How are Dec and RA affected by errors in angular reconstruction?

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December 17, 1998

Abstract

In this memo we study the effect of zenith and azimuth angle measurement errors on Dec and RA. As one of the possible sources of systematic errors in Milagrito, the effective speed of propagation of shower front is considered using Monte Carlo simulations. The effective speed is shown to be $28.8cm/ns$. The consequences of this type of systematic error are considered in detail. This systematic error leads to a shift and smearing of the Crab and to smearing of Mrk 501 only.

1 Effect of the zenith angle measurement error.

In order to study how an error in the measurement of the zenith distance affects the determination of the celestial coordinates of the source, we recall that (see, for example, "Pointing with Milagro" memo, or some other source):

$$\sin\delta = \sin\phi\cos z + \cos\phi\sin z\cos A \tag{1}$$

$$\cos z = \sin\delta\sin\phi + \cos\delta\cos\phi\cos H \tag{2}$$

$$\tan H = \frac{\sin z\sin A}{\cos\phi\cos z - \sin z\sin\phi\cos A} \tag{3}$$

where

ϕ is the latitude of the observer on the Earth

z the zenith distance of a star

A the azimuth of a star

H hour angle of a star, R.A.=sidereal time - H, $\Delta R.A. = -\Delta H$

δ declination of a star

Substituting $\sin\delta$ from (1) to (2) and using (3) we obtain:

$$\sin H = \frac{\sin z \sin A}{\cos\delta} \tag{4}$$

We will now use the above expressions to study the errors in declination and Right Ascension induced by the zenith angle measurement errors. Let us study declination first. After differentiating (1) we obtain:

$$\cos\delta\Delta\delta \approx -\sin\phi\sin z\Delta z + \cos\phi\cos z\cos A\Delta z$$

from (1)

$$\cos\phi\cos A = \frac{\sin\delta - \sin\phi\cos z}{\sin z}$$

Therefore

$$\frac{\Delta\delta}{\Delta z} = \cot z \tan\delta - \frac{\sin\phi}{\cos\delta\sin z} \tag{5}$$

The Right Ascension is also affected by the zenith error. From (3):

$$\cot H = \frac{\cos\phi}{\sin A}\cot z - \sin\phi\cot A$$

After differentiating it and using (4) we obtain:

$$\frac{\Delta H}{\Delta z} = \frac{\cos\phi\sin A}{\cos^2\delta} \tag{6}$$

2 Effect of the azimuth angle measurement error.

To study the effect of an azimuth angle error we, again, differentiate (4) and keep z constant:

$$\cos H \Delta H \approx \sin z \frac{\cos A \cos \delta \Delta A + \sin \delta \sin A \Delta \delta}{\cos^2 \delta} \tag{7}$$

and from (1):

$$\cos \delta \Delta \delta = -\cos \phi \sin z \sin A \Delta A \tag{8}$$

Then (7) becomes:

$$\frac{\cos H}{\sin z} \Delta H = \frac{\cos A}{\cos \delta} \Delta A - \frac{\tan \delta \sin^2 A \cos \phi \sin z}{\cos^2 \delta} \Delta A \tag{9}$$

Using (4):

$$\frac{\cos H}{\sin z} \frac{\Delta H}{\Delta A} = \frac{\cos A}{\cos \delta} - \frac{\tan \delta}{\sin z} \cos \phi \sin^2 H \tag{10}$$

Plugging $\cos A$ from (1) and $\cos z$ from (2) we obtain:

$$\frac{\Delta H}{\Delta A} = \tan \delta \cos \phi \cos H - \sin \phi \tag{11}$$

It is easy to obtain the shift in declination due to azimuth error by differentiating (1) and using (4)

$$\frac{\Delta \delta}{\Delta A} = -\cos \phi \sin H$$

3 Sine test.

The speed of propagation of a shower v is an "external" parameter in the reconstruction algorithm. The value of the azimuth A does not depend on the choice of v, the zenith distance z, however, does:

$$\sin z = v\sqrt{a^2 + b^2}$$

where a and b are the x and y components of the direction vector obtained from the fit, $A = atan2(b, a)$.

