Monte Carlo Study of GRB Location Resolution and Optimal Binsize

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1 Introduction

Presented here is a Monte Carlo study of the search for GRB 970417a (the big one). Specifically, the position resolution and the number of events from the GRB are estimated. Additionally, a study of the optimal binsize for delayed emission is estimated. The The following assumes that the GRB detection is “real”.

2 Characterization of the Angular Resolution

In this analysis the angular resolution is characterized by a Gaussian point spread function for each of many for narrow ranges of NFIT. This is similar to how Joe characterized the angular resolution for his optimal bin analysis. Unlike Joe's analysis, we determined the point spread function from γ-ray triggers from the Milagrito Monte Carlo simulation. We chose to use the Monte Carlo rather than DELEO/2 as Joe used for 2 reasons. First, DELEO does not account for systematic effects such as shower curvature that dominate the angular resolution of large NFIT events. Second, for very low NFIT events, the DELEO poorly represents the angular resolution because, one of the two fits to half of the pond PMTs often fails. For typical values of NFIT (from 30 to 80 for example), where most of the data resides, the Milagrito simulation and DELEO/2 agree well. The down side of the Monte Carlo is the paucity of data which makes precise estimation of the angular resolution difficult. Table 2 gives the point spread function used for each NFIT band.

NFIT Range	σ_{PSF}	NFIT Range	σ_{PSF}	NFIT Range	σ_{PSF}
00-09	5.0	70-79	1.0	140-149	0.7
10-19	5.0	80-89	1.0	150-159	0.7
20-29	1.4	90-99	1.0	160-169	0.6
30-39	1.4	100-109	0.9	170-179	0.6
40-49	1.3	110-119	0.9	180-189	0.6
50-59	1.3	120-129	0.8	190-199	0.6
60-69	1.0	130-139	0.8	200+	0.6

Table 1: Parameterization of Milagrito point spread function vs NFIT

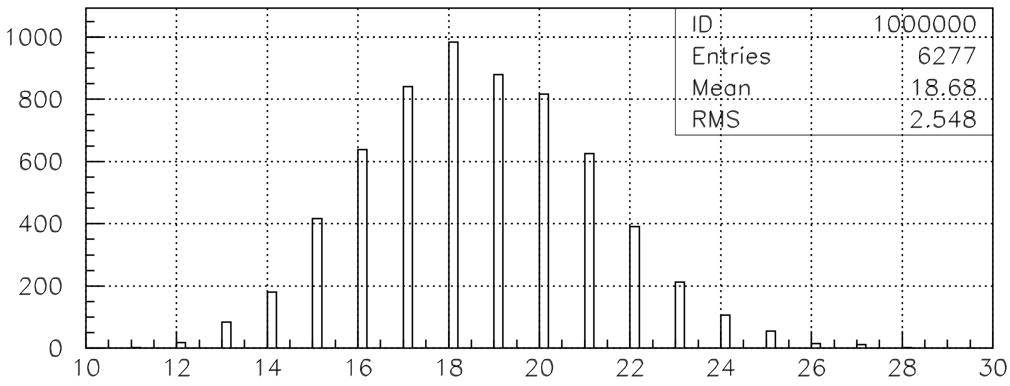

Figure 1: Distribution of number of events in the most significant bin for simulated searches with 21 signal events thrown.

GRB Search Simulation

Using this parameterization of the Milagrito angular resolution, we created a simple simulation to evaluate the search algorithm for GRB 190417a. The simulation was used to estimate the GRB optimal binsize for the exact background and signal levels as well as the specific search method used. The background was determined from the data to be 3.46 events in a 1.6 deg radius round bin or 0.43 events per degree squared. A 20x20 degree field defines the simulated search region, and 172 = (0.43*20*20) background events were randomly placed for each simulated GRB search. The position of the signal was randomly determined for each simulated search and ranged from 0 to 0.2 deg in x and y, where (0,0) defines the center of the search area. This guarantees that the phase of the GRB position was randomly distributed. For each simulated GRB search, 21 signal events were thrown by first randomly selecting NFIT from the NFIT distribution of the data and then generating a random (x,y) location from the point spread function appropriate for the selected NFIT. A search was then performed that mimics the GRB search done by Julie. Round bins were centered on a 0.2x0.1 deg grid with a bin radius of 1.6 degrees. The reason for selecting 21 signal events is that 21 is the mode number of signal events for simulated searched producing 18 events in the 1.6 degree bin of highest significance. Figure 1 shows the distribution of the number of events in the 1.6 degree bin with the highest significance. For searches that produces 18 events in the most significant bin. as was found in GRB 970417a, the average number of signal events in the bin was 13.3. The remaining 4.7 events were background. As predicted by Gus, the search method maximizes background+signal, so the typical number of background events in the most significant bin is larger than the average value of 3.46.

