GRB Sensitivity

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1 Introduction

I have been working on some calculations for the paper on untriggered bursts. The goal here is to calculate how many GRBs we expect to see with Milagro making some assuptions about burst parameters. Using our effective area, background rate, and a model for absorption of GeV-TeV photons by the IR background, I calculate the maximum observable redshift for different GRB durations and burst parameters.

2 Some Definitions

The differential energy spectrum (dN/dE) for a source tells us how much energy per unit of area, per unit of energy, is emitted by the source. For example, the spectrum of GRBs at BATSE energies is well modeled by a broken power law (the Band function):

$$\frac{dN}{dE} = \begin{cases} I_1(\frac{E}{E_0})^{-\alpha} & E_{min} < E < E_b \\ \\ I_2(\frac{E}{E_0})^{-\beta} & E_b \leq E < E_{max}, \end{cases} \tag{1}$$

where E_b is the break energy and E_0 is an aribitrary value where the spectrum is normalized and only effects the spectrum normaliztion, I. I has units of number per energy, per area, and the integral of dN/dE over energy gives the total number of photons per unit area, with energies between E_{min} and E_{max}.

The form of the high energy spectrum of GRBs is very uncertain. EGRET observations suggest that the power law measured by BATSE extends to at least a few GeV. Typical values of the high energy spectral index are ~ 2. However, GRB941017 exhibited a distinct high energy spectral component, with a power law index of ~ 1, up to at least 200 MeV. This must of course cutoff at some energy above 200 MeV, but how high in energy it extends is uncertain. Some models predict a significant sychrotron self compton (SSC) component from GRBs. If this is the case we might expect a high energy power law similar to the crab (spectral index = -2.49). For the purposes of calculating Milagro's sensitivity, a power law spectrum is assumed over some energy range, and calculations are done for a sample of power law indices and for different values of E_{min} and E_{max}. A band type function at higher energies may be more reasonable, but for this study, a simple power law is used.

The effective area is defined as:

$$A_{eff}(E,\theta) = A_{throw} \times \frac{N_{trig}(E,\theta)}{N_{throw}(E,\theta)}, \tag{2}$$

where A_{throw} is the throw area of the simulations ($\pi \times 10^{10} cm^2$), N_{trig} are the number of events that pass the trigger criteria (pass the VME trigger, pass an nfit cut, are fit within the search bin, and pass any other cuts), and N_{throw} are the number of events thrown at that energy and zenith angle. One oftens talks about an average effective area, which is just the total number of triggered events over the total number of events thrown, often given as a function of theta. For the purpose of computing the sensitivity as a function of redshift, we need the effective area vs energy. This is just the value given by Eqn. 2. For comparison I wanted to get the average energy from the area vs energy distributions. This is given by

$$A_{eff}(\theta) = \frac{\int_{E_{min}}^{E_{max}} A_{eff}(E,\theta)\frac{dN}{dE}dE}{\int_{E_{min}}^{E_{max}} \frac{dN}{dE}dE}, \tag{3}$$

which is just the spectrally weighted effective area. One point to note is that when the area is plotted as a function of energy, the dependence on the spectrum used to generate the Monte Carlo events falls out.

The effective area is used to calculate the total number of events seen in the detector. The effective area basically converts the number of particles arriving at the top of the atmosphere into the number seen in the detector. The number of events is given by

$$N_\gamma = \int_{E_{min}}^{E_{max}} A_{eff}(E,\theta)\frac{dN}{dE}dE = \int_{E_{min}}^{E_{max}} A_{eff}(E,\theta) I_o (\frac{E}{E_0})^{-\beta} dE. \tag{4}$$

Another important quantity is the energy fluence of an astrophysical source. Energy fluence is defined as:

$$S_{[E_{min},E_{max}]} = \int_{E_{min}}^{E_{max}} E\frac{dN}{dE}dE = \int_{E_{min}}^{E_{max}} E I_0 (\frac{E}{E_0})^{-\beta} dE. \tag{5}$$

Energy fluence has units of energy per unit area. This fluence is related to the total energy released by source as:

$$S_{[E_{min},E_{max}]} = \frac{1+z}{4\pi D_l^2} E_{iso_{[E_{min},E_{max}]}}, \tag{6}$$

where z is the redshift of the burst, E_{iso} is the total isotropic energy (integrated over 4π) released in photons between E_{min} and E_{max}, and D_l is the luminosity distance. The luminosity distance is defined as:

$$D_l = \frac{c}{H_0} \int_0^z \frac{dz'}{\sqrt{\Omega_M (1+z')^3 + \Omega_\Lambda}}, \tag{7}$$

and:

$$\frac{c}{H_0} = 9.2516 \times 10^{27} h^{-1} cm, \tag{8}$$

where $h = H_0/100$. In reality, the equations above should be corrected for the redshift, with $E \rightarrow E/(1+z)$, but I have not included this correction yet.

