EVENT ENERGY DETERMINATION

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March 14 2000

INTRODUCTION

Milagro by itself provides two variables from each hit photo-multiplier, which in principle depend on energy of the primary particle and its zenith angle. These are the number of tubes hit and their pulse-heights. These are not by themselves sufficient to estimate the energy as it is almost impossible to distinguish low energy triggers with cores on the pond and high energy triggers with cores far from the pond but with the same number of tubes hit. It is essential to know the core position. **Outriggers are necessary to determine core location.**

So I assume that for each event we know: Core location, zenith angle, coordinates of hit tubes(in pond and in outriggers), pulse height of each hit tube. I use MC gamma showers in this study which were: thrown out to 300 meters from the pond in x and y directions, lying between 0-45 degrees in zenith and with energies on a spectrum from 0.1 TeV threshold with spectral slope of -2.4. To be considered an event must have greater than 50 tubes hit in the air shower layer of Milagro.

MC data are then used to determine relation between most probable true energy of a triggered event as a function of zenith angle and core distance. This energy, E_{mp} is used to estimate an age, s, of the shower which triggers Milagro and also a shower size at detector level. If t is the slant depth in radiation length of air units(37 gm/cm^2) and β_0 is $\log(\frac{E_m}{E_c})$ where E_c is critical energy in air (about

80 MeV), then age,s, and size, Sz are calculated by

$$s = \frac{3t}{t + 2\beta_0} \tag{1}$$

$$Sz = \frac{0.31}{\sqrt{(\beta_0)}} e^{t(1-\frac{3}{2}log(s))} \tag{2}$$

Size including the shower gammas should be about a factor of 6 larger.

The knowledge of age and size makes it possible to guess at a lateral distribution function for shower particles that can cause PMT hits in Milagro.

ENERGY ESTIMATOR AND RESULTS OF MC STUDY

Procedure used for estimating energy of the event from MC Milagro hit data is as follows:

1. **Position of hit PMTs from core position:**

   Calculate the distance of each hit PMT from the core position: r_i.

2. **Lateral distribution function:**

   The probability that the tube records a hit should be proportional to the density of particles(electrons and gammas) near the tube. To get an estimate of the density we use the Nishimura-Kamata lateral distribution function:

   $$f(r) = C(s) \times (\frac{r}{r_0})^{s-2} \times (1 + r/r_0)^{s-4.5} \tag{3}$$

   C(s) is a constant which makes

   $$\int_0^\infty f(x) 2\pi x dx = 1 \tag{4}$$

where x = r/r_0. The constant C(s) can be approximated by:

$$C(s) = 0.443s^2(1.9 - s) \tag{5}$$

3. **Size estimate:**

   In Milagro, the pes recorded by a given tube is subject to rather large fluctuations. The origin of these fluctuations is of two types: (1) the distance of particle trajectory from the PMT recording a hit and (2) the energy of the particle which generates the hit. This makes it very difficult to interpret pulse height as a measure of density of shower particles which could be fitted by a lateral distribution function and related to shower size.

   I use instead just the fact that a tube is hit in trying to find an algorithm for determining shower size and consequently energy of the initiating particle. If the ' particle' (electron+gamma) density is small enough then a PMT 'hit' probability is proportional to shower size. This fact is used to estimate a quantity, N, which is related to shower size.

   For each event the following sum is calculated as an estimate of a variable which is proportional to shower size:

$$N = \sum_{1}^{nhit} \frac{1}{f_i(r_i)} \tag{6}$$

   This quantity gives an estimate of shower size at a slant depth(sld) which depends on zenith angle and is different for each shower. However, shower size at maximum of shower development is a better measure of the true energy. An estimate of shower size at maximum is obtained by propoagating N to depth of shower maximum. This is approximated by :

$$N_m = N \times e^{(sld - dmax)/\lambda} \tag{7}$$

Here depth of maximum, dmax, is estimated from the original estimate of the energy, E_{mp}. It is equal to β_0 in units of radiation lengths. The shower attenuation length past shower maximum, λ, is taken to be 80 gm/cm^2 for small N and increased for large N according to

$$\lambda = 80 + 20 \times log(N)/5 \tag{8}$$

4. **Correlation between true energy and estimate size at shower maximum:**

There should be a power law connection between shower size at maximum and energy of the primary. So we hope that N_m = Const E^{α}, where exponent α is close to unity. This is shown in Figure 1 which is a plot of $\log(E_{true})$ vs $\log(N_m)$.