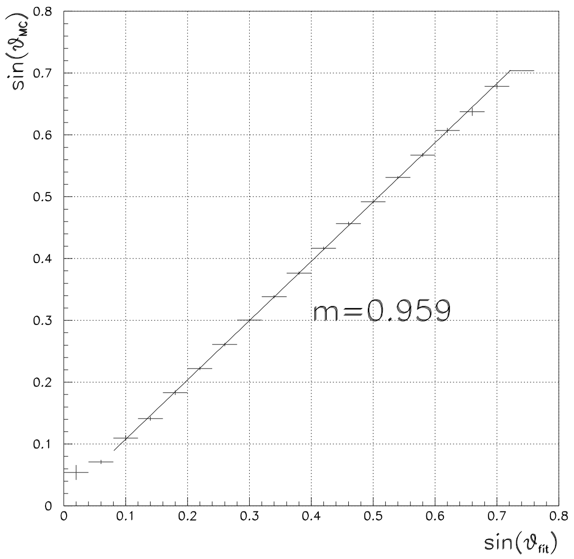
Thus, in order to correctly reconstruct shower direction, it is important to know the correct speed of shower front v. Currently used value of the speed is equal to the speed of light in air: $v_r = 30\ cm/ns$. Among the proposed reasons of why $v_r \neq 30\ cm/ns$ are the following: late tails in the absence of baffles (Cy H.), curvature and sampling corrections (Todd H.), "phase space" (David W.), two layer structure of the Milagrito... One of the ways to study it is to compare z_r and z_{mc} on the Monte Carlo data. Here we will use subscript "r" for values used/or obtained in the reconstruction, and subscript "mc" for "true" values:

$$\sin z_r = v_r\sqrt{a^2+b^2} \qquad \sin z_{mc} = v_{mc}\sqrt{a^2+b^2}$$

v_{mc} is the speed that Monte Carlo " wants" us to use in the reconstruction. Hence,

$$\sin z_{mc} = \frac{v_{mc}}{v_r}\sin z_r$$

The plot $\sin z_{mc}$ v.s. $\sin z_r$ provides information about the ratio of the speeds. Thanks to Stefan for generating this plot for Milagrito.



$$\frac{v_{mc}}{v_r} = 0.959$$

Therefore, Monte Carlo tells us that the correct effective speed of propagation of the shower front to be used in angular reconstruction is $30.0 \cdot 0.959 = 28.8\ cm/ns$. This plot exhibits a strange feature at small $\sin z_r$. Inasmuch as at small z_{mc}, z_r can only be bigger, we would expect $\sin z_r \geq \sin z_{mc}$ in this region. The plot shows opposite behaviour.

In the following discussion we are not concerned with the cause of this systematic error, can it be accommodated by the effective speed of propagation of the front or not. Understanding of this phenomenon is a subject for a separate investigation. We will use symbol η to characterize the phenomenon: $\sin z_{mc} = \eta \cdot \sin z_r$, $\eta = 0.959$.

4 Did Milagrito see the Crab?

The discovered systematics will cause an error in the measurement of the zenith distance. The magnitude of the error $\Delta z = z_r - z_{mc}$ can be estimated as:

$$\sin z_{mc} = \eta \cdot \sin(z_{mc} + \Delta z) \approx \eta\,(\sin z_{mc} + \cos z_{mc} \Delta z)$$

$$\Delta z \approx (1 - \eta) \tan z_{mc} = \Delta\eta \tan z_{mc} \tag{12}$$

where $\Delta\eta = 1 - \eta = 0.041$. This approximate formula is valid for small Δz which corresponds to $z < 60^\circ$ and $\Delta\eta \ll 1$. As one can see, the error increases with zenith distance.

It was pointed in section 1 that systematic zenith angle measurement error will lead to a drift of sources on the sky map. From (5), (6) and (12):