In the case that there is attenution by $\gamma\gamma$ absorption in the source or from the interaction of high energy photons with the cosmic infrared background (CIB), the spectrum is modified to be:

$$\frac{dN}{dE} = I_0 (\frac{E}{E_0})^{-\alpha} e^{-\tau(E,z,GRB)}, \tag{9}$$

where τ is the optical depth which depends on energy and redshift (CIB) and the GRB parameters (absorption at the source). For the calculations here, I have neglected absorption at the source, and used the models of Stecker for the CIB.

3 Sensitivity to GRBs at Different Redshifts

The first step in determining our sensitivity to a source is calculating the effective area for gamma rays from the simulations. This is given by Eqn. 2. A plot of the effective area is shown in Fig. 1 for zenith angles between 0 and 15 degrees. Fig. 3 shows the effective area vs. redshift for an $E^{-2.0}$ spectrum attenutated by IR absorption.

The next step is to determine the required number of events (N_r) for a 5σ detection given our measured background (N_b). For this we need to find the background vs duration and zenith angle. Since this is the untriggered search, the large number of trials results in a much smaller poisson probability required to reach 5σ. The probability for 5σ will be different for different durations due to the different number of trials. For now, I use a 1e-20 probability for a detection for all durations. I will figure out the correct probability in a later analysis. One point however is that the number of events required for 1e-20 and 1e-3 probabilities is different by less than an order of magnitude, so it will not drastically change these results.

Next I want to calculate the number of photons I get at the earth for GRBs at different redshifts, with different isotropic energies, and with different spectra. If I assume some power law spectrum over an energy E_{min} to E_{max}

and fix the total energy of the burst to be E_{iso}, I can find the intrinsic spectal normalization (not including absorption on the CIB) I_0 using Eqn. 5 and Eqn. 6

$$I_0(z, E_{iso}) = \frac{(1+z)}{4\pi D_l^2} \frac{E_{iso}}{\int_{E_{min}}^{E_{max}} E E^{-\beta} dE} \tag{10}$$

Now I can determine the number of photons at the earth for different redshifts using this value of I_0 in the following:

$$N_\gamma(z, E_{iso}, \theta) = I_o(z, E_{iso}) \int_{E_{min}}^{E_{max}} A_{eff}(E, \theta) (\frac{E}{E_0})^{-\beta} e^{-\tau(E,z)} dE. \tag{11}$$

This is shown in Figs. 3, 4, and 5 for power law spectral indices of -1.5, -2.0, and -2.5 respectively, keeping $E_{min} = 50 GeV$ and $E_{max} = 10 TeV$. In the figure, the dashed lines correspond to the number of photons detected by Milagro for different values of isotropic energy. The horizontal lines correspond to the minimum number of photons required ($N_\gamma = N_r - N_b$) for a 1e-20 probabilty for different durations. Where the dashed lines intersect the horizontal lines tells us the maximum observable redshift for that duration and amount of energy released by the burst.

In Fig. 6 is plotted the maximum observable redshift as a function of duration for different burst energies and spectra. As can be seen from the plot, the maximum redshift is not sensitive to the spectral index until you get to larger GRB energies. This is expected because at high energies one can see to larger redshifts, where the effects of the IR make you sensitive to the number of photons at the low end of the spectrum. So one would expect a steeper spectrum to allow observations to larger redshifts, as is seen in the plot.

Another way to look at this is to take a fixed duration, and plot the maximum observable redshift as a function of total burst energy. This is shown in Fig. 7 for an $E^{-2.0}$ spectrum, and a few different durations. For example, we can observe 10^{54} erg GRBs out to $z = 0.22$ for a 1 second duration. We can now use this to set limits on the number of bursts with emission in the 50 GeV to 10 TeV energy range. Since we have not observed a GRB at these energies, we have constrained the total energy of GRBs out the redshift indicated by Fig. 7 for each duration. This may be used to exclude a volume in which high energy emission by GRBs occur, for a given duration, and as a function of the total burst energy. This is shown in Fig. 8 for different spectral indices for a 100 second duration burst.

In all of the above a power law spectrum between 50 GeV and 10 TeV was used. I checked how sensitive these results are to changing the limits over which I assume the power law is valid. In Figs. 9 - 11 I plot the maximum observable redshift vs energy for a 1 second duration for the three different power law indices used, with a variety of values for E_{min} and E_{max}. As can be seen, the results are not too sensitive to the limits. This is due to the fact that I am fixing the total burst energy. So for example, if I change E_{min} from 50 GeV to 100 GeV, I have a smaller contribution from the low energy end of the spectrum, but this is compensated by the fact that the total energy is now concentrated in a smaller energy band.