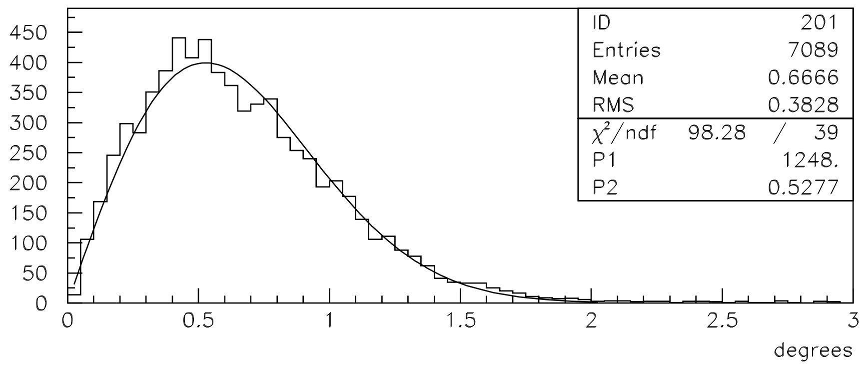

Figure 2: Position resolution of binned GRB search method.

GRB Localization

The simulation that we created is an ideal tool to for the study of the ability of the search algorithm to find the location of a GRB. The average distance between the most significant bin and the actual simulated GRB position is shown in figure 2. The distribution fits well to a 2D Gaussian with a width of 0.53 deg. 50% of the data fall within 0.60 degrees of the center of the bin of maximum significance, and 90% of the data fall within 1.16 degrees. Keep in mind that for a 2D Gaussian, 60% of the volume does not fall within 1σ. Do the integral if you don't believe us.

As the analysis that was performed was optimized to detect GRBs, and not to locate their position, we performed a second analysis to find the location of the GRB using Gaussian weighting. Each event contributes to each point on the search grid with a weight determined by its point spread function derived from NFIT. The point with the highest cumulative weight was identified as the candidate GRB position. The position resolution of this localization method is shown in figure 3. This distribution fits well to a 2D-Gaussian with width 0.43 deg.

3 Optimal Binsize

The simulation was modified to vary the search binsize. In Joe's analysis of optimal binsize, the statistic of merit for a selected binsize is the average of the log of the maximum probabilities for the simulated searches. We chose to identify the optimal bin in a different manner. The result presented here is completely consistent with Joe's, but we think illustrates the sensitivity of the search method to binsize better.

For each simulated GRB search, the highest significance bin was located for binsizes ranging from 0.1 to 5.0 degrees in steps of 0.1 degrees. The probabilities for each of the maximum significance points is calculated assuming Poisson statistics. The optimal binsize for each search is then identified as the binsize with the lowest probability. In figure 4, the solid line is the distribution of the binsize with the lowest probability. The distribution peaks at about 1.8 to 2.0 degrees. This result is completely consistent with the optimal binsize determined by young Joseph. Joe pointed out in an e-mail that smaller bins have more effective trials associated with a search. For a small

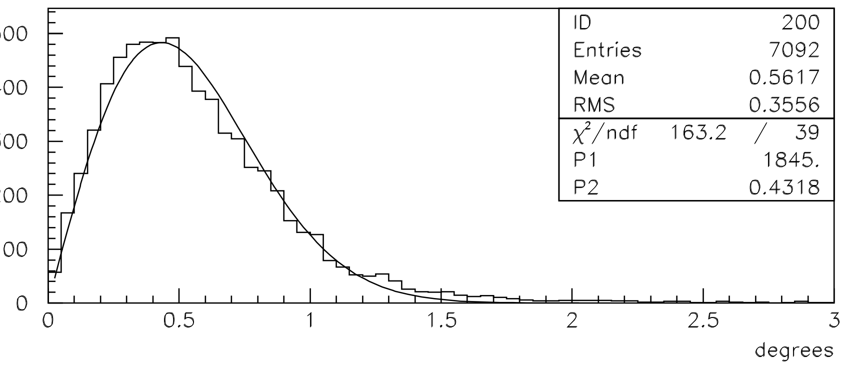

Figure 3: Position resolution of the Gaussian weighting localization method.

binsize and a larger binsize which give the same pre-trials probability, the smaller binsize analysis will have a larger, less optimal, post-trials probability. To compensate for this effect, the maximum probability for each binsize was divide by the bin area. This assumes that the number of effective trials is proportional to 1/(bin area) which seems rather reasonable to us. In figure 4, the dashed line is the distribution of binsizes with lowest probability after the area effect correction is made.