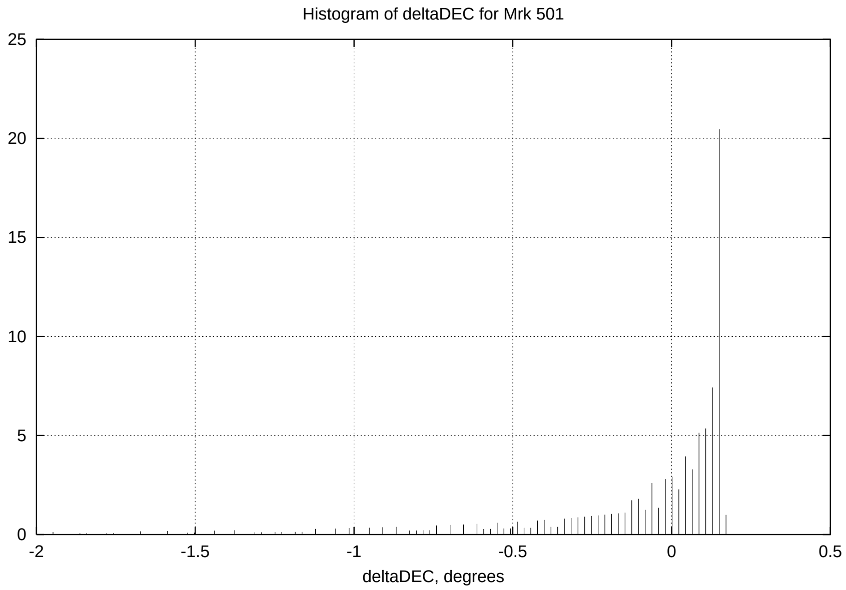
$$\frac{\Delta\delta}{\Delta\eta} = \tan\delta - \frac{\sin\phi}{\cos\delta\cos z} \tag{13}$$

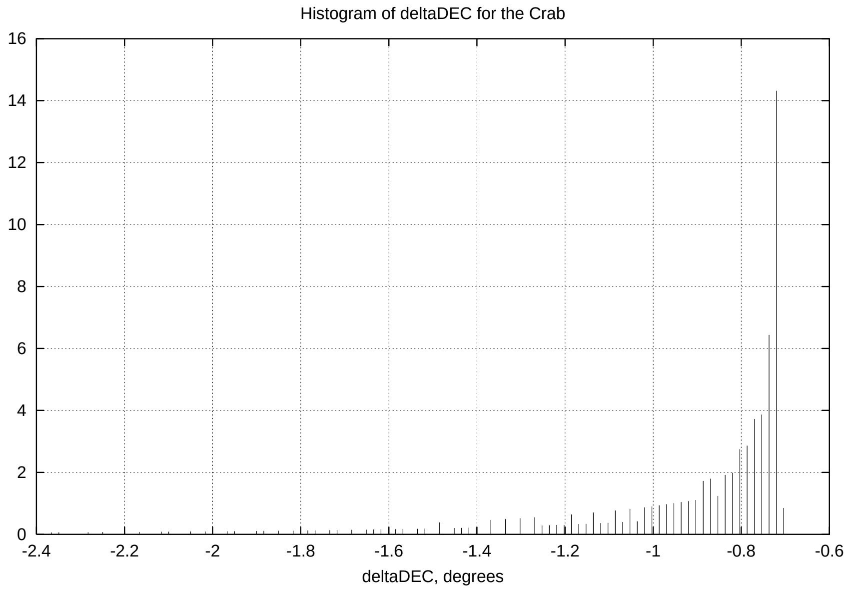
$$\frac{\Delta H}{\Delta\eta} = \frac{\cos\phi\sin A}{\cos^2\delta}\tan z \tag{14}$$

From (14) it is obvious that the average shift in R.A. $< \frac{\Delta H}{\Delta \eta} >= 0$ and this type of the zenith error causes an astigmatic type of aberration (elongation in Right Ascension). On the other hand, the most probable (significant) shift in declination is equal to (see appendix, (15)):

