Tuning the Angle Fitter, Sampling and Curvature Corrections for Milagro

by **J. F. McCullough and Javier Bussóns Gordo**

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Abstract

We have tuned angle fitter parameters such as weights, PE- and SIG-cuts and derived new sampling and curvature corrections for Milagro. In this memo, we present the techniques that we used and compare our results with the current standard fitter (version v41) in terms of angular resolution, significance, percentage of events fit and percentage of signal events kept as a function of bin size. In each and every aspect, our work represents a significant improvement over what is currently online.

1 Introduction

The current online angle fitter (v41) uses the arrival times to air-shower photomultiplier tubes (PMTs) and their pulse heights to find the direction (θ, ϕ) of the incoming primary by χ^2-fitting the PMT arrival times to a plane. In the fit, each PMT is weighted by a function which depends upon the number of photoelectrons (PEs) — the more PEs a PMT registers, the more weight it is given in the fit.

Currently, the angle-fitting routine does 5 passes, with cuts defined in two arrays called PE cut (indices 0 to 4) and SIG cut (indices 0 to 3, used in passes 2 to 5, respectively).

- First pass: only PMTs with PEs > PE cut[0], i.e. the best entries, are used in the fit. Since no fitted time (T_f) is available yet, the time of the hottest tube is used as reference when calculating the distribution of residual times ($T_\chi^i = T_f - T_i$ for each hit i). Then, the used hits are tagged, their weights calculated according to their pulse height and the fit is carried out.

- Passes 2 to 5: here, the fitter starts refining the fit on successive passes by throwing out PMTs whose T_χ values are too large (SIG cut) and by bringing in PMTs with smaller pulse heights (PE cut) but that still pass the SIG cut. Before each new pass, the code decides whether to increment the PE cut (to a lower PE cut) or stay with the same one. In version v41, the PE cut is incremented if $Nfit < 20$. However, we have found that relaxing the PE cut regardless of the number of PMTs left after the previous pass results in a more than 10% improvement in Δ_{eo} and almost twice as many tubes included in the fit.

The four increasingly more severe SIG cuts are used in passes 2 to 5. Late hits with respect to T_f (i.e., $T_\chi < 0$) pass the SIG cut if:

$$|T_\chi| \leq S(PE) \times SIGcut[\,], \tag{1}$$

where S(PE) is a PE-dependent measure of the spread of the T_χ distribution (either RMS or sigma, see below), whereas early hits ($T_\chi > 0$) pass if:

$$T_\chi \leq a \times S(PE) \times SIGcut[\,]. \tag{2}$$

We have allowed a to take different values and found that $a = 2.75$ is optimum (its value in v41 is 2.00). We define the weight of PMT i as:

$$w_i = \frac{1}{(S(PE)_i)^2}. \tag{3}$$

The (weighted) goodness of the fit (for each pass) is given by:

$$\chi^2_\nu = \sum_{j=0}^{Nfit} \frac{w_j \times (T_f - T_j)^2}{Nfit - 3}, \tag{4}$$

where the denominator is the number of degrees of freedom

Before the PMT arrival times can be used in the angle fitter, certain corrections have to be made. Aside from the slewing and pedestal corrections which are taken into account by the new laser calibrations, the PMT arrival times must be corrected for sampling and curvature. By sampling we mean the effect by which PMTs with low pulse heights are usually hit later than PMTs with large pulse heights. The curvature correction accounts for the fact that showers do not arrive in a flat plane but, rather, in a curved front, with the curvature depending upon the distance from the core.

The sampling and curvature corrections that are currently used online were derived more than a year ago from Monte Carlo. Although it has been known for some time that these corrections needed to be redone for the data, it did not seem useful to do so before we had calibrations (which we now have).

In this task we have worked simultaneously but separately. We both used the same data set (raw file GRB_2058_0017.dat, 480,753 events, ~ 1100 Hz) to optimize the angle fitter and derive new corrections, but we had different starting points and used different quantities for sampling, curvature and weights. For those interested in the technical details, in Appendices A (Joe) and B (Bussy) we have each described our technique in detail. The results from each technique and from the current online version are presented in the next section.

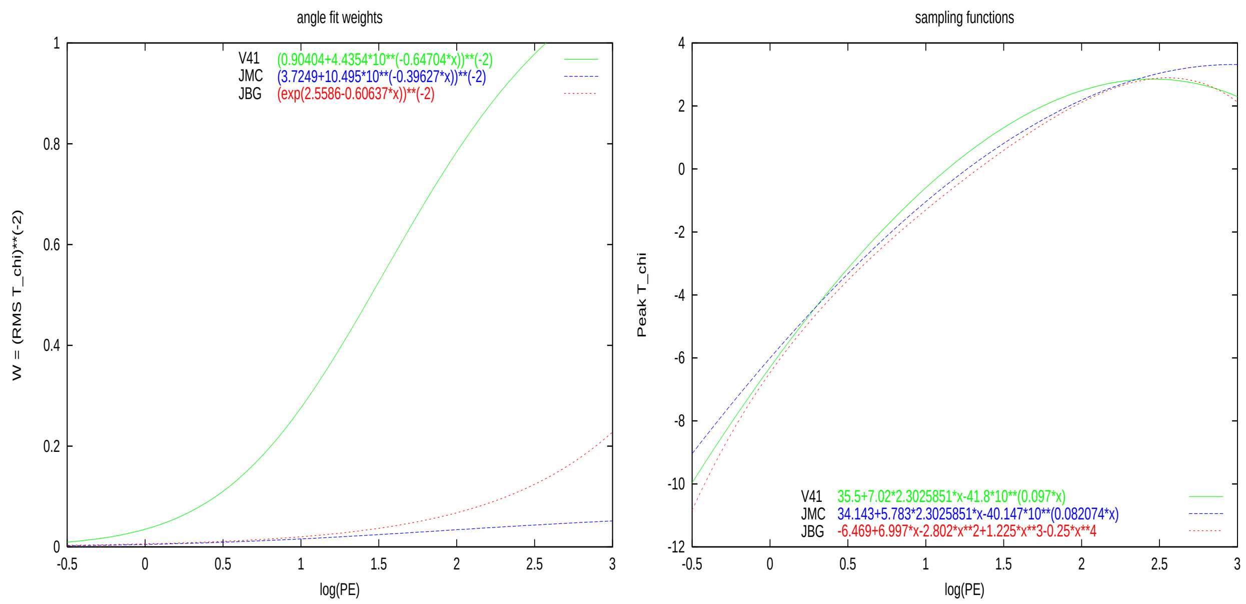

Figure 1: Comparison of weights (left) and sampling corrections (right): current standard code (V41), Joe's analysis (JMC) and Bussy's (JBG).