Although a binsize of about 1.8 degrees was found to be optimal, the improvement is very slight compared to the same analysis with a 1.6 degree bin. From figure 4 we can see that for any given simulated GRB search, the a 1.8 degree bin is only about 10% more likely to be optimal than a 1.6 degree bin. An optimal binsize analysis will produce an improved signal significance for only about 52% of the time even when the excess observed with a non-optimal bin size analysis was due to a GRB. One should be careful not to become to encouraged by the analysis done by Joe that shows an increase in the significance of GRB 970417a with a larger bin size and a modest NFIT cut. The changes in the analysis are sufficiently small that an observed improvement in the signal significance should not be interpreted as evidence that the GRB is "real".

4 Optimal Binsize for Searching for Delayed Emission

When searching the data for delayed emission, it makes sense to center the search bin at the point of highest significance for the original GRB detection. However, the localization of the actual burst position is not exact, so the bin is not guaranteed to be centered at the GRB position. As Joe found, for searching the sky for bursts with unknown positions, when the bin is not centered at the point of emission, the optimal binsize is larger than for that of a known source position. To determine the optimal binsize for the light curve for GRB 970417a, we simulated random GRB positions with a 2D Gaussian distribution where $\sigma = 0.53$ deg, the position resolution of the original GRB search. Delayed GRBs were simulated with 21 signal events and a background density that gives 3.46 events for a 1.6 degree radius bin, exactly the signal and background levels of the prompt emission. For each simulated delayed GRB, the Poisson probability was calculated. The binsize that had the least probable excess was identified as optimal. The distribution of optimal binsizes for 100000 simulated delayed bursts is shown in figure 5. This analysis shows that a 2.2 degree

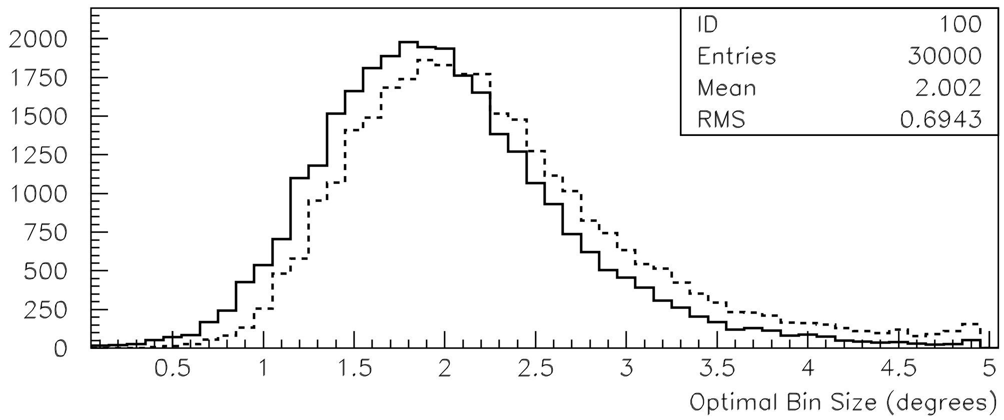

Figure 4: Optimal bin size for simulated GRB.

radius bin in optimal for the GRB light curve search.

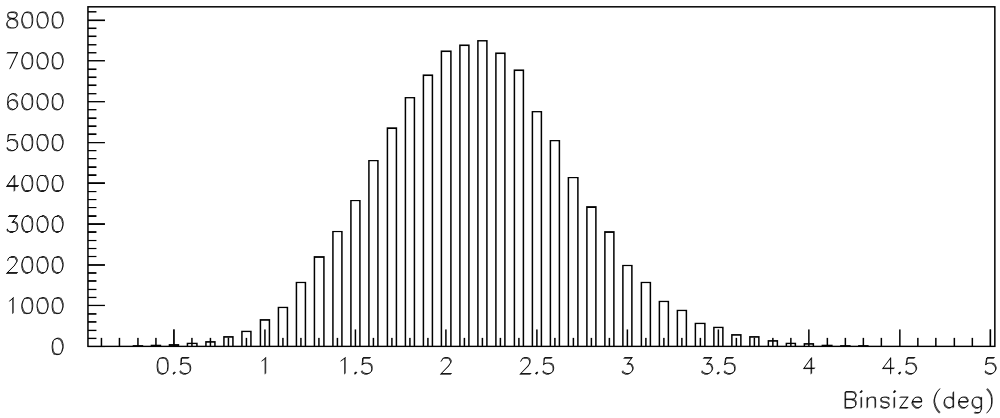

Figure 5: Optimal binsize search for delayed emission.