Large Scale Cosmic-Ray Anisotropy Analysis

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This memo contains the procedure used in the analysis of the cosmic-ray anisotropy. The various data cuts used and the estimation of errors are discussed. Also, the methods of forward-backward asymmetry and harmonic fitting are explained.

1 Organization of Event Data

The cosmic-ray events are binned in 2-D histograms according to their arrival direction from $-10°$ to $80°$ in declination and $-50°$ to $+50°$ in hour angle(see Fig. 1 for an example). These limits are imposed due to the fact that only events with a zenith angle $\leq 50°$ are accepted. This zenith angle cut is used to limit contamination from muons. The events are collected over 30 "minute" periods, where "minute" is defined in the following three time frames: sidereal (366.25 days/year), universal (365.25 days/year), anti-sidereal (364.25 days/year). These events are placed into histograms with $5° \times 5°$ bins giving us 48 half hour histograms per day (in one of the three time frames). This averaging scheme is used with the forward-backward asymmetry method (Sec. 3.1) in order to remove the effects of trigger rate variations due to changing atmospheric and detector conditions with time scales on the order of 30 minutes.

The time frames mentioned above correspond to different views of the sky. Universal time(UT) shows the sky in sun fixed coordinates(i.e. at Milagro longitude noon is at 195°). Sidereal time(ST) is the usual equatorial coordinate system of r.a. and dec. Anti-sidereal time(AST) corresponds to no physical viewpoint. As such it should have no signal present but is included for symmetry and as a check on systematics(more on this later). A ST day is 3 minutes and 56.56 seconds longer than a UT day. An AST day is shorter than a UT day by the same amount.

In order to minimize contamination of the signal between the three time frames we analyze the data in sets of an integral number of years. Take for example a fixed signal in sidereal time, this signal's position in UT shifts by about 4 minutes per day returning to the original postion after exactly one year. When the analysis is done the mean value is subtracted out and therefore this fixed ST signal will average to zero in the UT map. The same holds true for AST except in AST a fixed sidereal signal will transit twice in one year.

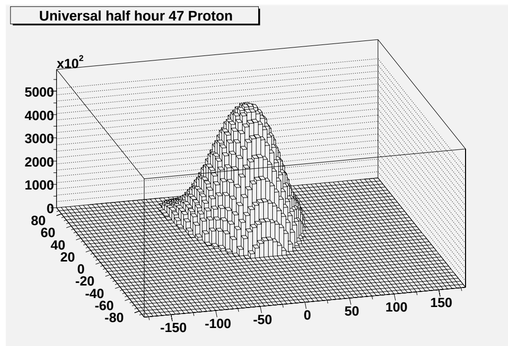


Figure 1: Sample of a histogram showing the number of events as a function of dec. vs. hour angle for a single 30 minute period.

2 Data Cuts

In addition to the aformentioned zenith angle cut we make the following cuts on the individual events:nFit$\geq$ 20 and nTop$\geq$ 90. Before the data is analyzed we also look through the data for trouble spots which will be excluded from the analysis. Beyond the obvious times where there were repairs we look for large deviations in the zenith(theta) and phi angle distributions. The way this is accomplished by using a program which reads through the data creating histograms of the theta and phi distributions containing a fixed number of events. The number of events is fixed to avoid picking out periods of dead time and corresponds to a $\sim$ 30 minute interval given our average trigger rate of $\sim$ 1700Hz. These intervals are collected over a period consisting of about three days. The individual phi and theta distributions are then compared to the three day average by computing the chi square difference between them after normalization. If the chi square is larger than 23 for the Theta dist. or larger than 5 for the Phi dist. the failing histo. is sent to a file along with the time interval it corresponds to. These cutoffs were chosen because it gives a reasonably low number of false failures without missing the problem spots. This procedure is repeated for the entire data set. The output intervals can then be inspected by hand and compared to the shift log etc. Many times these intervals will have associated log entries involving hardware failures, weather etc. If the interval appears to be legitimately corrupted it is excluded. Cuts made in this fashion correspond to $\sim$ 5% of the total number of events collected.

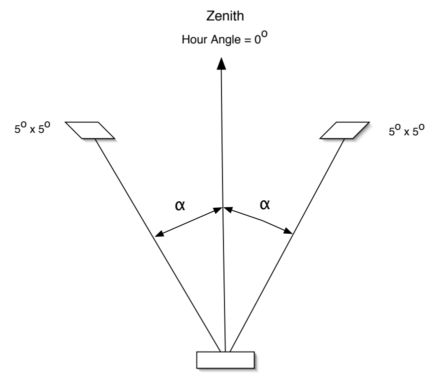


Figure 2: Diagram showing the definition of α used in the calculation of the forward-backward asymmetry for a single dec. band and a given 30 minute histogram. α is in the direction of hour angle.