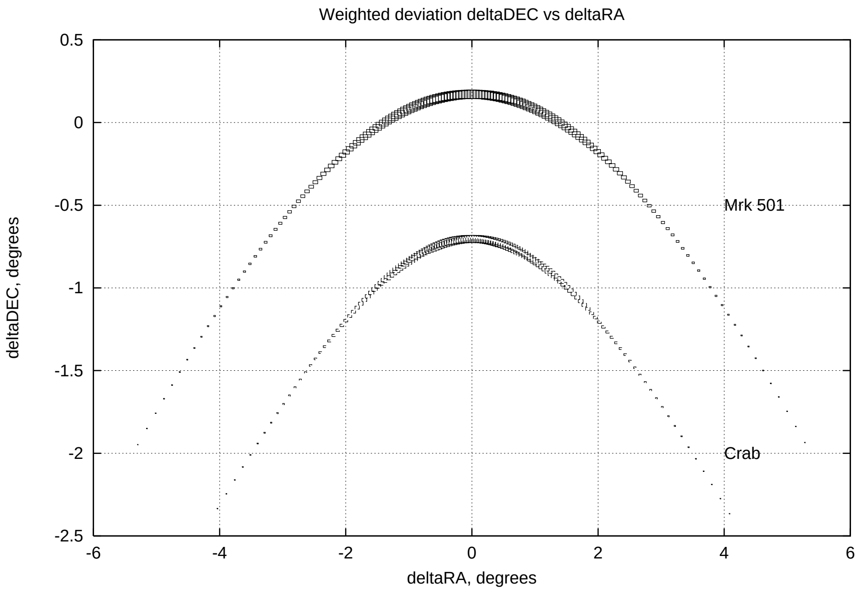
Mrk 501	$+0.2^\circ$
Crab	-0.7°

This is in complete agreement with the simulations (see plots below). In generating these plots we followed the sources on the sky and at each moment of time introduced the discussed error in the angular reconstruction. Number of events registered were assumed to be distributed as $\cos^4 z$.

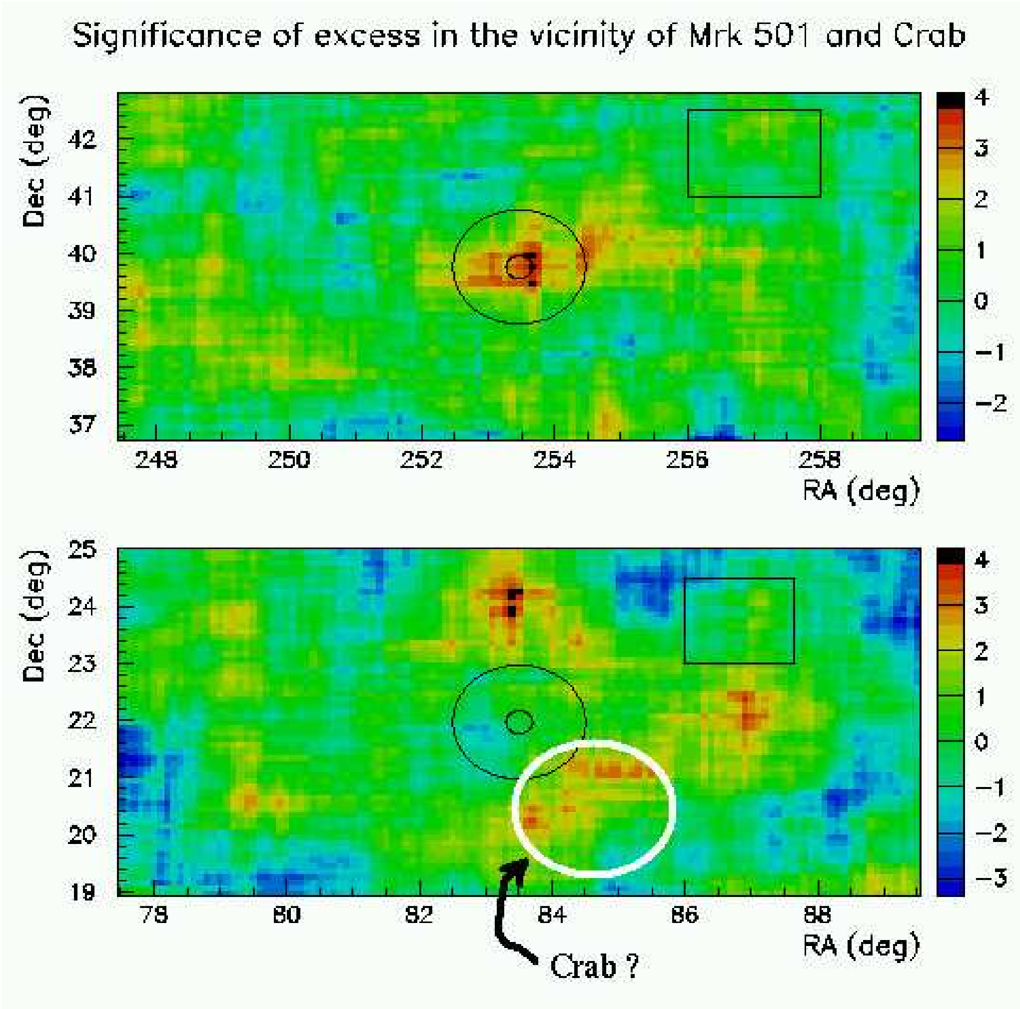




The plot below depicts the traces of the point sources on the sky relative to its nominal positions based on simulations. Squares represent the level of significance, generated using $\cos^4 z$ event distribution.



