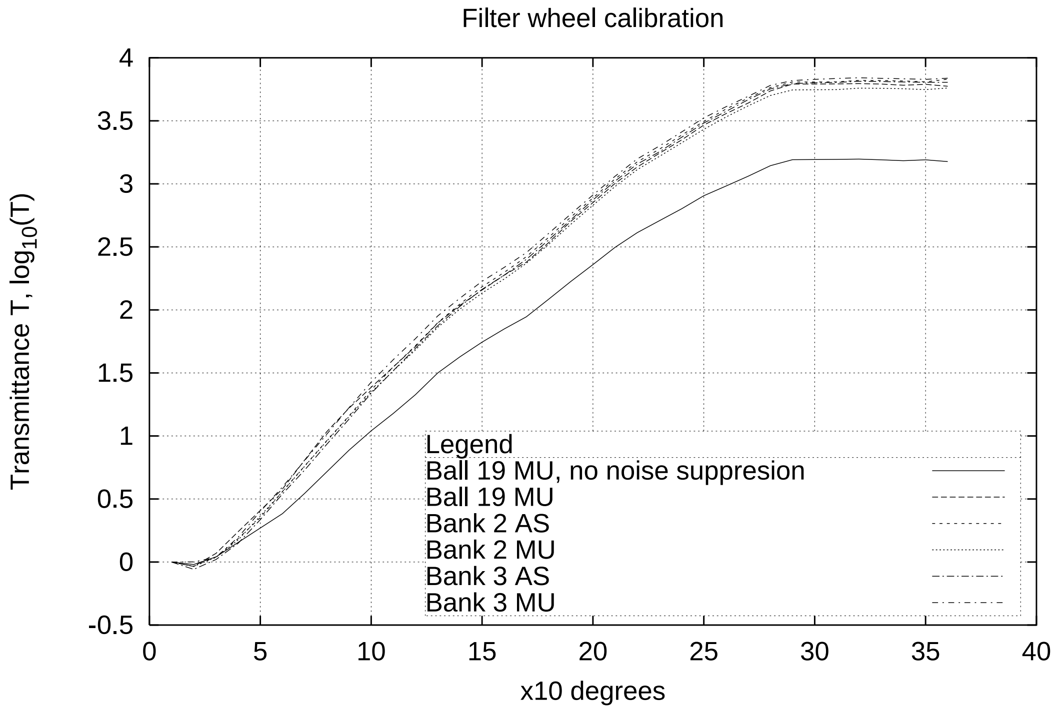

Figure 1: Filter wheel calibration curves obtained with different laser ball — PMT combinations are presented. Bank 2,3 represent balls 11-20 and 21-30 respectively. AS, MU —PMTs of air shower and muon layers respectively. Calibrations with ball 19 and muon layer PMTs are also shown to illustrate the effect of noise.

in a digit message is forced to the nearest integer multiple of 3 allowing for errors of ± 1 to be corrected. Furthermore, the first delimiter is longer in time and is at a higher frequency than the others, providing the beginning of the Morse code.

Besides its calibration settings information content needed to recognize the wheel and ball changes in the data, the Morse code pulses are used as "random" triggers in PMT noise estimation (See section 4), making it a very important part of the calibration process.

4 Dynamic Noise Suppression

Procedures, described in sections 1 and 2, rely on correct knowledge of PMT occupancy η. The total number of light pulses sent to the PMT is known exactly, but problems arise when trying to estimate the number of pulses registered by the PMT. If noise level is relatively high, noise (random) hits will increase the apparent occupancy. To cope with the problem, the following line of arguments is followed. An event, registered by a PMT, could have come from a laser pulse or from a noise hit which are assumed to be uncorrelated. Thus, the probability of observing a hit $P(any)$ is:

$$P(any) = P(laser + noise) = P(laser) + P(noise) - P(laser) \cdot P(noise)$$

where $P(laser)$ – is probability of observing a pulse due to laser (true occupancy η), $P(noise)$ – is probability of observing a pulse due to noise. $P(any)$ can be measured by sending laser pulses to a PMT (as described in section 1) and $P(noise)$ can be estimated by sending "random" triggers to the data acquisition system (see section 3), then