4 GRB Redshift Models

Knowing the maximum observable redshift, we can then use a model for the number of GRBs per year as a function of redshift and burst duration. The simplest case is to take the BATSE duration distribution and the measured redshift distribution (even though the measured redshifts are only for the long, $> 2s$, duration bursts). Fig. 12 shows the BATSE T90 distribution, Fig. 13 shows the measured redshift distribution (33 bursts with solid redshift measurements; see http://www.mpe.mpg.de/ jcg/grbgen.html), and Fig. 14 shows the measured cumulative redshift distribution. From the duration distribution I get the fraction that occur at each duration. Then I assume that bursts have a constant density (at least nearby):

$$N_{GRB} = \frac{4\pi}{3} D_l^3 n_0 \tag{12}$$

I fix n_0 from the measured redshift distribution. From the cumulative redshift distribution I find that 0.174 GRBs occur within a z of 0.5. If I then take the BATSE rate of $\sim$ 1 GRB per day, and correct for their FOV, I get an overall rate of GRBs in the universe of $\sim$ 700 GRB per year, or 123 per year within $z = 0.5$, which translates to a density of 0.919 $Gpc^{-3} yr^{-1}$. Now I can plot the number of GRB per year vs energy. This is shown in Fig. 15. This assumes that GRBs all release the same amount of energy in GeV-TeV photons. The next step is to assume a more reasonable redshift distribution and a luminosity function. There are a number of models for both. I will also convert the results into upper limits, study any errors in the calculations, and consider different cuts on the data.

5 Checking the Results

I wanted to do a few checks of my analysis to make sure that nothing was really way off. For this purpose I took out the absorption due to the CIB and put in a hard cutoff. I then put in the spectrum for the crab ($2.56e{-}7(E/1TeV)^{-2.59} s^{-1} m^{-2} TeV^{-1}$), and calculated the number of events. The number of events from a crab spectrum versus cut off is plotted in Fig. 16. This plot checks two things: that I can predict approximately the correct number of events for the crab, and that the calculation without including the CIB makes sense. With no cutoff I predict 16.3, 8.0, and 1.3 events per hour for zenith angle ranges of 0-15, 15-30, 30-45 degrees respectively. If I take the 15-30 degree range to calculate the number of events per day, I get 8 events/hr*6hr = 48 events per source day. Andy calculates about 15 events/day, but that is with an x2 cut, a 2.1 degree bin instead of the 3 degree bin which I used here, including dead time and dead tubes, and calculating the time spent at each zenith angle. So this result is consistent.

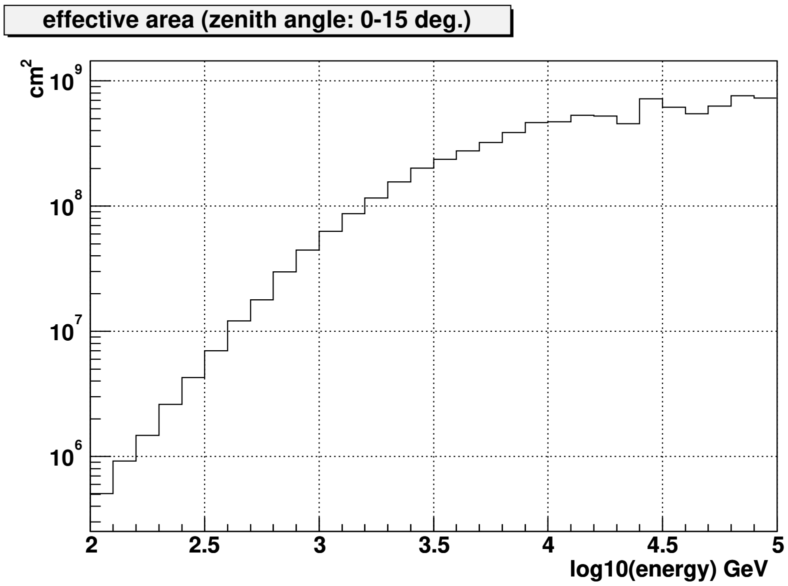

Figure 1: Effective Area for gamma-rays vs energy, zenith angle: 0-15°

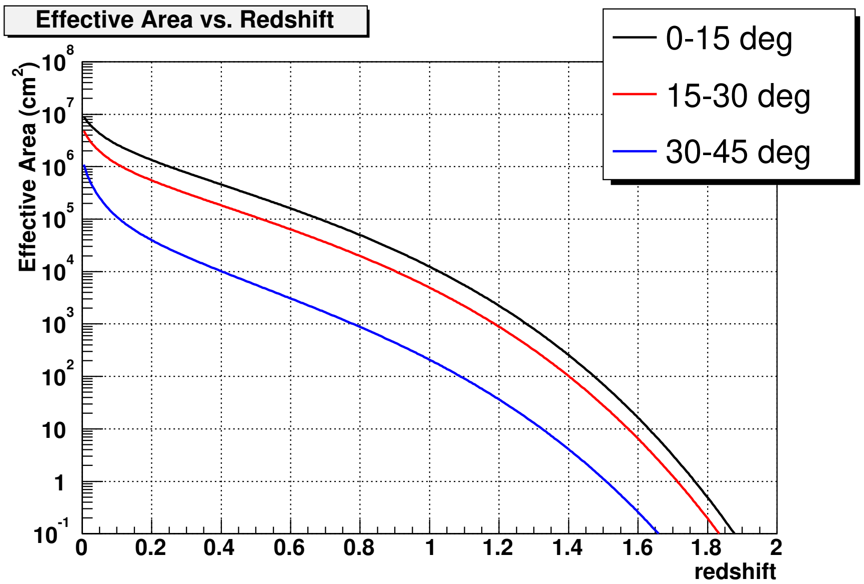

Figure 2: Spectrally weighted effective area vs redshift for an $E^{-2.0}$ spectrum. For three different zenith angle ranges.