Thus, the calculations show that this type of systematic error does not shift the Mrk 501, but does move the Crab. The following significance plots provided by Andy show the regions of the sky around the sources. We circled the possible Crab location.



5 Conclusion.

In this memo we studied the general question of how measurement errors of angles in local coordinate system affect the reconstructed source locations. Sine test was motivated and proposed as a check of the presence of systematic zenith angle measurement errors. It was shown that the discovered systematic error leads to a drift of the Crab nebula as reconstructed by the present Milagrito routines while keeping the location of the Mrk 501 practically intact. In general, the sources with declination $\delta \approx \phi$ are least affected by the errors (15,16).

Several sources of potential problems have been mentioned. Another one is a strange behaviour of Δz vs z plot (not shown here) which has the expected qualitative shape (12) but Δz ranges only between 0° and 0.8° instead of predicted 0° and 2.3° when z is between 0° and 45°. We believe that these topics deserve separate studies because it might impact the quality of Milagro event reconstruction.

6 Acknowledgment.

We would like to thank everybody who took part in discussion of the matter during preparation of this memo. In particular, we express our gratitude to Todd H. and Stefan W. for fruitful discussions, suggestions and words of encouragement. We also would like to thank Peter N. and Allen M. for useful comments.

A Properties of deviation distribution function.

In (13) let us denote $\Psi = \frac{\Delta\delta}{\Delta\eta}$ and solve it for $\cos z$:

$$\frac{\Delta\delta}{\Delta\eta} = \tan\delta - \frac{\sin\phi}{\cos\delta\cos z}$$

$$\cos z = \frac{\sin\phi}{\sin\delta - \Psi\cos\delta}$$

Since Ψ is a monotonously decreasing function of z, and denoting the upper culmination by $z_{upper} = |\delta - \phi|$:

$$\Psi_{min} \stackrel{z \to \pi/2}{\longrightarrow} -\infty \qquad \Psi_{max} \stackrel{z \to z_{upper}}{\longrightarrow} \left(\tan\delta - \frac{\sin\phi}{\cos\delta \cos z_{upper}} \right)$$

Probability of getting $\Psi \leq \Psi_0$ is equal to the probability of getting $z \in [z_{min}(\Psi_0) = \arccos \frac{\sin\phi}{\sin\delta - \Psi_0 \cos\delta},\ \pi/2]$ e.i. assuming $\cos^n z$ event distribution:

$$P(\Psi \leq \Psi_0) = N \int_{z_{min}(\Psi_0)}^{\pi/2} \cos^n z dz$$

where N is normalization constant.

Differentiating the above with respect to Ψ_0 we obtain probability density distribution function:

$$\rho(\Psi) = N \frac{\cos^{n+2} z}{\sin z} \cdot \frac{\cos\delta}{\sin\phi} \Big|_{\cos z = \frac{\sin\phi}{\sin\delta - \Psi\cos\delta}}$$

Now one can see that $\rho(\Psi)$ grows monotonously at least for $n \geq -2$, because $\cos z$ and $\frac{1}{\sin z}$ increase with Ψ.

Therefore, most probable value of Ψ is reached when $\Psi = \Psi_{max}$. Thus,

$$\frac{\Delta\delta}{\Delta\eta} \Big|_{most\ probable} = \tan\delta - \frac{\sin\phi}{\cos\delta \cos(\delta - \phi)} \tag{15}$$

Similar calculations can be done in order to estimate the most probable error in RA due to constant systematic error in azimuth (11). The distribution function can be shown to be of the form:

$$\rho(\Phi) = \frac{1}{\sin\delta} \cdot \frac{\cos^n z}{\sin z} \Big|_{\cos z = \frac{\Phi + \sin\phi}{\sin\delta}}$$

where $\Phi = \cos^2\delta \cdot \frac{\Delta H}{\Delta A}$.

From the above we see that $\rho(\Phi)$ grows monotonously at least for $n \geq 0$, and, therefore, the most probable value of Φ is reached when $\Phi = \Phi_{max} = \sin\delta \cos z_{upper} - \sin\phi$, thus:

$$\frac{\Delta H}{\Delta A} \Big|_{most\ probable} = \frac{\sin\delta \cos(\delta - \phi) - \sin\phi}{\cos^2\delta} \tag{16}$$