Does the Crab Data Agree with the γ Monte Carlo?

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Abstract

The agreement is less than ideal, though the statistics are marginal. As $X_2 = 2.5$ happens to give the largest excess, we might have over-estimated the flux.

1 Introduction

The thrust of the Crab paper was to show that our apparatus was working well enough that we could see the Crab; that we owned a variable (Compactness, or colloquially, X_2) which, according to our Monte Carlo, enhances gammas with respect to hadrons; and that such an analysis yielded a reasonable flux (with large uncertainties dominated by lack of energy resolution). We argue that the gamma MC describes X_2 well enough that we can use the MC distribution to calculate the X_2 cut efficiency. However, to my disappointment, we never actually presented for $X_2 > 2.5$ a direct comparison of the gamma Monte Carlo curves to what the data tells us. I attempt such a comparison here and try to draw conclusions.

2 Inputs to the Comparison

Andy Smith kindly supplied me with the data and significance for the Crab photon excess for X_2 from 0.0 to 8.50; these are corrected for the spread of signal into the background region, since we agree there really is a signal to correct. I calculated error bars for bin contents assuming I could get by with $\mathcal{S} = x/\sigma$, where $\mathcal{S}$ = significance in standard deviation units, x is the observed excess in the bin, and σ is the error bar I sought for x, the contents of the data bin.

Gus Sinnis kindly supplied me with the data for the gamma Monte Carlo as a function of X_2 (we ran enough Monte Carlo so that the error bars are small compared to the data). So I plotted them on the same graph for the range over which both were available. The differential data spectrum contains X_2 bins with a negative photon excess for $X_2 < 2.5$: this is one reason the intended comparison of the photon efficiency for $(X_2 > 2.5)/(X_2 > 0)$ didn't work very well. Beyond $X_2 = 8.5$, I didn't have data values. The Crab paper reports $X_2 > 2.5$, the entire range.

3 Normalization

One issue is how to normalize the Gamma MC shape to the data. Here I'll show a couple of alternatives. The table shows the relative normalization factors: larger factors mean the MC has to be scaled up higher to meet the listed normalization condition. The table also shows the implied

X_2 *Range*	$MC/MC(0-8.5)$	Gammas (0-8.5)	$\gamma/\gamma(2.5-8.5)$
0-8.5	1.0000	8202	.432
2.5-8.5	2.315	18991	1.000
3-4	2.319	19020	1.002
4-8.5	1.782	14618	0.770
min Chisq			
0-8.5	1.463	12000	0.632

Table 1: Relative Normalization Factors and Deduced Gamma Excesses

excess number of gammas, and the ratio of these deduced gamma for the various schemes. A natural normalization is to simply scale the Monte Carlo to the same number of events as the data (in net excess for all X_2), or a bit more carefully, for the range $0.0 < X_2 < 8.5$. However, because of the large uncertainties in gamma excesses for $X_2 < 2.5$, we chose instead to deduce the flux from the excess after the cut, $X_2 > 2.5$, and to use the gamma MC to deduce the cut efficiency. Since I don't have the data for $X_2 > 8.5$, I approximate this normalization by matching integrals on $2.5 < X_2 < 8.5$.

Before showing Brenda Dingus the resulting plots, we'd agreed on one other normalization to consider: $3 < X_2 < 4$; however this turned out by coincidence to have a relative factor between data and Monte Carlo within 1% of the $2.5 < X_2 < 8.5$ normalization, thus providing no usefully-different information. So I added another plausible choice: normalizing in the upper tail, $4 < X_2 < 8.5$, since the MC indicated that a cut at 4.0 or 2.5 should lead to a similar significance. The last line in the table is discussed in the penultimate section.

4 The Comparisons

From the table, and more detailed checks, I found that the cut we chose, $X_2 > 2.5$, yields the largest estimated flux of any lower-limit cut on a 0.5 bin boundary. With our data, almost any other normalization of MC to data would have given us a lower flux estimate (significance too? not obvious). If the cut value was chosen without looking at the data (just by studying the Monte Carlo expected significance as a function of cut value), we were lucky; if not, we have likely overestimated the flux, as we are implicitly using the MC X_2 distribution shape to calculate the efficiency of our cut.

Now let's look at some plots comparing the differential and integral shapes. Among the candidate normalizations, the extremes are well represented by $X_2 > 0$ and $X_2 > 2.5$, so I'll only show plots using those.

