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Simulation of the Lunar and Solar Shadows

- Discussion of Integration Methods
- Status of Shadow Results

Tracing of Particles in B Field

General Statement of the problem:

Integrate the equations:

$$\frac{d\vec{r}}{dt} = \vec{v}(t)$$
$$\frac{d\vec{v}}{dt} = \vec{a}(t, \vec{r})$$

General Numerical Approach:

Let $(y_1, y_2, ...y_6) = (\vec{r}, \vec{v})$

Starting values y_i^0 are known.

Break the motion down into steps such that the size Δt of the nth step does some sort of optimization of time versus accuracy.

For each step n, $y_i^{n+1} = y_i^n + \Delta t \cdot f_i(t, y)$.

AIM Stepping Routine Description:

- At any point, pick Δt such that:

$$(\Delta v_i)/|v| < 2.0 \cdot 10^{-4} \quad i = 1, 2, 3 \quad \text{and}$$
$$\Delta B_i < 2.0 \cdot 10^{-7}\mathrm{T} \quad i = 1, 2, 3 \quad \text{and}$$
$$(\Delta v_i)/|v| > 1.0 \cdot 10^{-4} \quad \text{or} \quad \Delta B_i > 1.0 \cdot 10^{-7}\mathrm{T}$$
$$i = 1 \;\; or \;\; i = 2 \;\; or \;\; i = 3$$

- For each Δt

$$\vec{r}(t^n + \Delta t/2) = \vec{r}(t^n) + \vec{v}(t^n)\Delta t/2 + \vec{a}(t^n)(\Delta t)^2/2$$
$$\vec{v}(t_n + \Delta t/2) = \vec{v}(t^n) + \vec{a}(t^n)\Delta t/2$$
$$\vec{a}(t^n + \Delta t/2) = q\vec{v}(t^n + \Delta t/2) \times \vec{B}(\vec{r}(t^n + \Delta t/2))/\gamma m$$
$$\vec{v}(t_0 + \Delta t) = \vec{v}(t_0) + \vec{a}(t^n + \Delta t/2)\Delta t$$
$$\vec{r}(t_0 + \Delta t) = \vec{r}(t_0) + \vec{v}(t_0)\Delta t + \vec{a}(t^n + \Delta t/2)(\Delta t)^2/2$$

Tracing of Particles in B Field

4th Order Runge-Kutta:

$$
\begin{aligned}
k_i^1 &= \Delta t f_i(t^n, y^n) \\
k_i^2 &= \Delta t f_i(t^n + \Delta t/2, y^n + k^1/2) \\
k_i^3 &= \Delta t f_i(t^n + \Delta t/2, y^n + k^2/2) \\
k_i^4 &= \Delta t f_i(t^n + \Delta t, y^n + k^3)
\end{aligned}
$$

$$
y_i^{n+1} = y_i^n + k_1/6 + k2/3 + k_3/3 + k_4/6 + O(\Delta t^5)
$$

Result: Slightly worse than AIM method.

Tracing of Particles in B Field

Stoermer rule, as developed by Henrici:

For a system of equations in the form:

$$\begin{aligned} y'' &= f(t, y) \\ y(t^0) &= y^0 \\ y'(t^0) &= z^0 \end{aligned}$$

use:

$$\begin{aligned} h &\equiv \Delta t/m \\ \Delta^0 &\equiv h[z^0 + hf(t^0, y^0)/2] \\ y^1 &\equiv y^0 + \Delta^0 \end{aligned}$$

then for $k = 1, 2, ...m - 1$

$$\begin{aligned} \Delta^k &= \Delta^{k-1} + h^2 f(t^0 + kh, y^k) \\ y^{k+1} &= y^k + \Delta^k \end{aligned}$$

and finally,

$$z^m = \Delta^{m-1}/h + hf(t^0 + \Delta t, y^m)/2$$

Modified version of Stoermer rule

For us, there is one problem, in that $y'' = f(t, y, y')$ so use AIM modified version:

$$\begin{aligned} h &= \Delta t/m \\ \text{we are given } r^0,\ v^0 & \\ \Delta^0 &\equiv h[v^0 + ha(r^0, v^0)/2] \end{aligned}$$

$$\text{then for } k = 1, 2, ...m$$

$$\begin{aligned} r^k &= r^{k-1} + \Delta^{k-1} \\ v^k &= \Delta^{k-1}/h + ha(r^{k-1}, v^{k-1})/2 \\ \Delta^k &= \Delta^{k-1} + h^2 a(r^k, v^k) \end{aligned}$$

$$\text{and finally,}$$

$$v^m = \Delta^m/h - ha(r^m, v^m)/2$$

Result: 0.5 TeV particles come within $\pm \sim 3km$ of starting point in round trip to sun for earth dipole field. Particle loss from sun solved.

But statistics needed scale as ratio of sun/moon distance squared!!!

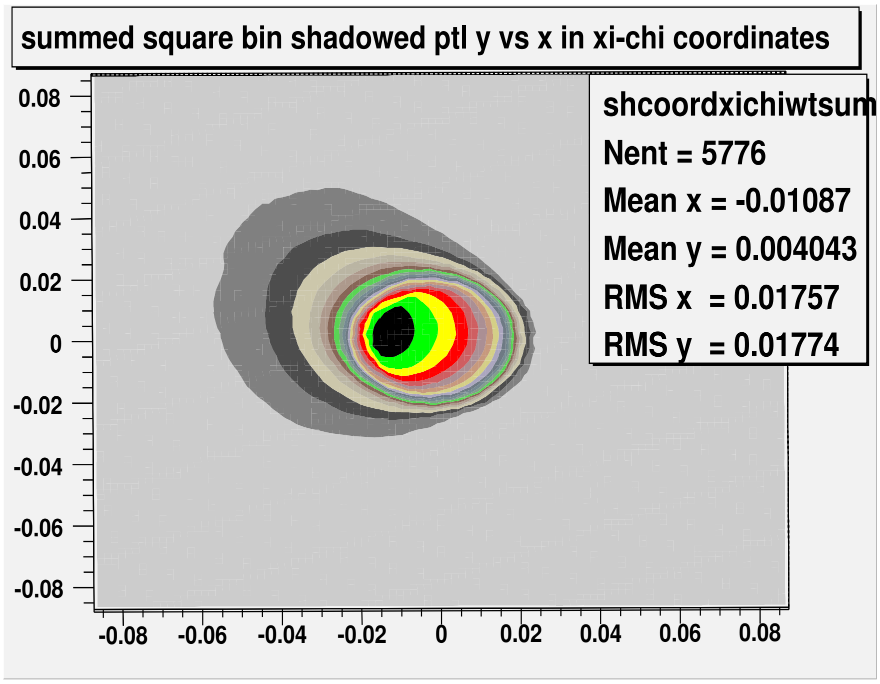
Simulated Moon shadow with original AIM method

Method:

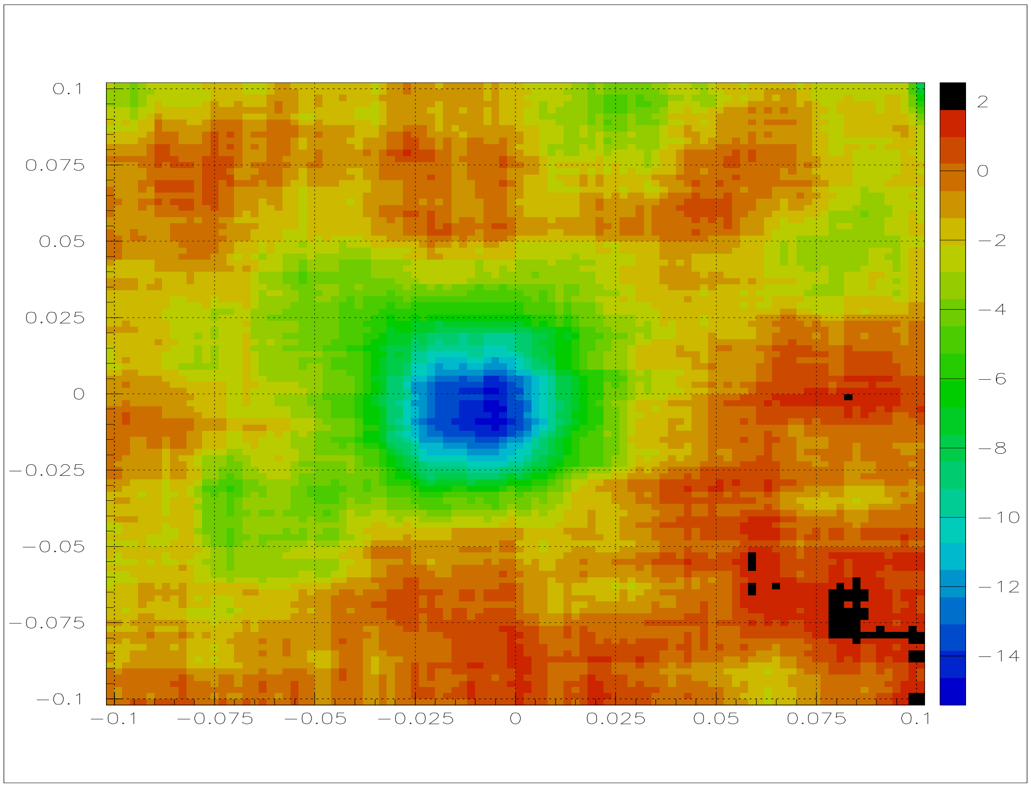
- Simulate one source location every 100 seconds for 1 year.
- For each bin, generate particle energy on $\mathrm{E}^{-2.7}$ spectrum form 100 GeV.
- Send particles from Milagro toward source in 0.1 degree steps around source up to ± 5 degrees.
- Record θ, ϕ, θ_v, and ϕ_v, when particle reaches source distance.
- Generate 500K events coming from source distance randomly in ± 5 degrees around source position and ± 5 degrees in direction toward earth.
- Propagate events which fall in bins.
- Store separately:
  - Particles which intersected source (shadowed particles).
  - Particles which didn't intersect source (non-shadowed particles).
- Save events which hit within ± 10 degrees of earth Milagro location (ie., big detector).
- Weight with effective Milagro area.
- Bin in 0.1° by 0.1° bins, then integrate over 12 bin radius.

Simulated Moon shadow in Lazar coordinates

Comparison with data:



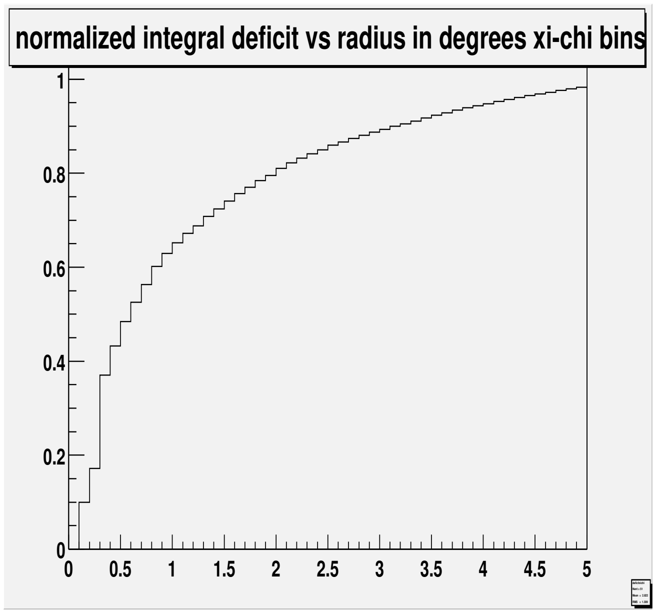
Simulation: 1 year.