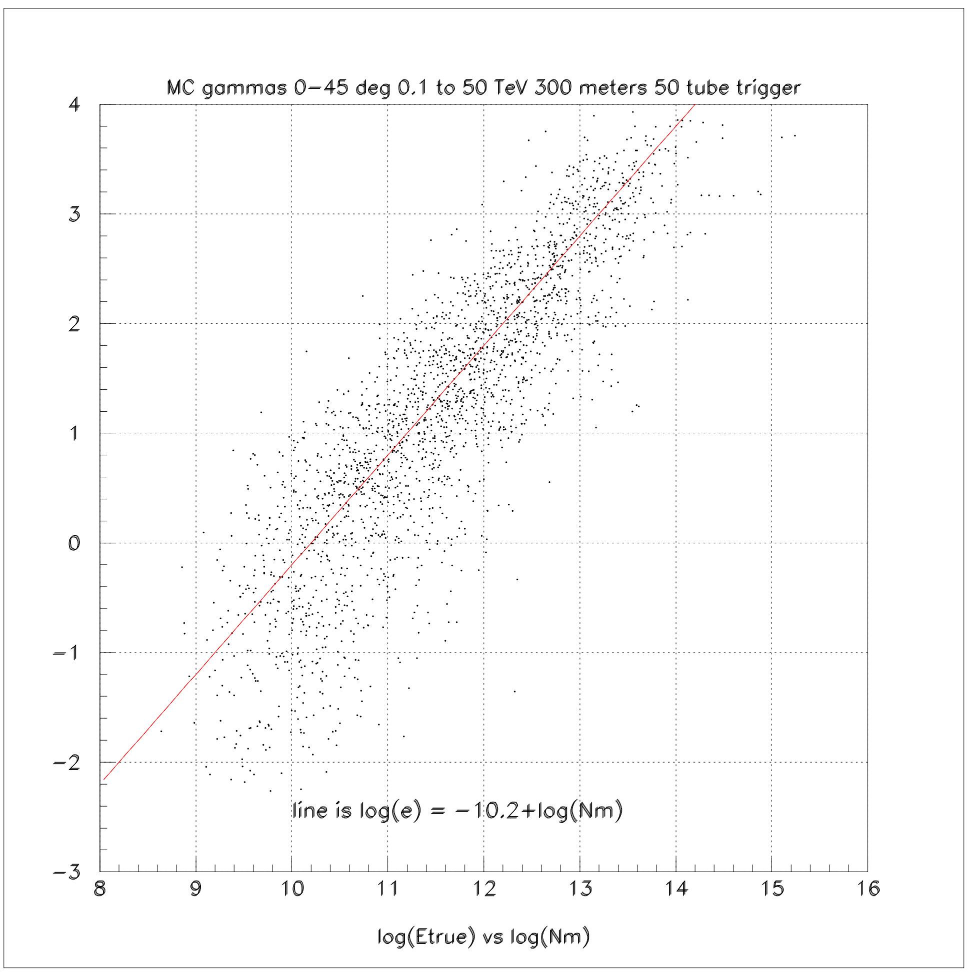

Figure 1: Correlation between log(E_{true}) and log(N_m)

This graph can be used to relate log(N_m) to log(E_{true}) as shown by the line. I note that the straight line does not give a good representation of scatter below about 1 TeV. To do a better job we must use a a more complicated polynomial than a linear line on this log-log plot.

However, for the present, for each event I calculate log(N_m) and then estimate its energy using this estimator, which I call E_{fit}. E_{fit} = exp(-10.2+log(N_m)). This is shown in Figure 2.