2 Results

The optimized parameters arrived at are as follows (first expression listed is Joe's (JMC), second one is Bussy's (JBG)):

- **Function for Weights** (Figure 1, left plot):

  if (PEs > 300) PEs = 300 (Joe's saturation);

$$S(PE)^{JMC} = 10.495\ (PE)^{-0.39627} + 3.7249, \tag{5}$$

$$S(PE)^{JBG} = exp(2.5586 - 0.60637\ \log(PE)). \tag{6}$$

- **Sampling Function** (Figure 1, right plot):

$$T_s^{JMC} = 5.7830\ \log(PE) - 40.147\ (PE)^{0.082074} + 34.143, \tag{7}$$

$$T_s^{JBG} = -0.25\ \log^4(PE) + 1.225\ \log^3(PE) - 2.802\ \log^2(PE) + 6.997\ \log(PE) - 6.469. \tag{8}$$

- **PE and SIG cuts** (Table 1).

- **Curvature Function**:

$$T_c^{JMC} = 0.04\ R_c, \tag{9}$$

$$T_c^{JBG} = -0.445 + 0.0174\ R_c - 0.00147\ R_c^2. \tag{10}$$

Method	Pass:	1	2	3	4	5
JBG:	PE cut	2.00	1.25	1.00	0.50	0.50
	SIG cut		2.00	1.00	0.75	0.50
JMC:	PE cut	1.75	1.25	0.75	0.75	0.50
	SIG cut		2.00	1.25	0.75	0.50
v41:	PE cut	1.75	1.50	1.25	0.75	0.50
	SIG cut		4.00	3.50	3.00	3.00

Table 1: PE and SIG cuts for this work (JBG, JMC) and for the standard code (v41).

	JBG	JMC	V41
Events fit (%)	97.1	97.5	96.8
failed deleo (%)	4.4	3.4	5.0
unweighted χ^2	13.05±6.94	14.54±7.56	15.87±9.16
σ (Nfit> 0)	11.52	11.42	9.71
$\Delta_{eo}(^o)$	1.77	1.77	2.17
corrected $\Delta_{eo}(^o)$	1.85	1.84	2.28
median Nfit	50.63	52.64	33.58
Opt. bin size (^{o})	0.8	0.8	0.9
Events in bin (%)	46.13	45.72	43.74
σ (Nfit> 20)	11.22	11.17	9.46
$\Delta_{eo}(^o)$	1.69	1.71	2.11
Opt. bin size (^{o})	0.8	0.8	0.9
Events in bin (%)	42.88	42.96	41.08
σ (Nfit> 40)	11.65	11.63	9.35
$\Delta_{eo}(^o)$	1.36	1.40	1.28
Opt. bin size (^{o})	0.7	0.7	0.7
Events in bin (%)	33.50	34.26	20.31
σ (Nfit> 50)	11.60	11.65	9.24
$\Delta_{eo}(^o)$	1.17	1.21	1.10
Opt. bin size (^{o})	0.7	0.7	0.6
Events in bin (%)	28.20	29.83	14.73

Table 2: Comparison of the three angle fitters. Unweighted χ^2 is the goodness-of-the-fit measure of equation 4 without weights (w_j) in order to make it method-independent. For Nfit> 0, corrected Δ_{eo} is Δ_{eo} that has been corrected for showers which have failed Δ_{eo} (see section A.3). The significance quoted is for a simulated 10σ source thrown in a 2^o bin. The angular resolution is taken to be $\Delta_{eo}/2$ (showers that failed Δ_{eo} are not accounted for). Note: when comparing Δ_{eo} (or sigma) for Nfit $\neq 0$, one cannot make a direct comparison between V41 and JBG or JMC because JBG and JMC fit $\sim$55% more PMTs.