3 Reconstruction Method

3.1 Forward-Backward Asymmetry

The forward-backward asymmetry method(FB) as mentioned previously is used to remove problems rising from short term variations in trigger rates due to effects which cannot be accurately modelled (weather, detector etc.). On time scales of around 30 minutes the observatory conditions are assumed to be stable and thus FB allows us to analyze each of these periods independently effectively removing variations occurring between these periods. To limit the effects of random variations during these periods many days are averaged together.

Since Milagro scans the sky with the motion of the Earth, we have no information about the modulation in the declination direction. For this reason each dec. band is treated as an independent observation and is analyzed separately. We make the assumption that the large scale anisotropy in any given dec. band can be modelled by a fourier series and that it is a small modulation of a nearly isotropic signal. Three harmonics are used in this analysis (See Sec. 3.2) which allows us to see large scale effects having a width in r. a. of greater than $\sim 40°$.

Using this model, the equation for the (normalized) rate is:

$$R(\theta) = 1 + \sum_{n=1}^{3} \gamma_n \cos n(\theta - \phi_n) \tag{1}$$

$$\gamma_n \ll 1$$

The first step in finding the fourier coefficients is calculating the FB asymmetry for each half hour histogram as a function of α (see Figure 2).

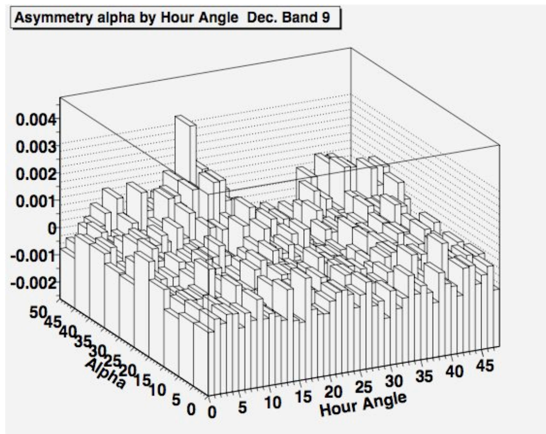


Figure 3: Sample histogram to be fit of the forward-backward asymmetry as a function of α and hour angle for a single dec. band. The mean is set to zero.

$$FB(\theta,\alpha) = \frac{R(\theta+\alpha) - R(\theta-\alpha)}{R(\theta+\alpha) + R(\theta-\alpha)} \quad (2)$$

where θ = mean time in degrees of the half hour histogram and α ranges from 2.5° to 47.5° in 5° steps. These values of FB are binned in a 2-D histogram of α vs. θ which then has the mean value subtracted out (See Fig. 3).

3.2 Reconstruction

The fourier coefficients are obtained from these histograms by fitting to the following function function obtained by substituting (1) in (2), applying the appropriate trigonometric identities and using the fact that $\gamma_n \ll 1$.

$$FB(\theta,\alpha) \approx \sum_{n=1}^{3} -\gamma_n \sin(n\alpha)\sin(n(\theta-\phi_n)) \quad (3)$$

The coefficients thus obtained are used to reconstruct the anisotropy as a fractional difference from isotropic in a given dec. band as follows:

$$A(\theta) = \sum_{n=1}^{3} \gamma_n \cos(n(\theta-\phi_n)) \quad (4)$$

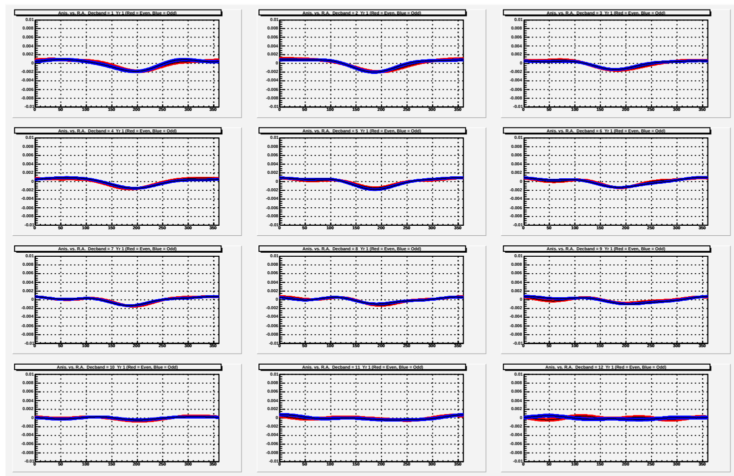


Figure 4: Reconstructed dec. profiles for the first year of data. The red curves are the "even" numbered events the blue "odd". The width of the curves are the statistical error.