Figure 1 shows the differential spectrum: a histogram of events per bin of X_2. Here I chose to normalize the gamma Monte Carlo to the observed net data excess in the range $0 < X_2 < 8.5$, so the areas of the two curves are equal, after counting the negative data bin contents as negative area. This normalization fortuitously forces the high tail $X_2 > 5$ to be roughly equivalent.

Figure 2 shows the same data plotted as an integral spectrum, showing the percent of the data greater than the X_2 cut value. Again, the normalization is chosen for $X_2 > 0$; this forces equal values at 0. This is essentially a plot of efficiency vs cut, but with the 100% point scaled up to a number of events.

Figure 3 shows the integral spectrum on a semi-log scale; this time I chose the normalization with the gamma MC integral for $2.5 < X_2 < 8.5$ set equal to the observed net data excess in the

range $0 < X_2 < 8.5$, as we implicitly did in the Crab paper. Since it's a semi-log scale, you can imagine the curves sliding up and down with respect to each other if you want to choose a different normalization. That would be the same as multiplying one curve by a constant.

Figure 4 shows the differential spectrum on a semi-log scale, again with the to the gamma MC integral for $X_2 > 2.5$ set equal to the observed net data excess in the range $0 < X_2 < 8.5$; but to make the plot I substituted a small positive value for the two negative bins below $X_2 = 2.5$.

4.1 Conclusions from the Plots

What I conclude from these plots is that

- The data and MC spectra shapes are different, even if you restrict attention to the region $X_2 > 2.5$ (but see the discussion of fits below on statistical significance).
- Thus, you can't both match $X_2 > 2.5$ and larger X_2; we already gave up on demonstrating a nice match to $X_2 < 2.50$. As we use the MC shape to calculate efficiency, we are assuming this is a statistical fluke, due to uncertainties becoming large at low X_2.
- One could reasonably conclude that either the X_2 distribution for the Monte Carlo gammas is an incorrect representation of our data (and its calibration constants), or that the excess for the chosen cut of $X_2 > 2.5$ is a-typically large.
- If you prefer to believe that the Monte Carlo shape matches the data well, then you might also conclude that we may have overestimated the flux for the Crab somewhat.
- Where we set the cut for X_2 within a reasonable range of $2.5 - 4.0$ leads to a 30% (downward) variation in the deduced flux. That's still less than the uncertainty due to energy scale, but not entirely negligible.

5 Other Comparisons Between MC and Data

5.1 Relative Efficiency for Harder Cuts

Another way to look at the data is to compare the excess for $X_2 > 4.0$ with the excess for $X_2 > 2.5$. Since I have data only out to $X_2 = 8.5$, I'll instead compare the ratio $\mathcal{E}$ of integrals in the ranges $4.0 < X_2 < 8.5$ to $2.5 < X_2 < 8.5$, the relative efficiencies. For data, $\mathcal{E} = .37 \pm .09$, and for Monte Carlo, $\mathcal{E} = .48$. The uncertainty in the data ratio was calculated assuming un-correlated errors between the two regions, which will overestimate the actual uncertainty. So this comparison again says we are somewhat lower than Monte Carlo expects, by 1-2 standard deviations. This comparison didn't make it into the paper because it is not much more statistically powerful than the $X_2 > 0$ to $X_2 > 2.5$ comparison we did cite.

5.2 Chi-Squared Fits

Finally, I calculated the Chi-squared for the data vs MC, ignoring the errors on the MC (which are small), and scaling the MC to the integrated total data excess (summing negative and positive) over the range $0 < X_2 < 8.5$. The result is a Chi-squared of 28.2 for 16 degrees of freedom (17 bins - 1 normalization constant); the probability of getting a worse Chi-squared if the MC really describes the data is .03, equivalent to roughly 1.9 standard deviations from a perfect fit: not what you wanted, but certainly not enough to make you suspect you really can't fit the data with the

MC. Of course, Chi-squared is blind to the signs of deviations, or runs of deviations of the same sign.