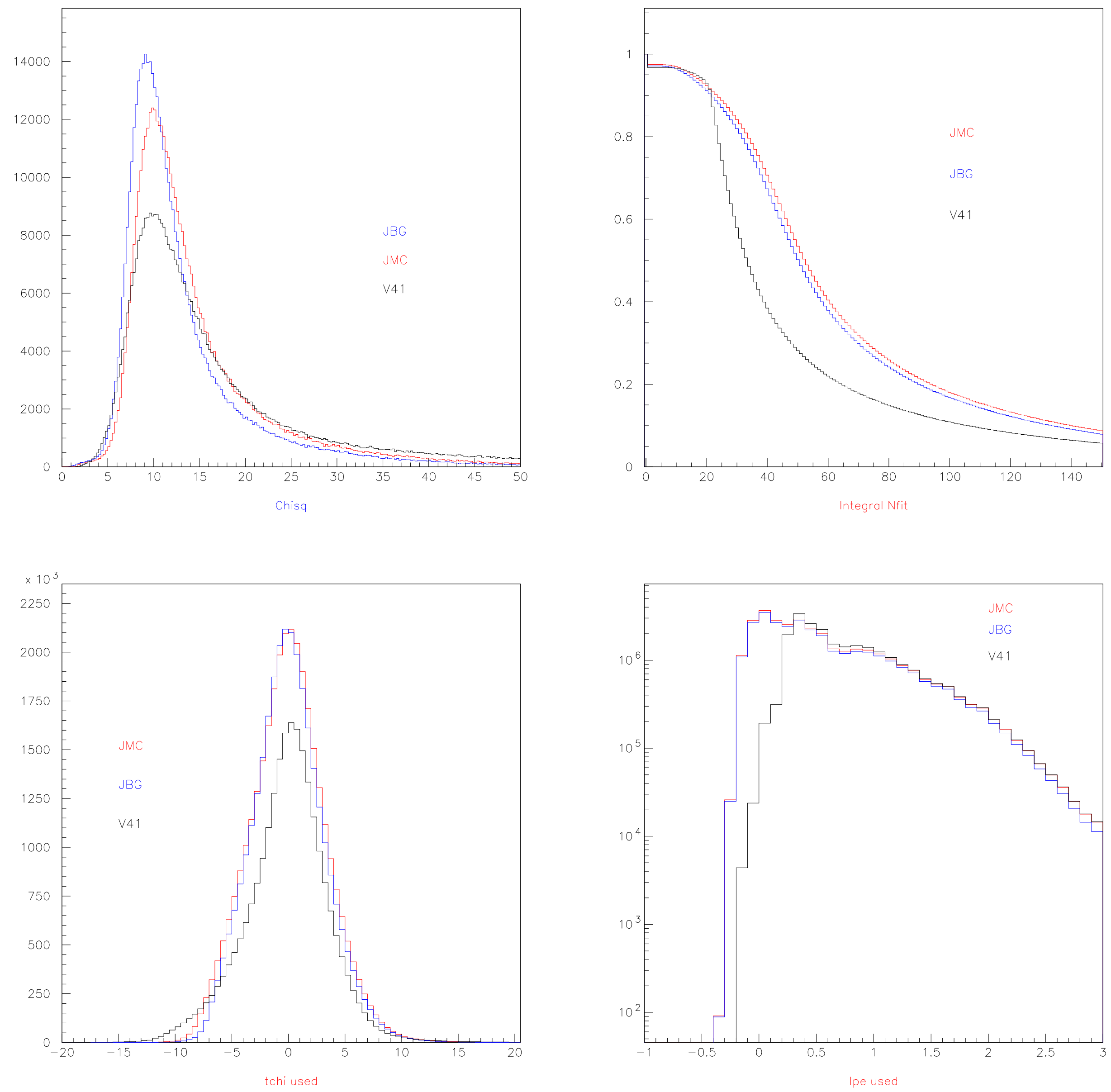

Figure 2: Top: unweighted (method-independent) χ^2 (left) and integral Nfit (right). Bottom: time (left) and pulse height (right) distributions for used hits.

- **Angle Fitter Modifications** (common to JMC and JBG):

  $a = 2.75$ (time window for early hits),

  Relax PE cut if $Nfit < 450$ (i.e. every time).

Finally, we tested these sets of values on an independent raw file (GRB_2058_0018.dat) and compared our results with each other's and with what is presently online (see Table 2 and Figures 2, 3, 4 and 5). In particular, JBG achieves better quality fits (lower χ^2) than