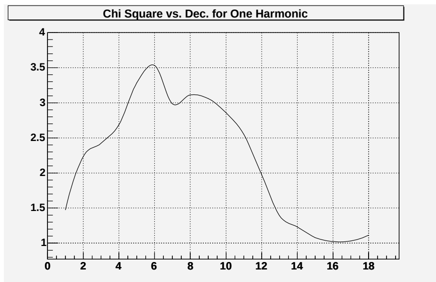
The statistical errors are obtained by the usual propagation method. Assuming the error during event collection obeys a poisson distribution, for a bin with N events the error is $\sqrt{N}$. From the fit we obtain the errors on the fit parameters from which the error in the reconstructed signal is calculated using:

$$\sigma^2_{signal}(\theta) = \sum_{n=1}^{3} \left[\sigma^2_{\gamma_n} \left(\frac{\partial A(\theta)}{\partial \gamma_n} \right)^2 + \sigma^2_{\phi_n} \left(\frac{\partial A(\theta)}{\partial \phi_n} \right)^2 \right] \tag{5}$$

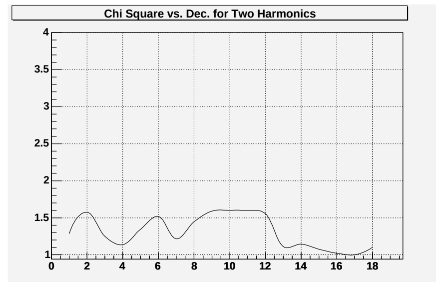
A check on the stability and validity of the statistical errors of this method was performed by creating two sets of data corresponding to the same period of time. The way this was done was by alternating the set into which a event is placed. For example the first event is put in the "odd" set, the second event is put in the "even" set etc. Figure 4 shows the result of the analysis for the "even" and "odd" sets superimposed for twelve different declination bands. As can be seen the profiles match very well within the statistical errors.

3.3 Number of Harmonics

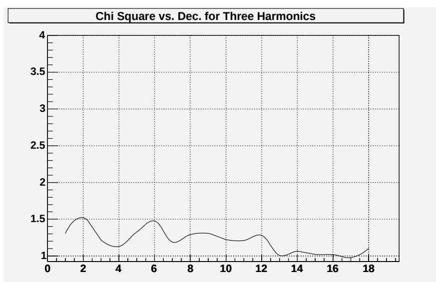
The optimal number of fourier harmonics was determined by examining the chi square per degree of freedom for the 2-D fits as a function of declination. These results are plotted in Fig. 5. As can be seen the three harmonic fit gives a chi square/ndf of ~ 1 with no significant improvement for four harmonics. To avoid over fitting three was deemed sufficient.



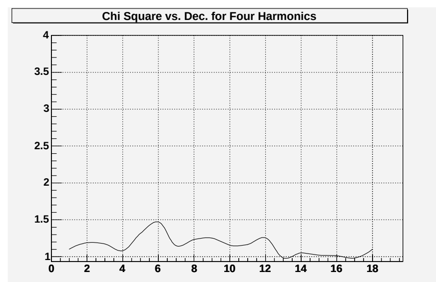
(a) One Harmonic



(b) Two Harmonics



(c) Three Harmonics



(d) Four Harmonics

Figure 5: χ^2 vs. dec. for different number of fit harmonics.

3.4 Systematic Errors

The systematic errors are estimated by examining the ATS time plots for an integral number of years. For an integral number of years a static signal in one time frame will not affect the others. The reason for this is explained in section 1. Monte carlos shows that if there is a time varying signal in one frame it will induce a signal in the adjacent frames since it doesn't average out over the course of the year. The induced signal is attenuated greatly for slowly varying signals. Furthermore, MCs show that if the signal varies in universal time it will affect both anti-sidereal and sidereal time with equal magnitude but not necessarily the same phase. Given that anti-sidereal time corresponds to no physical frame of reference we expect to see no signal here. If a signal does appear here the cause is assumed to be temporal variations in the universal time signal. We can rule out the sidereal time variation as the origin of the AST signal because the sidereal signal, although not constant, only varies on the order of a factor of two over the entire 6 yr. data set and therefore cannot induce the AST variations at the level we do see. Since we know this same (ATS) signal will be superimposed on the sidereal signal but with unknown phase, we use the r.m.s. of these fluctuations to estimate the systematic errors of the sidereal anisotropy.

One check of this procedure we can do is to look at magnitude of the errors for a number of set sizes. Since the systematics are expected to vary randomly in time the error should go like $\sigma/\sqrt{N}$ with N being the size of the data sample. The six and one year systematic errors can be estimated from the AST plots. For a two month period (our usual smallest averaging period) we have to use a different approach. Fig. 6 shows a sample of single day slices corresponding to the dec. band at our zenith.