$$\eta = P(laser) = \frac{P(any) - P(noise)}{1 - P(noise)}$$

This scheme was applied for low light level PE calibration (section 1) and for the filter wheel calibration (section 2). Figure 1 shows an example of a filter wheel calibration curve obtained without application of the noise suppression algorithm (Ball 19, muon layer PMT's combination). As seen on the plot, the curve obtained from the same data agrees well with the rest when noise suppression is applied. In as much as muon layer PMT's suffer more from the accidental hits than air shower ones, the effect is more prominent for tubes of this layer. Noise suppression is performed on a tube by tube basis.

5 Statistical error of the occupancy method

In order to address the question of accuracy of the occupancy method we suppose that n shots of the laser beam were sent, out of which m were detected by the PMT. Occupancy η is estimated as:

$$\eta = \frac{m}{n}$$

The question of the accuracy of this estimation is that of confidence interval. Since the probability of a PMT registering a signal in a shot is equal to η, then the number of detected pulses m is distributed according to binomial distribution:

$$P_\eta(m,n) = C_n^m \eta^m (1-\eta)^{n-m}, \quad C_n^m = \frac{n!}{m!(n-m)!}$$

The upper bound of the confidence interval η_{upper} corresponding to the significance α (we choose α to be at least 95 %) is defined as such, that probability of detecting $k \leq m$ pulses is less than $(1-\alpha)$:

$$P_{\eta_{upper}}(k \leq m, n) = \sum_{k=0}^{m} C_n^k \eta_{upper}^k (1-\eta_{upper})^{n-k} \leq 1-\alpha$$

Correspondingly, the lower bound η_{lower} is such that

$$P_{\eta_{lower}}(k \geq m, n) = \sum_{k=m}^{n} C_n^k \eta_{lower}^k (1-\eta_{lower})^{n-k} \leq 1-\alpha$$

or

$$\sum_{k=0}^{m-1} C_n^k \eta_{lower}^k (1-\eta_{lower})^{n-k} \geq \alpha$$

If the required relative error on the value of occupancy is $\delta(\eta) = \frac{\Delta\eta}{\eta}$ then η_{upper} should be no greater than $(1+\delta(\eta))\eta$ and η_{lower} should be no less than $(1-\delta(\eta))\eta$. Thus, for given number of laser shots and number of registered pulses we can check if the required accuracy is met. We require the accuracy $\delta(\eta)$ to be such as to provide $q = \frac{\Delta\lambda}{\lambda} = 0.01$ relative error of the PE determination:

$$\lambda = -\ln(1-\eta)$$

$$\Delta\lambda = \frac{1}{1-\eta}\Delta\eta$$

$$q = \frac{\Delta\lambda}{\lambda} = -\frac{\Delta\eta}{(1-\eta)\ln(1-\eta)}$$

Thus, $\delta(\eta) = -q\frac{1-\eta}{\eta}\ln(1-\eta)$ and we look for the range of occupancy levels η for which the following criteria are satisfied ($n = 2000$ laser beam shots sent):

for $\eta < 0.5$:

$$\sum_{k=0}^{\eta * n} C_n^k \eta_{upper}^k (1-\eta_{upper})^{n-k} \leq 1-\alpha, \quad \eta_{upper} = (1+\delta(\eta))\eta$$

for $\eta > 0.5$

$$\sum_{k=0}^{\eta * n-1} C_n^k \eta_{lower}^k (1-\eta_{lower})^{n-k} \geq \alpha, \quad \eta_{lower} = (1-\delta(\eta))\eta$$

The results of these tests for a list of η's are as follows:

η	$\delta(\eta)$	pass/fail
0.1	0.009	fail
0.2	0.009	pass
0.3	0.008	pass
...	...	...
0.8	0.004	pass
0.9	0.002	fail

Therefore, the allowed range of occupancies to be used is $0.2 < \eta < 0.8$. For the phototube noise measurements ($\eta \sim 0.03$) the requirements are eased: $q = 0.1$. We also verify that using 60000 random "shots" is just enough to reach the goal. (The requirement of $q = 0.01$ is met at 66 % confidence level only.)