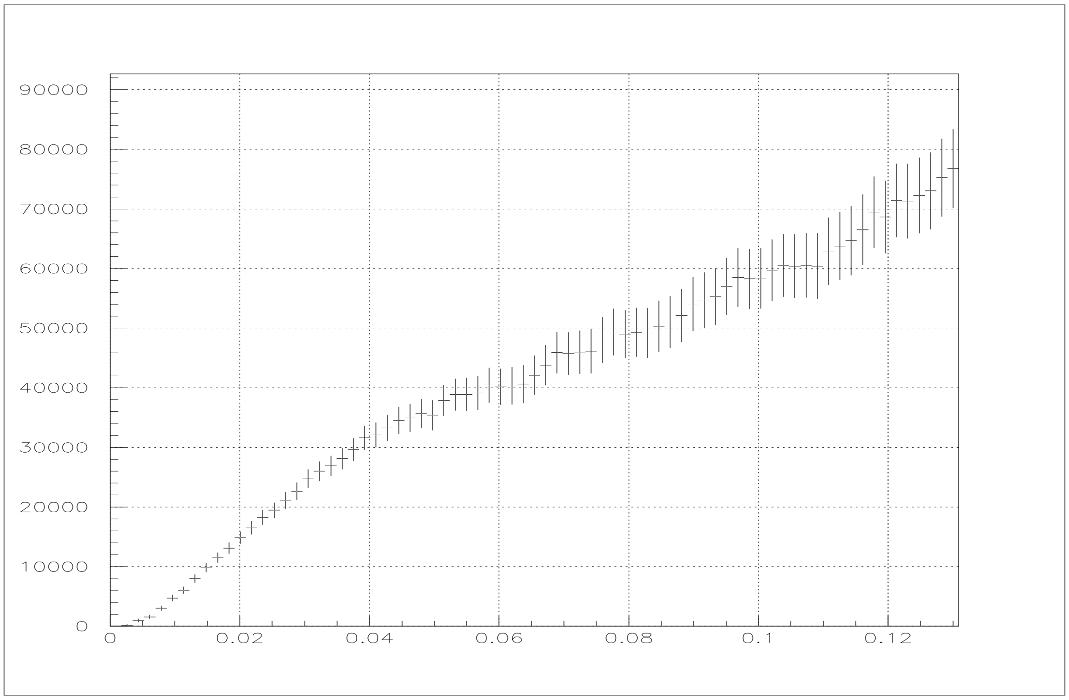
Data: 6 March, 2000 to 28 January, 2001 (Lazar Fleysher)

Simulated Moon shadow deficit

Comparison of measured and simulated deficit:



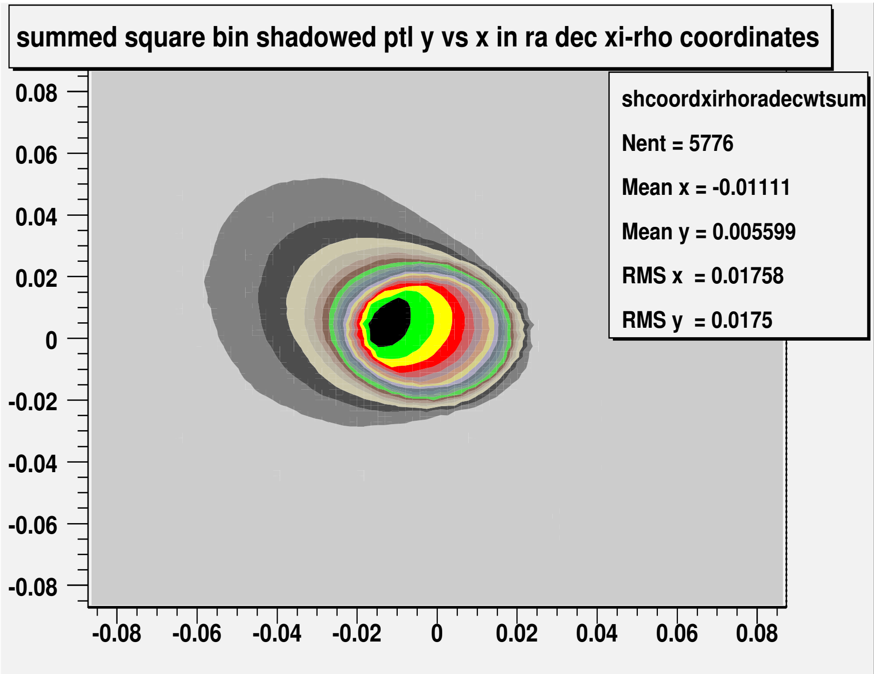
Simulation:
1 year



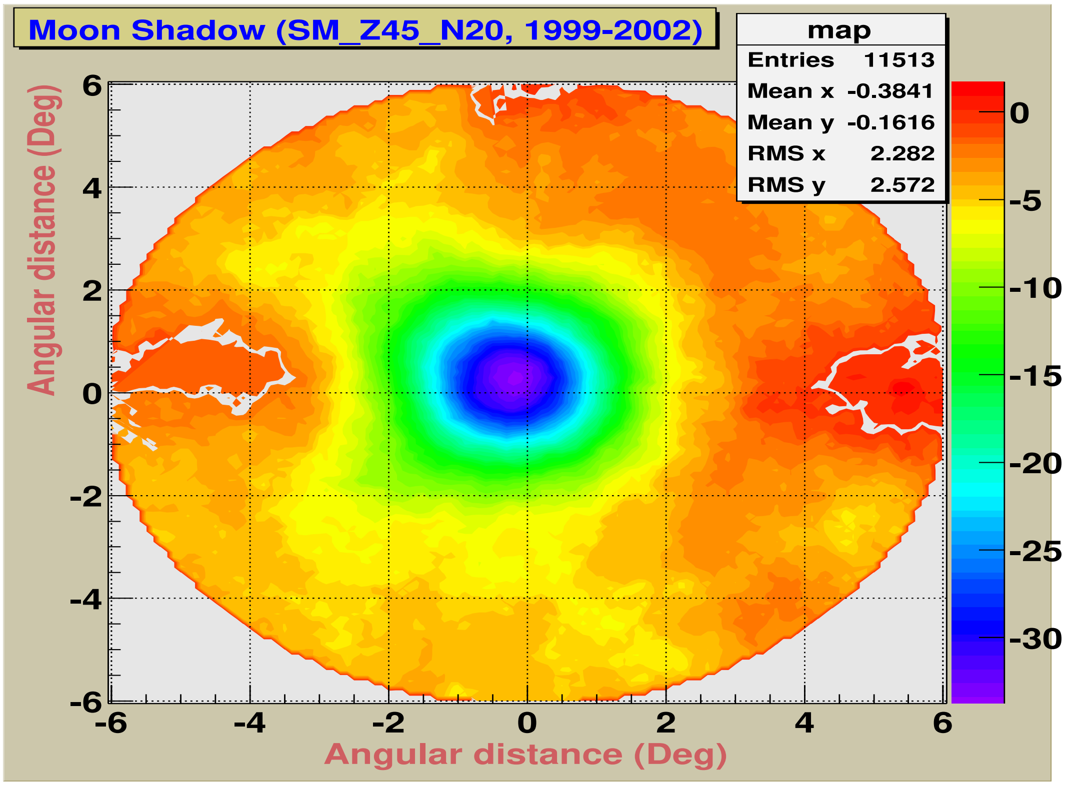
Data: 6 March, 2000 to 28 January, 2001 (Lazar Fleysher)

Simulated Moon shadow in Xu coordinates

Comparison with data:



Simulation: 1 year.



Data: 1999 – 2002 (Xianwu Xu, Andy corrected)

Work Remaining

- Move to matrix inversion for each time bin?
- Model Solar field.