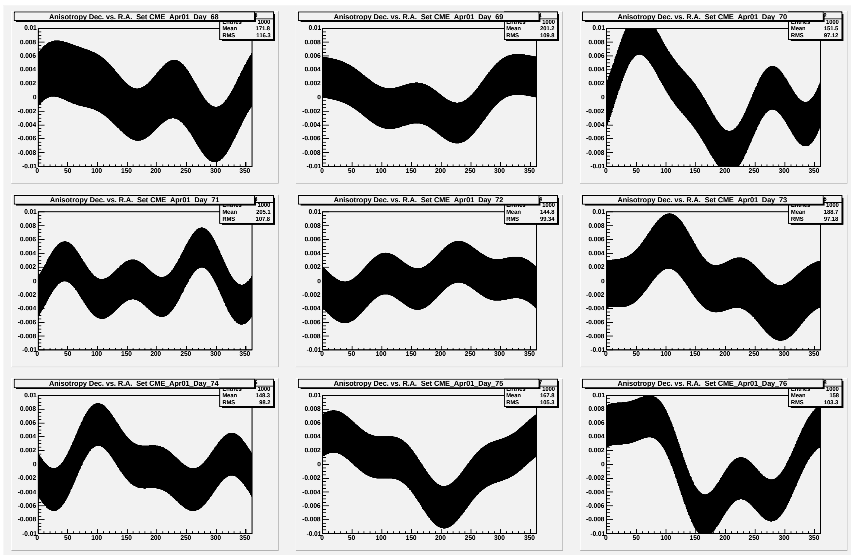


Figure 6: Anisotropy analysis for dec. $\sim 36^\circ$ for a 9 consecutive day sample. The width of the lines correspond to the statistical errors. The scale is $\pm 1\%$.

As can be seen there is a great deal of variation on this time scale which completely washes out the sidereal signal. Also on a day to day basis the difference between ST, UT and AST is non-existant in this analysis since they differ by $\ll 1^\circ$ in hour angle. Assuming these variations are largely due to systematics we can try to estimate the sys. error for a two month period by taking 60 single day plots and averaging them together after randomizing their phases. In doing this any signal present should be washed out and only the variations should remain. Taking the r.m.s. of this variation will be interpreted as the sys. error. Figure 7 shows a plot of the estimated systematic error inverse squared for two months, one year and six years for our data. As can be seen the errors obey the linearity one expects from from standard error analysis.

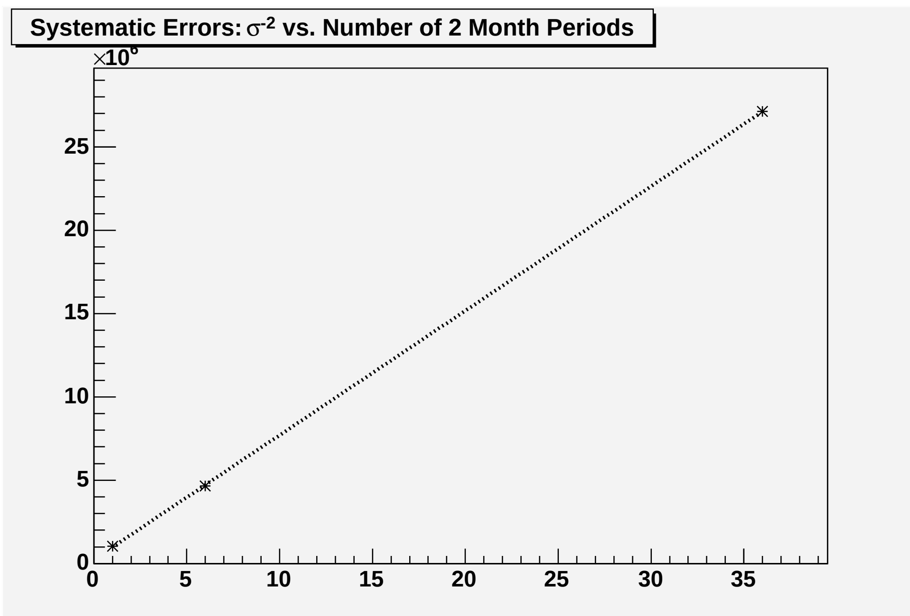


Figure 7: Plot of σ_{sys}^{-2} vs. number of two month periods. The 12 and 72 month errors were obtained using the AST estimation procedure. The 2 month error was found by averaging the daily variations. The dotted line is a linear fit.