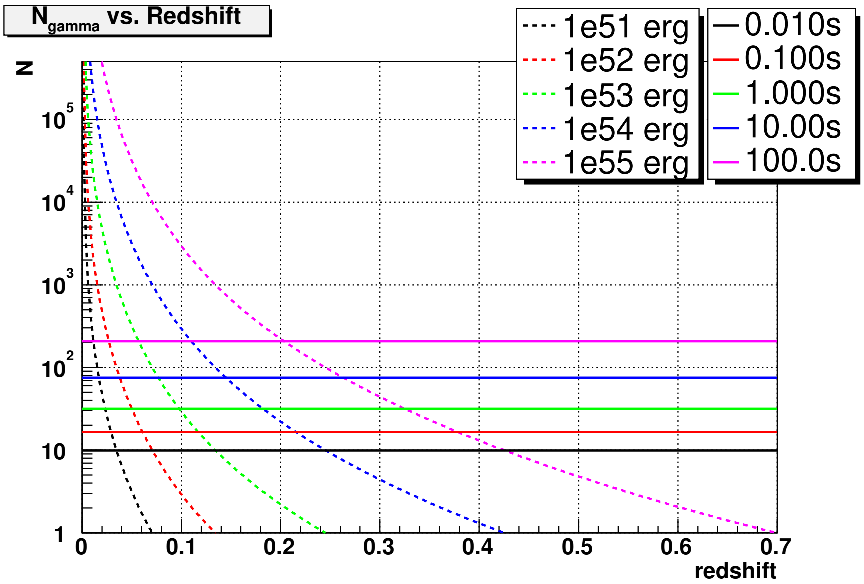

Figure 3: Number of photons detected for an $E^{-1.5}$ spectrum from 50 GeV to 10 TeV vs redshift for different amounts of isotropic energy. The horizontal lines are the minimum number of photons required for a 1e-20 probability for different durations.

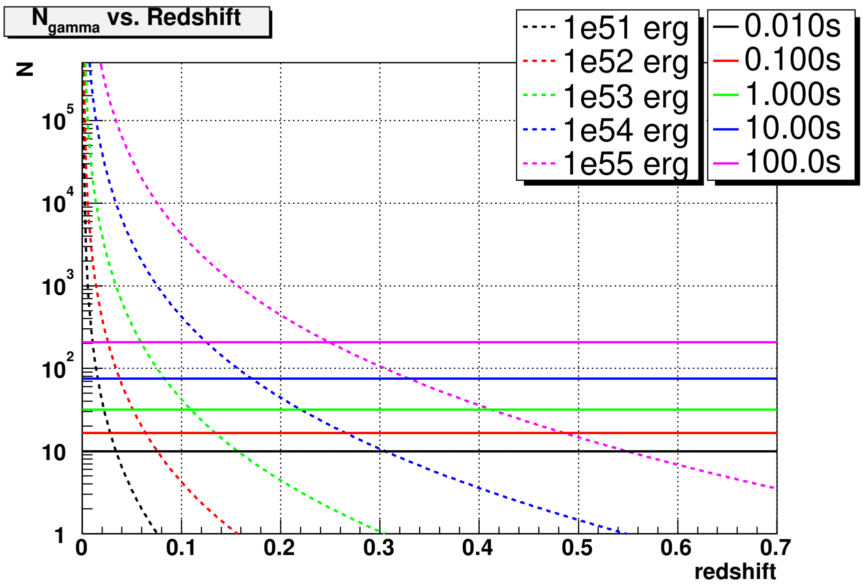

Figure 4: Number of photons detected for an $E^{-2.0}$ spectrum 50 GeV to 10 TeV vs redshift for different amounts of isotropic energy. The horizontal lines are the minimum number of photons required for a 1e-20 probability for different durations.