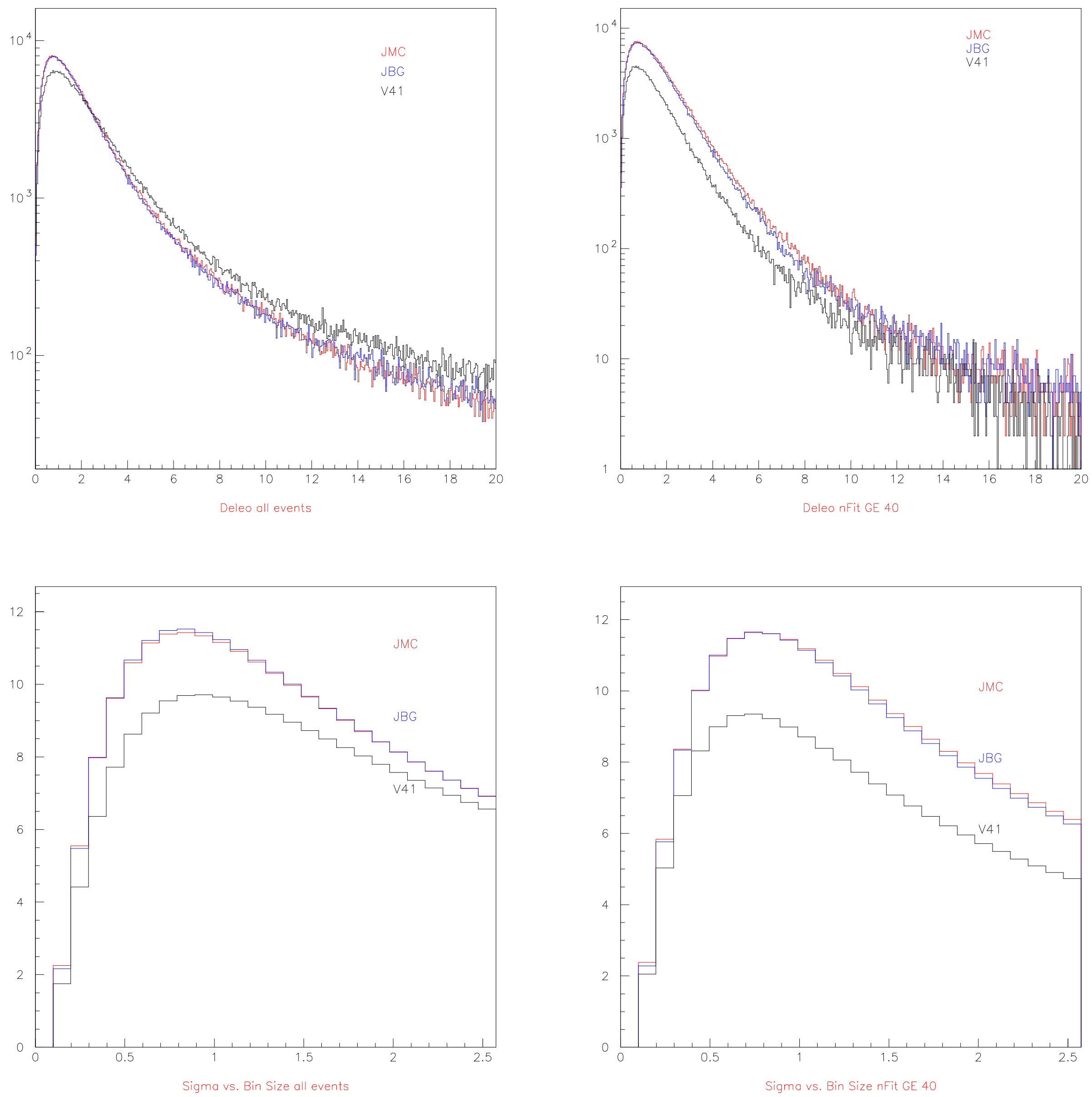

Figure 3: Uncorrected deleo distribution (top) and significance vs bin size (bottom) for $Nfit > 0$ (left) and for $Nfit > 40$ (right).

JMC, which in turn results in a better Δ_{eo}. (However, events that failed deleo have not been accounted for). On the other hand, JMC fits and deleo's more events. The maximum significance and optimum bin size are identical for both (JBG at $Nfit > 40$ and JMC at $Nfit > 50$). In general, both new fitters are very similar and constitute a grand improvement over v41: roughly speaking, 0.5% more events fit (also more events deleo'ed and more kept in the optimum bin), 54% improvement in nFit, 18% higher significance, 16% lower Δ_{eo} and 11% better χ^2. Either of them should be implemented immediately.

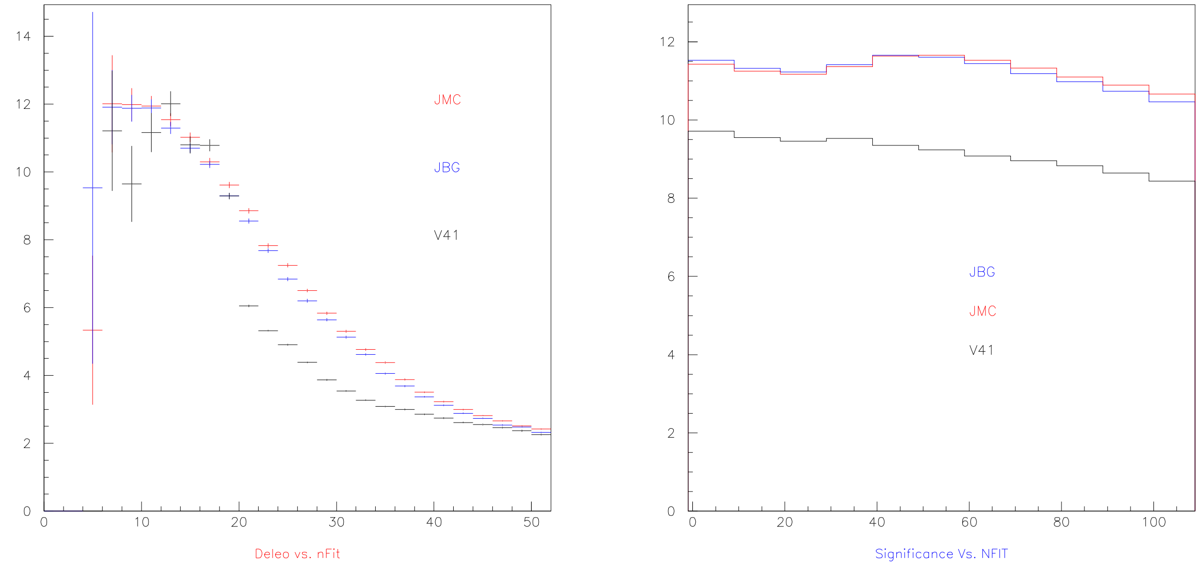

Figure 4: Uncorrected deleo (left) and significance (right) as a function of Nfit cut. Note: One cannot directly compare deleo (or significance) as a function of Nfit because JMC and JBG fit ∼55% more PMTs than V41.

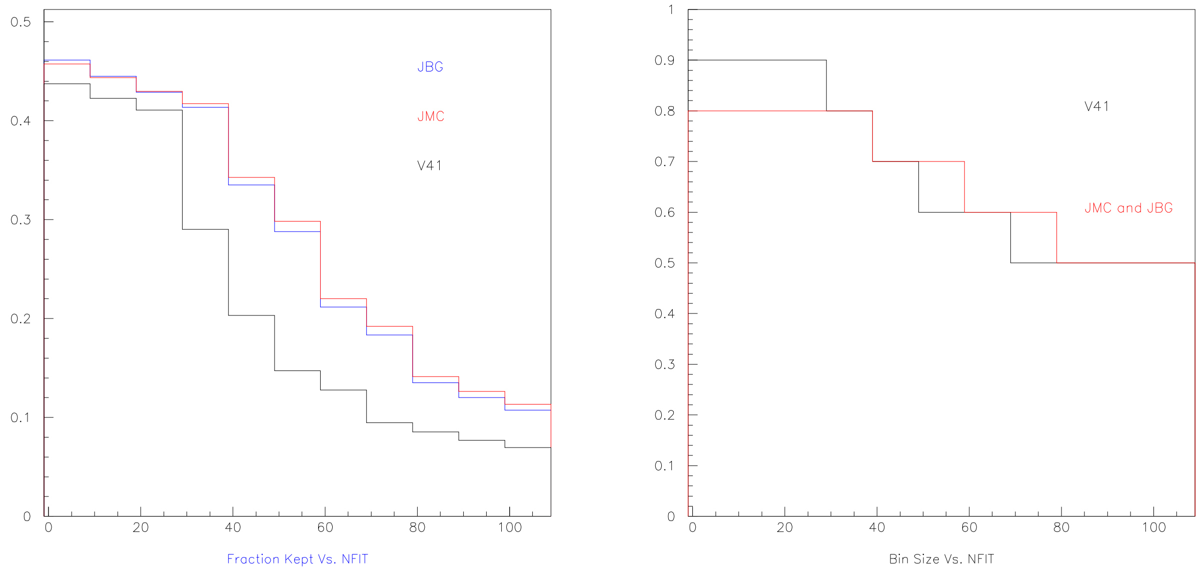

Figure 5: Fraction of signal events kept (left) and optimum bin size (right) as a function of Nfit cut.