6 Threshold effect on the occupancy measurement

The problem we address in this section is that of a finite threshold level which phototube signal should cross in order to be "seen" by the electronics. The presence of the finite threshold leads to an under estimation of the occupancy and we intend to investigate by how much. If the final effect on the number of PEs is bigger that q than we need to correct for this effect.

The assumption we make here is that electron multiplication in each stage is a statistical process with some average gain. This assumption is similar to that of occupancy method and is based on proposition that probability of emitting an "amplified" electron does not depend on other electrons being emitted. Then, the number of electrons on the output of the first amplification stage obeys Poisson distribution with average gain $g = 5.3$[3] (for one electron on the input):

$$P(n, a) = \frac{a^n}{n!} e^{-a}$$

It is the number of electrons on the output of the first stage that dominates the fluctuations on the output of the entire cascade. Thus, the ability of the signal to cross the discriminator threshold level is governed by the output of the first stage. If less that $g/4 \sim 1$ electrons are emitted the threshold level will not be crossed. (The discriminator threshold is set to about $\frac{1}{4}$ of 1 PE in amplitude.) The probability of that happening given one electron (1PE) on the input is:

$$\beta = P(k \leq 1, a = g) = \sum_{k=0}^{1} \frac{a^k}{k!} e^{-a}$$

For $a = 5.3$ $\beta = 3.0 \cdot 10^{-2}$. (For two electrons on the input $a = 2g = 10.6$ $\beta = 2.5 \cdot 10^{-4}$ and can be neglected.)

The probability of observing a signal is decreased by the amount that some 1 PEs will be missed:

$$occupancy = 1 - P_\lambda(0) - P_\lambda(1)\beta$$

$$\eta = 1 - e^{-\lambda} - \beta\lambda e^{-\lambda}$$

The magnitude of the correction $\beta\lambda e^{-\lambda}$ has to be compared with the required accuracy on η which leads to the direct comparison of β and q.

[3] The typical gain of the entire cascade is $2 \cdot 10^7$ and consists of 10 stages. Thus, the gain of a single stage is $(2 \cdot 10^7)^{\frac{1}{10}} = 5.3$. (see Milagrito NIM paper for the measured gain of PMT's at the operated voltage.)

Thus we conclude that the systematic error in occupancy is comparable to the statistical one, therefore, for the desired accuracy of PE determination (few per cent) the threshold effect can be neglected. If one knows the probabilities β for each tube and wishes to make the correction, then it can be done with the following approximate formula:

$$\lambda = -\frac{1}{1-\beta}\ln(1-\eta)$$

Also note, the filter wheel calibration is approximately immune from this problem because it is based on the ratio of the λ's for a given PMT and the $\frac{1}{1-\beta}$ factor cancels.

7 Conclusion

In this memo we had reviewed the basic idea of the occupancy method used for PE calibration in Milagro. Innovative methods of filter wheel calibration and dynamic noise suppression were developed. Their use is illustrated. The introduced method of filter wheel calibration has several advantages over previously used one:

- Does not require additional hardware.
- No additional data taking is required.
- Eliminates the worry of loosing the wheel orientation because the filter calibration is obtained from the same data as the PMT one.
- Less time consuming, reducing the down time of the experiment.

The importance of the dynamic noise suppression based on the "Morse code" is illustrated and leads to an improved accuracy of the calibration compared to the previously used methods. The statistical error and threshold effect studies indicate that the accuracy of the PE calibration is on the level of few percent.

8 References

1. Isabel Leonor, TOT-PE Conversion using Occupancy Method. Milagro memo #30, 4/22/98.

2. R. Becker-Szendy et al., Calibration of the IMB detector. NIM, A 352, 629-639, (1995)

3. Gus Sinnis, Effect of Laser Fluctuations on the Occupancy Method. Milagro memo #25, 2/18/98.