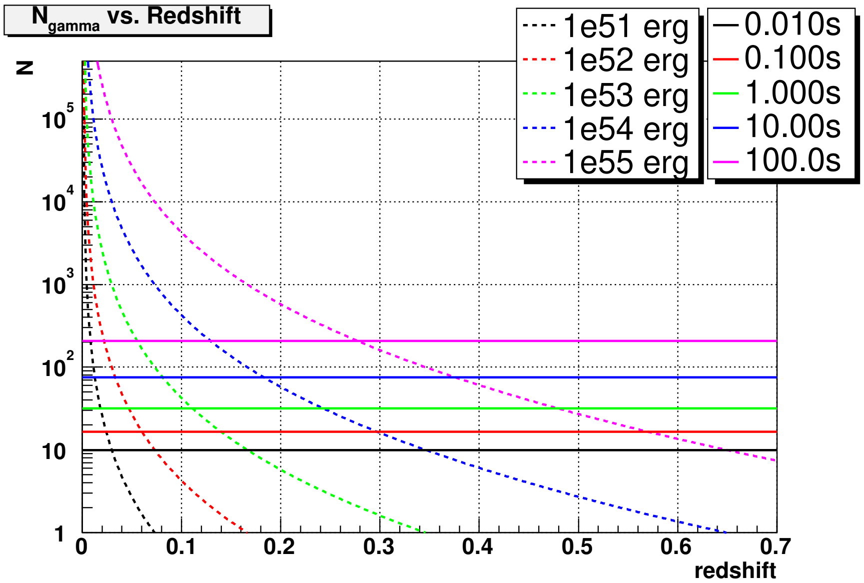

Figure 5: Number of photons detected for an $E^{-2.5}$ spectrum 50 GeV to 10 TeV vs redshift for different amounts of isotropic energy. The horizontal lines are the minimum number of photons required for a 1e-20 probability for different durations.

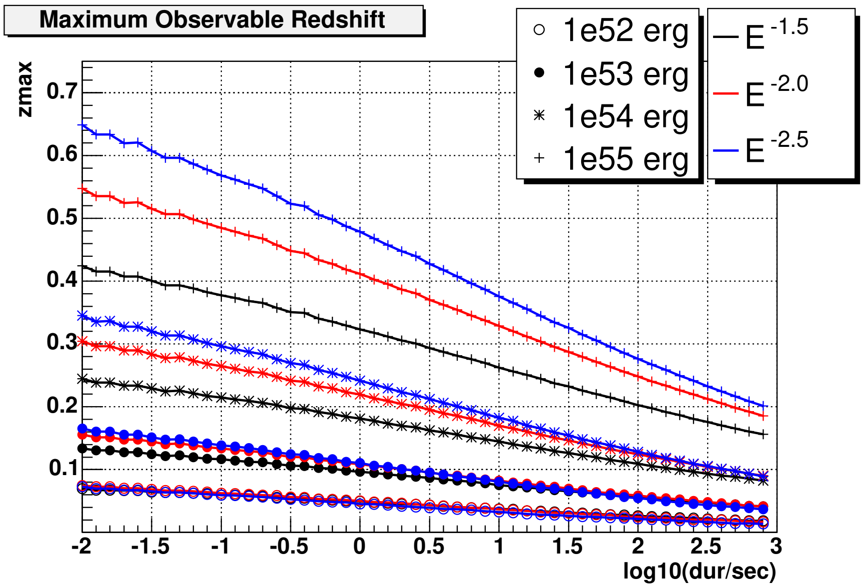

Figure 6: Maximum observable redshift versus burst duration for different burst energies and different burst spectra.

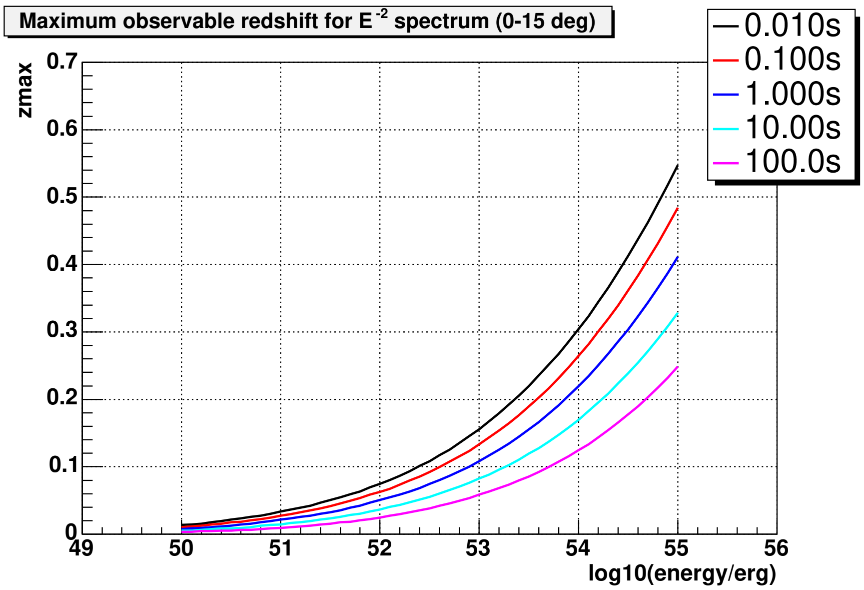

Figure 7: Maximum observable redshift versus burst energy with an $E^{-2.0}$ spectrum for different burst durations.