A Joe's Method

When deriving new corrections and cuts used in the angle fitter, it is important to start off with as unbiased a weighting scheme as possible or else the final results may not be optimal. I decided to start with the weights (and SIG and PE cuts) that I derived from γ Monte Carlo showers (these are currently used in online version 41). I will therefore give a brief description of how these weights and cuts (and sampling) were derived from the MC.

I first investigated the best quantity to use in each case. For the sampling correction, I looked at the median, the mean, and the peak (of fit gaussian) of the T_χ distributions. For the weights, I looked at the FWHM, the RMS, and the sigma (of fit gaussian) of the T_χ distributions. I performed three iterations fitting T_χ distributions for 20 different PE bins using each of the three quantities for the weights and the sampling correction. Based upon the median of delangle on the third pass, sigma was the best quantity to use for the weights and the fitted peak was the best quantity to use for sampling.

Next I derived an unbiased weight to use as a first pass. This was done by weighting all the PMTs equally and calculating T_χ's (for 20 PE bins) from the true (MC) plane rather than the fitted plane. I then calculated the FWHM of the T_χ distributions and used these as an unbiased weight for the first pass (sigmas could not be used because the T_χ distributions were non-gaussian). I then performed four iterations fitting the T_χ distribution to gaussians and using the peaks for sampling and the sigmas for weights. I tried both 1D (only dependent on PEs) and 2D (dependent on PEs and distance from core) weights and sampling. After four iterations, based on the median of delangle, the 1D weights and sampling gave much better results. This was most likely because we do not find the core very well. After the sampling correction and weights were derived, I then modified the SIG and PE cuts. This was done by plugging in different values and seeing which values gave the best median delangle. Finally, the curvature correction was derived using this same procedure.

I used these weights and SIG and PE cuts derived from the MC as the starting point for deriving new corrections and cuts for the data. I imposed an Nfit cut of 30 to try and use the best showers for the fits. I broke the T_χ distributions up into 35 different PE bins and fit each T_χ distribution to a gaussian. For the sampling I used the peaks of the T_χ distribution and for the weights I used the sigmas.

A.1 Sampling Correction and Weights

When I first looked at the T_χ distributions using the sampling correction and weights derived from the Monte Carlo, they seemed to be fairly well centered around zero. I therefore decided to do two sets of corrections simultaneously. For the first set of corrections, which I call Start_old, I started with the sampling correction derived from MC and then made corrections to it. For the second set of corrections, which I call Start_new, I started from scratch with no sampling correction.

Deriving the sampling corrections and weights was an iterative process. I did a first pass using the weights derived from MC and fit the peaks of the T_χ's to derive a sampling correction (or a correction to the sampling correction for Start_old). I then did a second pass using the new sampling correction to derive new weights from the sigmas of the T_χ distributions. This process of fitting peaks (for sampling) and then sigmas (for weights) continued until I no longer saw any improvement in the fits. I judged how well each new pass was doing by using $\sqrt{showersfit}/(median\ \Delta_{eo})$ as my figure of merit. The results of each pass for the two different starting points are shown in Tables 3 and 4.

Pass	New quantity	Number of showers fit	Median of Deleo	$\sqrt{showersfit}/(median\ \Delta_{eo})$
0	—	125,523	1.062049	333.59
1	sampling	125,354	1.050996	336.87
2	weights	138,245	1.057543	351.58
3	sampling	138,022	1.063057	349.48
4	weights	138,143	1.061261	350.22

Table 3: Results of the different passes for Start_old - starting with the sampling correction derived from MC.

Pass	New quantity	Number of showers fit	Median of Deleo	$\sqrt{showersfit}/(median\ \Delta_{eo})$
0	—	99,267	1.437146	219.23
1	sampling	118,423	1.105792	311.21
2	weights	131,897	1.100422	330.03
3	sampling	134,416	1.057642	346.65
4	weights	138,504	1.068200	348.40
5	sampling	139,623	1.061247	352.10
6	weights	140,021	1.057458	353.86

Table 4: Results of the different passes for Start_new - starting with no sampling correction.

When starting with the sampling correction derived from MC (Start_old), the best results were obtained using the sampling correction from pass 1 and the weights from pass 2. When starting with no sampling correction (Start_old), the best results were obtained using the sampling correction from pass 5 and the weights from pass 6. The next step was to determine the curvature correction for both starting points.

A.2 Curvature Correction

My initial hope was to calculate the curvature correction from the data. My plan was to break up the T_χ distributions into different bins depending on the distance from the core (0-5m, 5-10m, ...). I would then determine the curvature correction by adjusting the peaks of the T_χ distributions so they were centered at zero (similar to how the sampling correction was done). To see if this method would work, I first looked at MC proton showers (version 2.2). Since the center-of-mass core fitter (which is currently used in online version 41) only fits cores on the pond, I tried to devise cuts that would ensure that the core was on the pond. I used an Nhot cut similar to that used by Bussy in his previous memo (JBG991221). I required that the 5, 7, and 9 hottest tubes hit were adjacent to each other. I tried requiring that the hottest tubes were within 10, 15, and 20 meters of each other. My hope was that these cuts would ensure a good sample of showers where the core was on the pond and well fit. As you can see in Table 5, this was not the case.

Nhot	Diameter	Number of showers fit	Median of Delta Radius
—	—	10,623	64.57
5	10m	587	73.63
5	15m	979	68.19
5	20m	1445	63.25
7	10m	344	77.75
7	15m	652	72.75
7	20m	955	68.13
9	10m	176	77.00
9	15m	481	76.38
9	20m	722	70.25

Table 5: Results of Delta Radius (the difference between the fit core position and the actual core position) using different Nhot cuts. The first row is the results with no Nhot cut.