If you take the relative normalization as a totally free parameter, but still look at fitting the whole range of X_2 (recall we maintain in the paper that the MC describes the whole range), the minimum Chi-squared drops to about 24.7, or a probability of .075–a 1.4 standard deviation equivalent. This normalization, which you'd get if you used the MC shape to fit all the data for the number of gammas, corresponds to a gamma excess of 12K events in the X_2 range 0 to 8.5, compared with the observed excess in that range of 8.2K, and the value of 19K from considering only $X_2 > 2.5$, which is essentially what we reported in the paper. Note that each value, except for the 8.2K, implicitly assumes that the gamma MC is correct.

6 Conclusions

Overall, the Crab source strength, and our analysis, are still too statistically weak to give a crisp answer to the question of whether the gamma MC shape is correct. This is a bit discouraging, as this is the best source we're likely to get. If we're convinced we can maintain calibration, we might wait for more statistics to do a better test. But we can't point to the Crab paper as having cleanly shown that we can use the gamma MC as an accurate representation of the X2 distribution for photons; we're compatible at the 2 standard deviation level, hardly a ringing endorsement of our MC-data agreement.

On the basis of this data, one could even argue that we have a residual flux uncertainty of some 30% due to choice of where to put the cut. However, one could also argue if you really expected no difference between MC and data (a fairly bold position), then the flux is also consistent with a constant value independent of the cut–within 1 to two standard deviations.

I thank Gus, Andy, and Brenda for many conversations and useful suggestions.

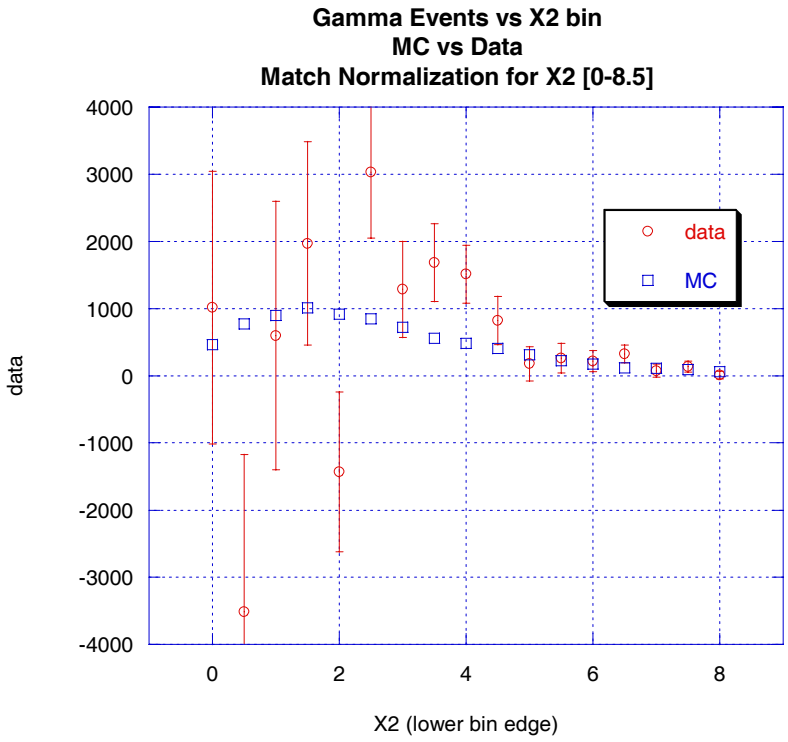

Figure 1: Distribution of Gamma Excess vs. X2 Bin for Data and Gamma MC.