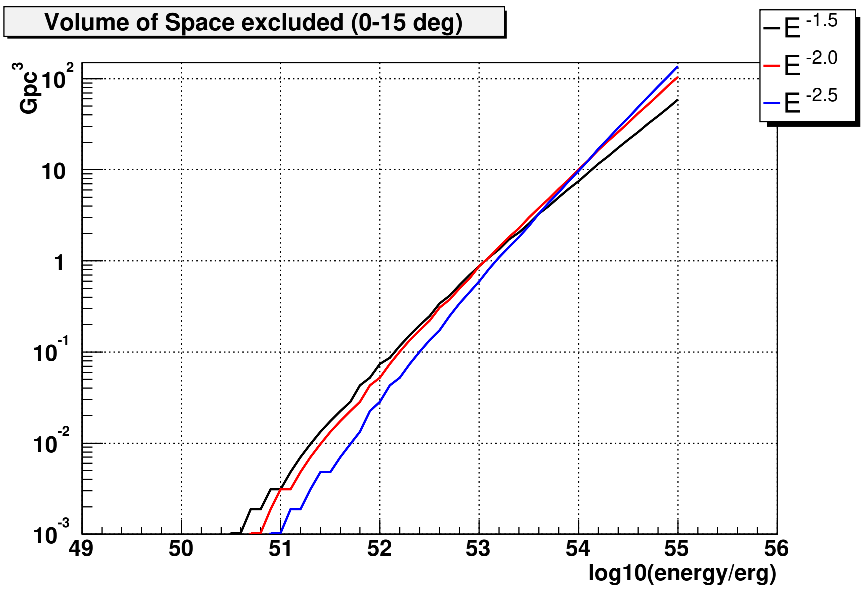

Figure 8: Volume of space excluded for bursts of 100 second duration versus burst energy. Zenith angle range of 0-15 degrees, and three different spectral indices.

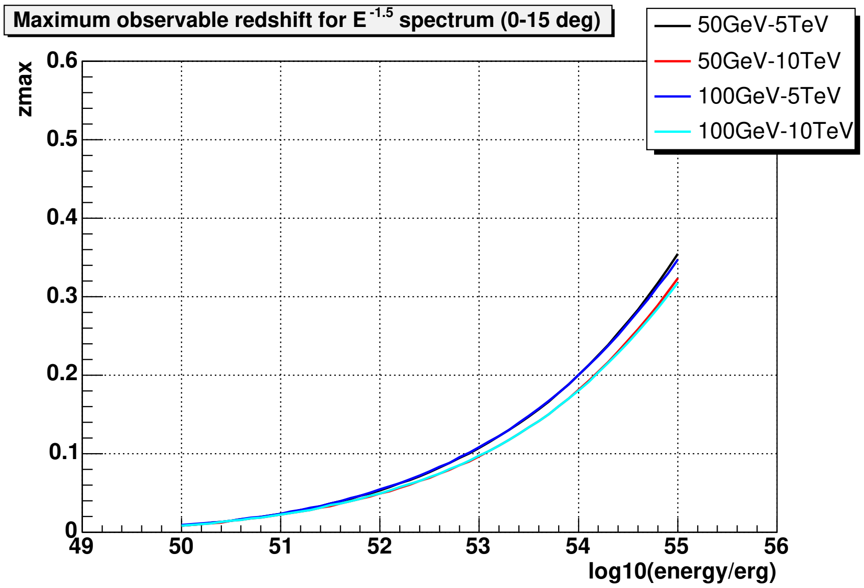

Figure 9: Sensitivity of this analysis to the minimum and maximum energies, for an $E^{-1.5}$ spectrum.

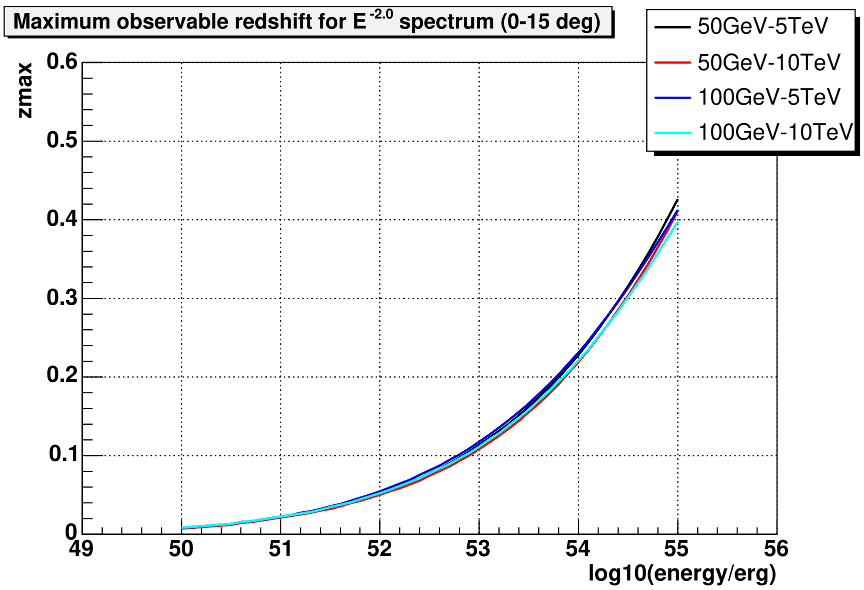

Figure 10: Sensitivity of this analysis to the minimum and maximum energies, for an $E^{-2.0}$ spectrum.