Its seems as if I have derived an anti-core fitter. With these Nhot cuts, we actually seem to fit the core worse than with no cuts. I believe the reason the core fit actually got worse (although I have not investigated this) is that these Nhot cuts are finding muons that hit the pond and not the core. Also, low energy proton showers do not have well defined cores.

I therefore decided to use MC gamma showers to derive the curvature correction. I plugged in different (linear) curvature corrections and used the median of delangle to determine which curvature correction was best. In both cases, the best curvature correction is given by 4ns/100m.

A.3 SIG cuts, PE cuts, and Modifications to the Angle Fitter

The next step was to determine the best SIG and PE cuts to use. I used the same raw data file as before but I did not impose an Nfit cut. I again used $\sqrt{showersfit}/(median\ \Delta_{eo})$ as my figure of merit. However, when doing PE and SIG cuts, one must be careful with Δ_{eo} because many showers fail Δ_{eo} if the cuts are made too restrictive. In this case, Δ_{eo} may improve although you may not have improved the angular resolution. This is because most of the showers that failed Δ_{eo} probably had small Nfit values and large Δ_{eo}. To try and account for this, any shower that failed Δ_{eo} was assigned a Δ_{eo} by throwing a random number between 0 and 20. Although this may not be the best approach, I assumed that any shower that failed Δ_{eo} most likely had a Δ_{eo} somewhere near the tail of the distribution. I then used $\sqrt{showersfit}/(median\ corrected\ \Delta_{eo})$ as my figure of merit.

To determine the best PE and SIG cuts, I initially tried to look at each cut on a pass by pass basis. I started with doing two passes of the angle fitter and determining the best PE cuts and the best SIG cut (there is no SIG cut on the first pass) for those two passes. I then added a third pass and determined the best PE cut and SIG cut for that pass. However, after thinking about this I decided that this may not be the best approach. The best PE and SIG cuts determined after 2 or 3 passes of the angle fitter are probably not going to be the best cuts after all five passes are incorporated. The first few cuts will probably be too restrictive if you are only doing 2 or 3 passes of the angle fitter. What you want to do is gradually make the cuts more restrictive so that on the final pass through the angle fitter you have kept the best tubes.

I therefore modified my approach to incorporate my "scientific" method of plugging in different values and seeing which gave the best figure of merit. The drawback of this approach is that it could go on indefinitely. With 5 PE cuts and 4 SIG cuts, the phase space to be investigated is huge. After 2 days of intense scientific analysis, I reached the point of diminishing returns.

Having derived the best PE and SIG cuts, I then looked at one final aspect of the angle fitter. I investigated the SIG cut condition for early tubes ($T_\chi > 0$) described in the introduction (i.e. the value of a). I again plugged in different values for a and used the above figure of merit to determine which was best. For both starting points, the best results were obtained when this condition was relaxed even further (from $a = 2.0$) to $a = 3.0$. At this point, the results obtained with Start_new were better than those from Start_old so I only used Start_new for the final step.

A.4 One final step

The last step was to do one final pass using all the new corrections, cuts, and modifications to derive new weights. This last pass is necessary because anytime you make changes to the angle fitter (in this case new PE and SIG cuts and the above modification to the angle fitter), you expect the T_χ distributions to change (hopefully becoming narrower). You would expect to improve the fit even more by modifying the weights to reflect the new T_χ distributions.

I therefore did one final pass using the results derived from Start_new (starting with no sampling correction). I fit these final T_χ distributions to gaussians and used the sigma to derive a final set of weights. This final set of weights did not improve the figure of merit, instead it actually made things slightly worse. I therefore decided to try using the RMS of the T_χ distributions instead of the sigma of the fit gaussian (this is what Bussy was using for his weights). Because the RMS and sigma are off by a factor of 2.35 (for a gaussian), I also had to redo all the SIG cuts. I ended up redoing everything described in the previous section for this new set of weights derived from the RMS. After deriving new SIG cuts, new PE cuts, and a new condition for early tubes ($a = 2.75$), I did see an improvement in the results from this final pass.

B Bussy's Method

Some details of my approach have been explained elsewhere (JBG991221) so they will be omitted here except maybe to say that I divide the relevant phase space into 40 logarithmic PE bins and 20 linear R_c bins, where R_c is the distance to the core. Then I build the resulting eight-hundred T_χ distributions and use:

- the root-mean-square (RMS) to calculate the weighting function (only dependent on PE, averaged over R_c),
- the peaks to draw the sampling function (again, only PE-dependent) and
- the medians to estimate the curvature (only R_c-dependent).

My whole optimization process consisted of five phases. Most of this process can be followed graphically through Figure 6.

PHASE 1: Find time offsets (if necessary), weights and sampling function.

The distribution of time offsets (TPed) was found to be good even for non-optimal angle fit parameters ($\sum_{i=1}^{418} |TPed_i| = 441$ns, i.e. ~ 1ns offset per PMT on average; centering of these offsets can be achieved with the sampling correction) and it did not improve with the iterative application of extra TPed corrections so these were abandoned. This is an indication of the better quality of the new laser timing calibrations.

The (brief) details of this phase are as follows:

- step 1a) set initial (unbiased) weights to unity all across the PE range, no sampling, no curvature correction, default PE and SIG cuts: work out first set of weights (W1).

- step 1b) same as above but with $Nfit > 50$ and $\theta < 30$: work out first set of sampling corrections (S1).

- step 2a) weights and sampling as per W1 and S1, no Nfit cut, no θ cut, everything else the same: work out second set of weights (W2).

- step 2b) W1, S1, now with $Nfit > 50$ and $\theta < 30$: work out second set of sampling corrections (S2).

- step 3a) W2, S2, no Nfit cut, no θ cut: get W3.

- step 3b) W2, S2, with $Nfit > 50$ and $\theta < 30$: get S3.