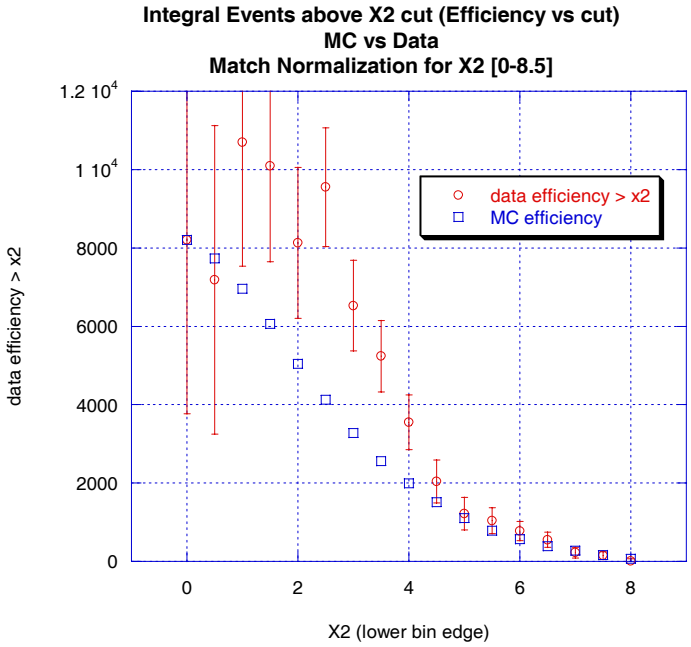

Figure 2: Integrated Distribution of Gamma Excess vs. X2 Cut for Data and Gamma MC.

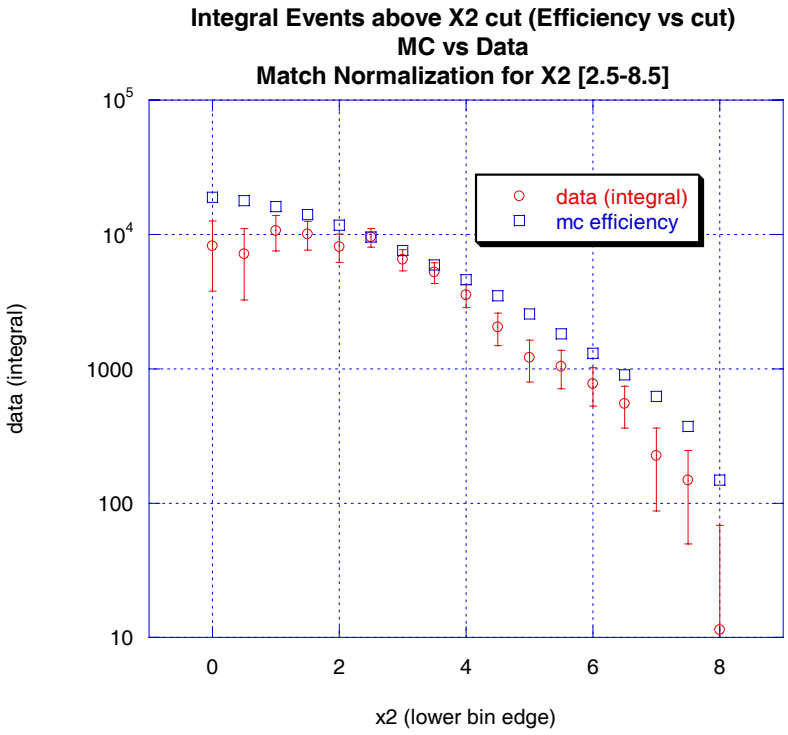

Figure 3: Integrated Distribution of Gamma Excess vs. X2 Cut for Data and Gamma MC.

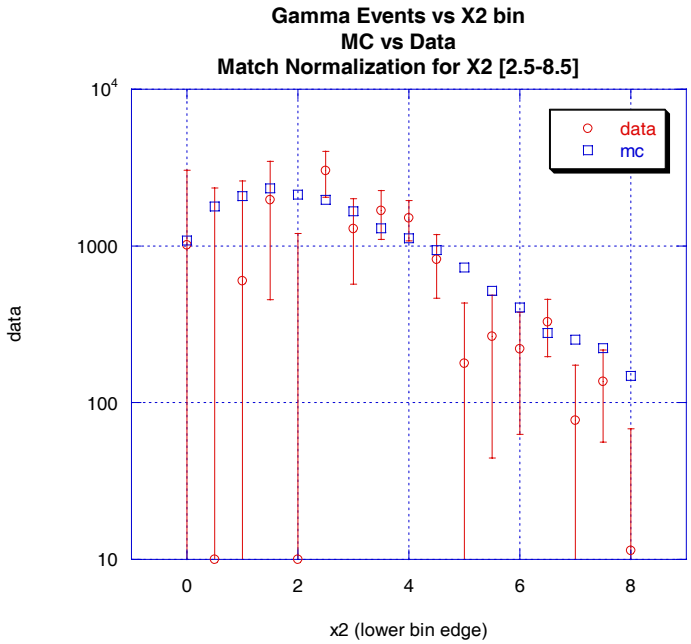

Figure 4: Integrated Distribution of Gamma Excess vs. X2 Cut for Data and Gamma MC.