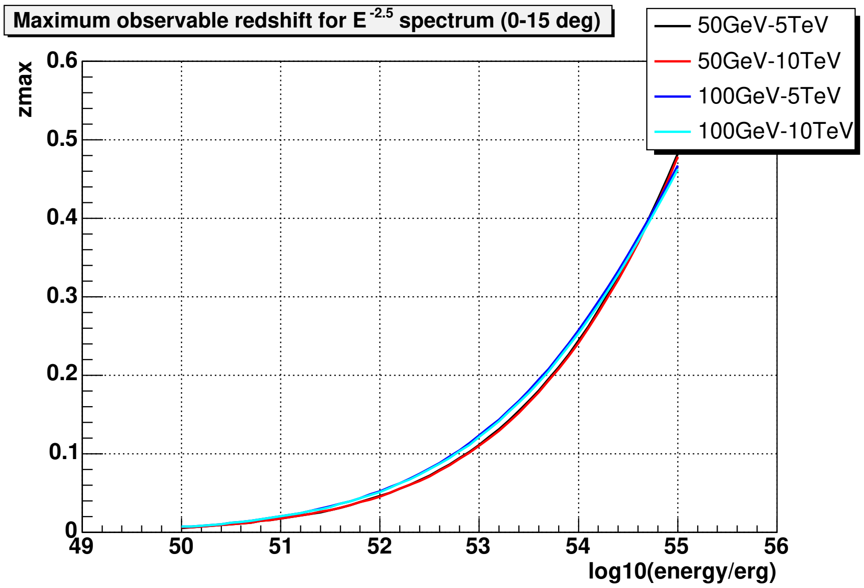

Figure 11: Sensitivity of this analysis to the minimum and maximum energies, for an $E^{-2.5}$ spectrum.

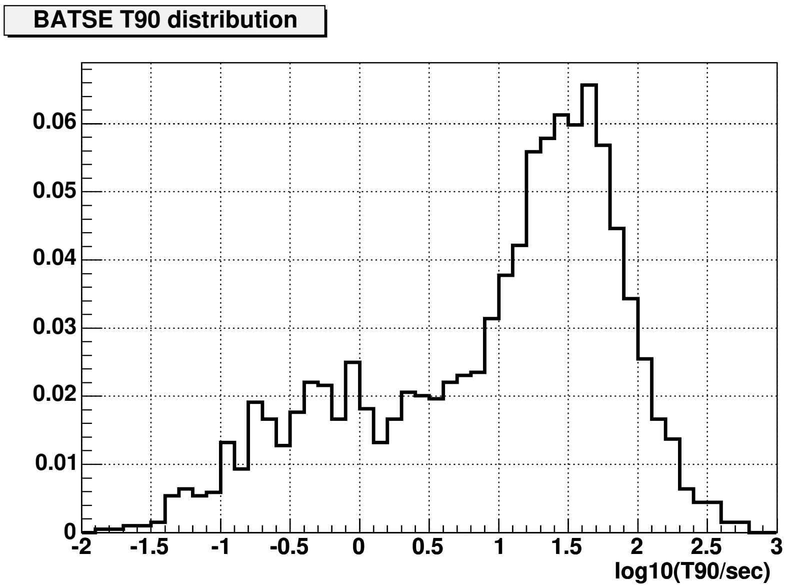

Figure 12: BATSE T90 distribution.

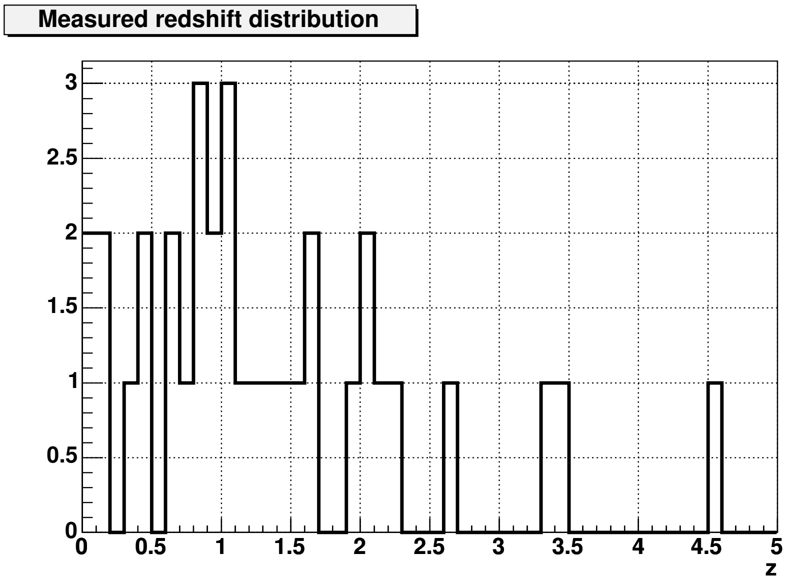

Figure 13: Measured GRB redshift distribution.