After some such iterations, both the weights (fifth iteration, W5) and the sampling corrections (third iteration, S3) converged so I stopped at the following expressions for the RMS and sampling functions:

$$RMS = -0.87 \ \log^3(PE) + 5.16 \ \log^2(PE) - 11.60 \ \log(PE) + 14.08 \ , \tag{11}$$

$$T_s = -0.25 \ \log^4(PE) + 1.49 \ \log^3(PE) - 5.21 \ \log^2(PE) + 12.33 \ \log(PE) - 9.13 \ . \tag{12}$$

A fake 10σ signal consisting of 316 events over a 1000-event background was thrown on a 2^o round bin with a view to optimizing the fitter not only in terms of angular resolution, percentage of events fit and number of hits in the fit but also in terms of signal significance, bin size and fraction of signal kept. See Figure 6 (top) and Table 6 for results of this phase:

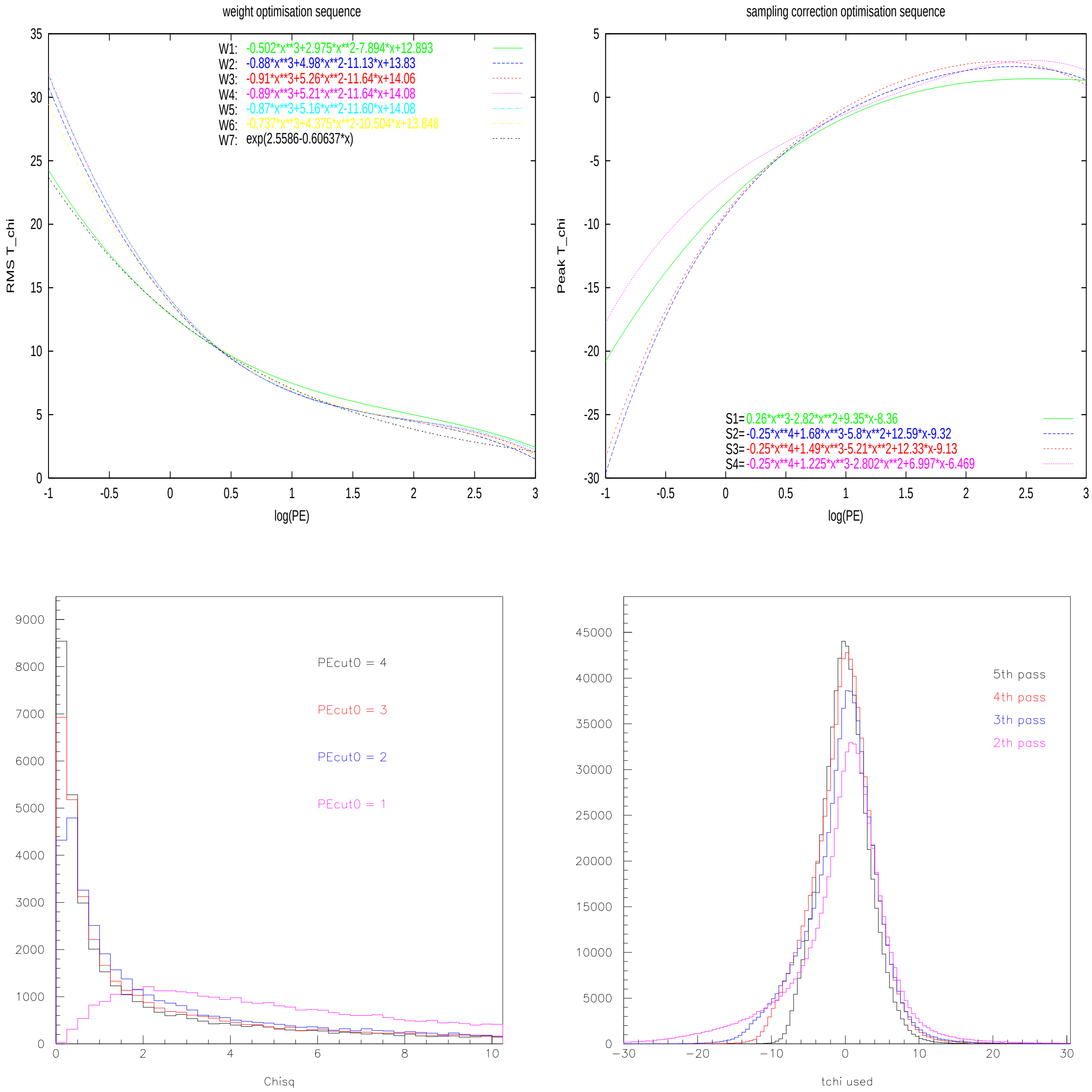

Figure 6: Top: Sequence in the optimization of angle fit weights (left) and sampling corrections (right).
Bottom: Effect of various PE cuts in the 1st pass (left), where PE cut[0] = 2 p.e. was taken as optimum (little improvement at higher values is at the expense of losing events. The right panel shows the evolution of the T_χ distribution of hits used in the angle fit in passes 2 through 5: we manage to include a larger and larger population of hits while at the same time making the time distribution tighter.

reduction of 13 points in the percentage of non-fit and/or non-deleo'ed events, 0.4ns reduction of the time offset per pmt, 7.1 σ significance, a median Δ_{eo} of 2.34^o, keeping 32.7% of the signal events in a 1.1^o bin with an Nfit cut of 30.

Iteration:	1	2	3	4	5
noΔ_{eo} (%)	19.4	6.1	6.1	6.1	6.1
$\sum \lvert TPed \rvert$ (ns)	406	362	243	234	236
max σ	7.0	7.0	7.1	7.1	7.1
bin size (^{o})	0.8	1.1	1.1	1.1	1.1
events in bin (%)	17.6	32.1	32.7	32.7	32.7
$\Delta_{eo}(^o)$	2.62 av	3.36 av	2.34 mdn	2.34 mdn	2.34 mdn

Table 6: Phase 1, iterations 1 to 5: percentage of non-deleo'ed events (includes no-fits), sum of top-layer time offsets (over 418 pmts), maximum significance (it occurred at $Nfit > 30$ in all five cases), bin with max. significance, events kept in such bin and Δ_{eo} for $Nfit > 30$ (av=mean; mdn=median).