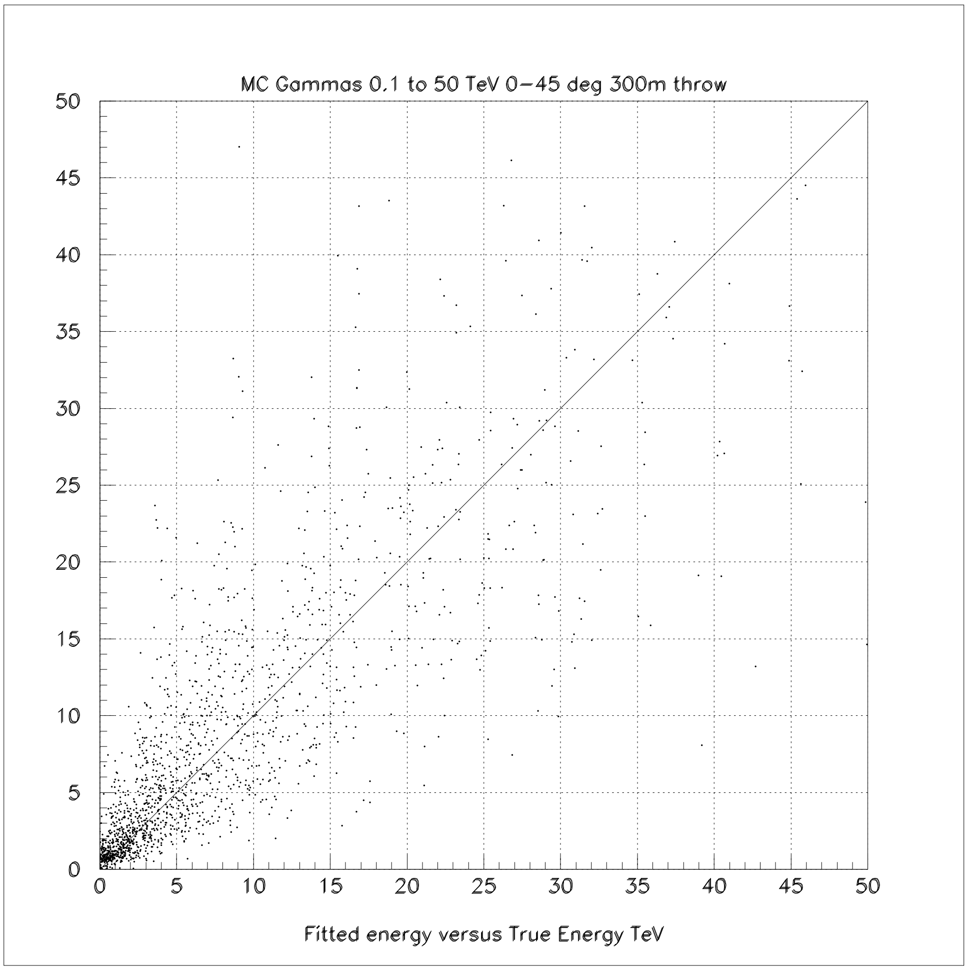

Figure 2: Correlation between fitted and true energy

5. **Energy resolution:**

Now we can ask how well do we determine true energy by this technique. As the MC data sample was not large enough, I use fractional energy deviation from true energy as indicator of energy resolution. The variable used, y , is

$$y = \frac{E_{fit} - E_{true}}{E_{true}} \tag{9}$$

The results are summarized in Figures 3, 4 and 5. Figures 3 and 4 show the distributions of y for different energy ranges of true energy from 0.1 TeV to 50 TeV. The bin sizes were chosen so that the distributions contain similar number of events. Deviation from the straightline fit of the log-log plot of figure 2 at small energies is seen in Figure 3 for true energy bins between 0.1 and 0.5 TeV and between 0.5 and 1 TeV. It is also seen that the energy resolution imporoves signficantly with energy.