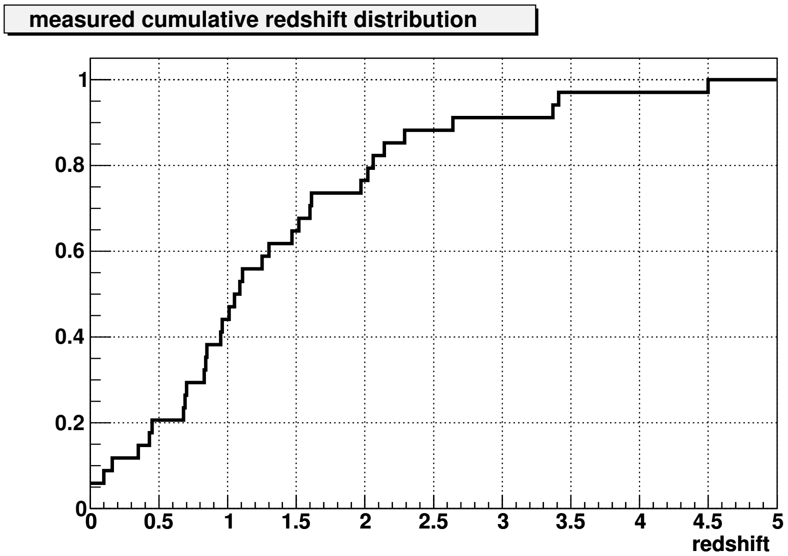

Figure 14: Measured cumulative redshift distribution.

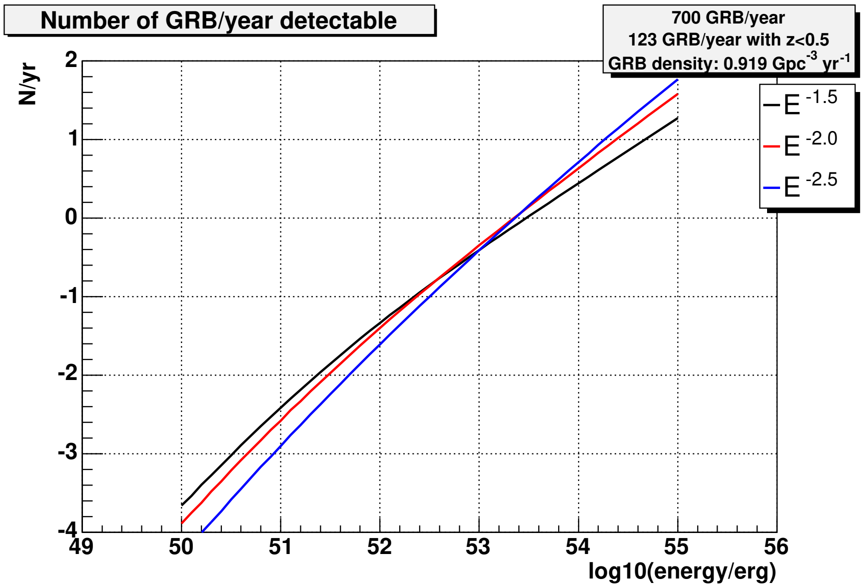

Figure 15: Number of GRBs per year expected to be observed by Milagro versus burst energy. I assumed a constant GRB density, and used the measured redshift distribution normalized by the BATSE rate of $\sim$ 700 GRB per year. Everything was calculated in three zenith angle ranges (0-15, 15-30, 30-45 degrees), and the results were combined to get the total number. Also, a standard candle luminosity function was used to calculate the total number.

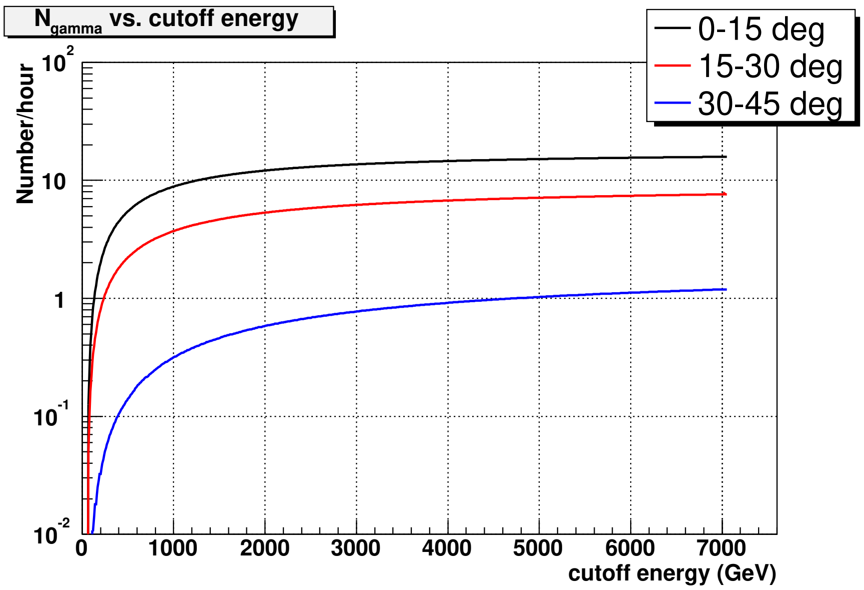

Figure 16: Number of events per hour expected from a crab spectrum for different zenith angle ranges. This result is consistent with the number of events expected from the crab.