PHASE 2: First optimization of PE cut[0] and, subsequently, of the other PE and SIG cuts on a pass-by-pass basis.

The following values of PE cut[0] were tried with an angle fitter consisting of just the first pass: from 0.5 up to 5.0 photoelectrons in steps of 0.5 and the results were compared. When in doubt,

$$q = \frac{\sigma}{\Delta_{eo}} \; N(\%) \tag{13}$$

was used as an aid in selecting the optimum case, where σ is the maximum significance, Δ_{eo} is the median Δ_{eo} in the bin with maximum significance and N is the percentage of events fitted. In trying not to lose very many fitted events, PE cut[0]=2 was judged to be the best cut (see Figure 6 (bottom left), where cuts of 1, 2, 3 and 4 photoelectrons are compared and Table 7). Notice that the growth of q slows down after 2 p.e..

PE cut[0]:	1.00	1.50	2.00	2.50	3.00
Events fit (%)	96.2	94.7	93.9	93.0	92.2
max σ	3.42	4.41	4.73	4.78	4.84
$\Delta_{eo}(^o)$	5.93	4.71	4.41	4.34	4.30
q	55.5	88.7	100.7	102.4	103.8
σ (Nfit$>$ 30)	3.33	4.17	4.27	4.09	3.93
$\Delta_{eo}(^o)$	5.90	4.44	3.84	3.52	3.31

Table 7: Phase 2, optimization of PE cut[0] (only one pass through angle fitter) maximum significance was achieved for Nfit > 0. At Nfit > 20 and higher cuts, PE cut[0]=2.00 is always the best.

With PE cut[0] fixed to 2 photoelectrons, I asked the code to try several combinations of SIG cut[0] and PE cut[1] in the second pass of a two-pass angle-fitting routine. Similarly, I obtained the best SIG and PE cuts for passes 3 and 4 (no improvement was made with a fifth pass) using a three- and a four-pass routine respectively. The optimum values appear in Table 8 together with the results they give rise to: at the end of this phase we gain another 2 points of improvement in the percentage of events fit and reach a maximum significance of 9.57σ, 35% higher with just four passes than in with the 5 passes of phase 1, and a median Δ_{eo} which is 40% lower (at $Nfit > 30$).

At this stage, the average time offset has already gone well below 1 ns per PMT.

optimum SIG cut[ ]: optimum PE cut[ ]:	[0]=2.00 [1]=1.25	[1]=0.75 [2]=0.75	[2]=0.50 [3]=0.50
Events fit (%)	94.6	96.4	96.6
max σ	7.13	9.26	9.57
$\Delta_{eo}(^o)$	2.88	2.14	2.05
σ (Nfit> 30)	6.82	9.06	9.19
$\Delta_{eo}(^o)$	2.08	1.54	1.41

Table 8: Phase 2: evolution of percentage of events fit, significance and Δ_{eo} as PE cut[1,2,3] and SIG cut[0,1,2] are optimized (angle fit here consisted of two, three and four passes, respectively): maximum significance was always achieved for Nfit > 0.

PHASE 3: Re-calculate weights (W6) and sampling corrections (S4), estimate curvature correction and optimize parameter a in SIG cut for early hits, using all along the recently found PE and SIG cuts.

We tried several values of a (it determines the width of the time window allowed for hits early with respect to the fitted time) ranging from 1 to 5 and adopted $a = 2.75$ as the optimum. Here is also where the new relax criterion for PE cuts was tested and implemented. These changes, put together, brought us to the yet again improved results shown in Table 9.

Events fit (%)	96.9
σ (Nfit> 0)	11.30
$\Delta_{eo}(^o)$	1.74
max σ	11.37
$\Delta_{eo}(^o)$	1.41

Table 9: Phase 3: new weights, new sampling, new curvature, new a and new RELAX: maximum significance has moved to Nfit> 40.

PHASE 4: Re-optimize the PE and SIG cuts pass by pass: using the latest weights, sampling and curvature corrections, re-do phase 2 (this time a fifth pass bought us some improvement). The results arrived at in phase 4 are listed in Table 10, namely 97.1% events fit, 11.44σ and $\Delta_{eo} = 1.39^o$ in a 0.8^o bin for $Nfit > 40$. These will change to 97.2% events,

11.53σ and $\Delta_{eo} = 1.36^o$ in a 0.7^o bin for $Nfit > 40$ in the final phase.

optimum SIG cut[ ]:		[0]=2.00	[1]=1.00	[2]=0.75	[3]=0.50
optimum PE cut[ ]:	[0]=2.00	[1]=1.25	[2]=1.00	[3]=0.50	[4]=0.50
Events fit (%)	93.9	96.5	96.9	97.0	97.1
max σ	4.71	7.42	9.50	10.75	11.38*
bin size (^{o})	1.5	1.0	0.9	0.8	0.8
$\Delta_{eo}(^o)$	4.43	2.74	2.09	1.82	1.72*
σ (Nfit> 40)	3.98	7.27	9.47	10.71	11.44
$\Delta_{eo}(^o)$	3.39	2.10	1.71	1.58	1.39

Table 10: Phase 4, evolution after each step in the re-optimization of PE cuts and SIG cuts (angle fit here consisted of one, two, three, four and five passes, respectively): maximum significance stayed at Nfit> 0 except for the last pass; for this pass, maximum is at Nfit> 40 and values marked (*) are for Nfit> 0.

PHASE 5: Finally, recalculate weights (W7) and sampling function (this did not change). Results appear in Table 11.

Events fit (%)	97.2
σ (Nfit> 0)	11.45
$\Delta_{eo}(^o)$	1.71
max σ	11.53
bin size (^{o})	0.7
$\Delta_{eo}(^o)$	1.36

Table 11: Phase 5, new weights: maximum significance is at Nfit> 40 whereas the optimum bin size has moved to 0.7^o.