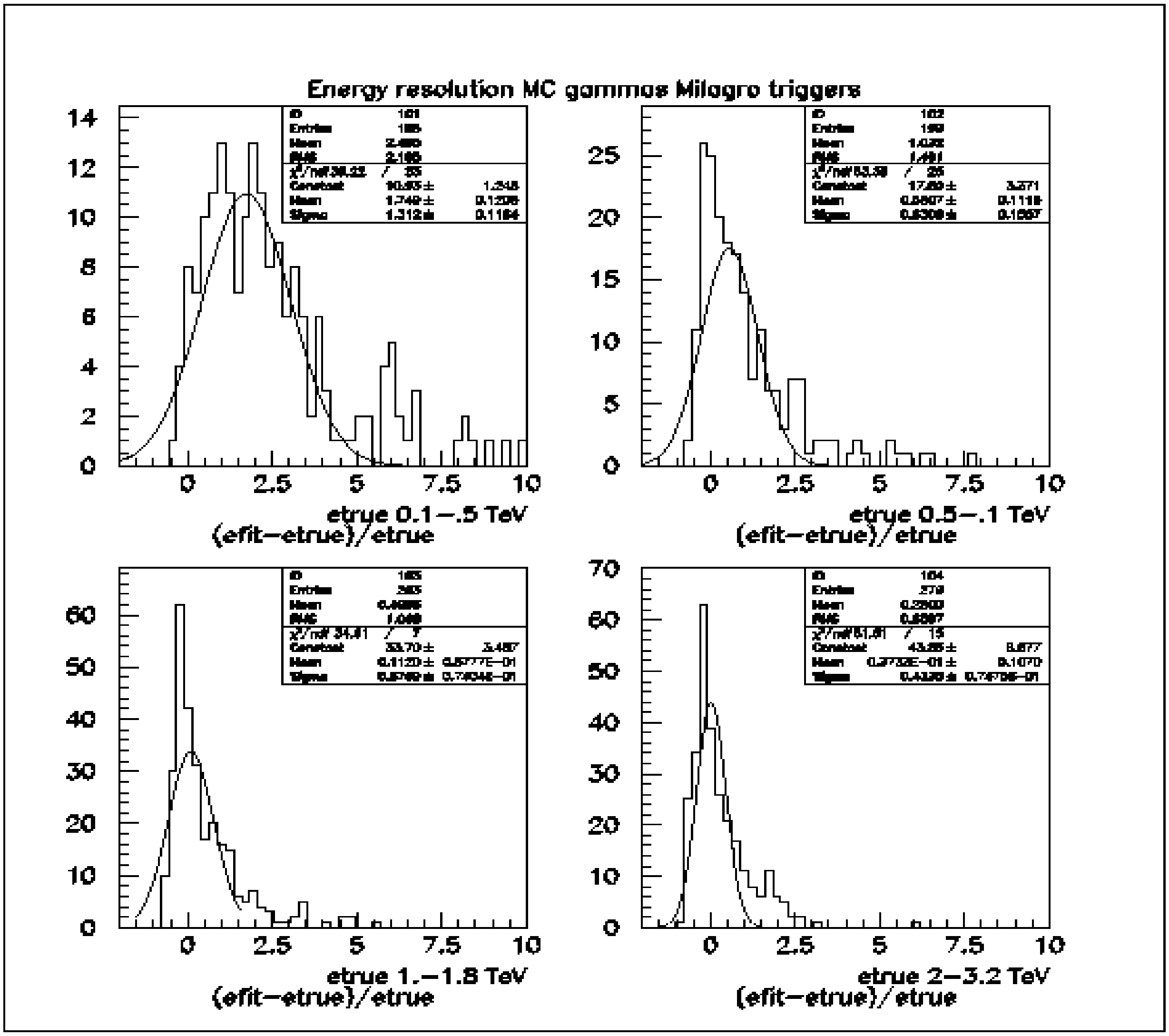

Figure 3: Fractional energy resolution for different energy bins,

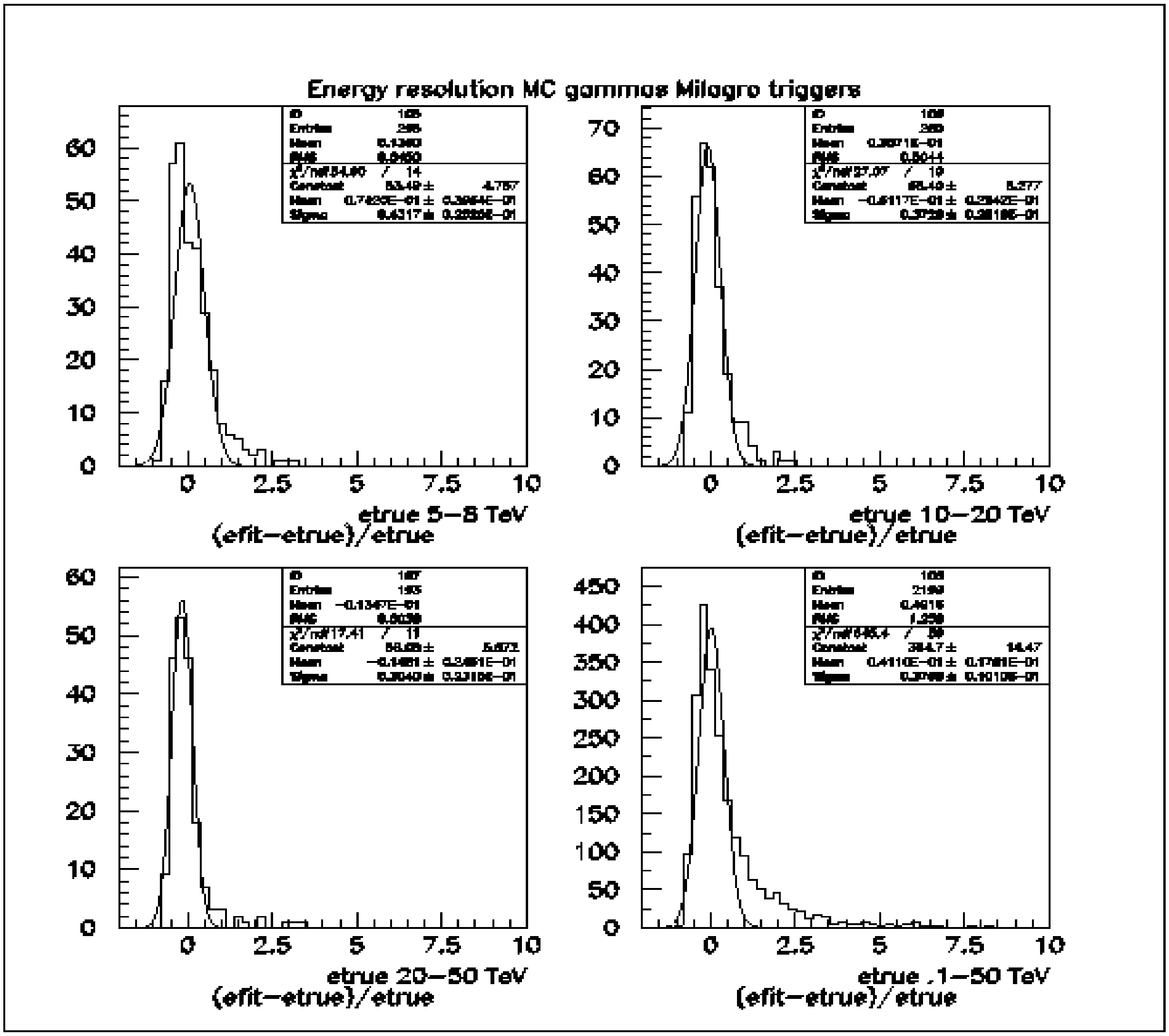

Figure 4: Fractional energy resolution for different energy bins

Figure 5 shows the fractional energy widths as function of true energy. Above about 1 TeV the energy resolution improves and becomes constant at about 30 percent. Note that there are tails to the distributions in Figures 3 and 4 and the number of badly fitted events decrease with energy.

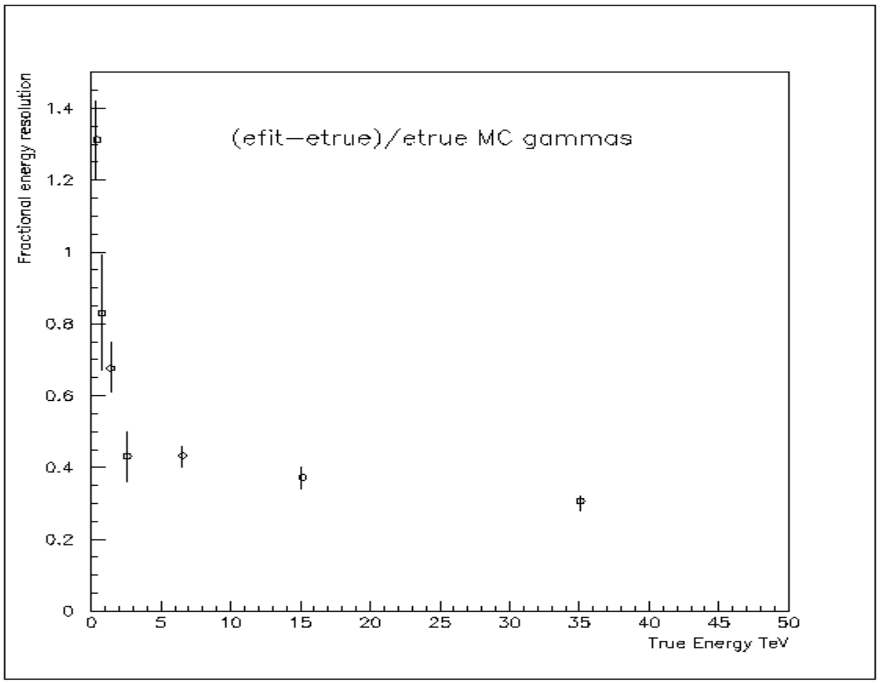

Figure 5: Fractional energy resolution as function of true energy

A fair contribution to this energy resolution comes from fluctuations in the starting depths of these showers and the rest is due to other fluctuations in detector response to the showers and due to the simplicity of our model to find energies. The expected spread due to first interaction depth fluctuations was simulated to be about 45 percent for the lowest energy bin decreasing to less than 30 percent for the higher energies.

If we had no knowledge of the core position, energy resolution can be shown to deteriorate to close to 100 percent. In fact, if core position is known then just weighting each hit tube by distance from the core and summing over all the hit tubes we can get an energy estimator. This energy estimator is not nearly as good as the one discussed in previous sections. The energy resolutions are of the order of 50 percent even at the highest energies for this estimator.