The Big Idea

When any two bodies in the universe interact, they can exchange energy, momentum or both. The law of *conservation of energy* states that in any closed system (including the universe) the total quantity of energy remains fixed. Energy is transferred from one form to another but none is lost or gained. If energy is put into a system from the outside or vice versa it is often in the form of *work*, which is a transfer of energy between bodies. (Calculated as force times displacement, provided the force is in the direction of displacement)
The law of *conservation of momentum* states that in any closed system (including the universe) the total quantity of momentum is invariant. Momentum can be transferred from one body to another but none is lost or gained. If a system has its momentum changed from the outside it is caused by an *impulse,* which transfers momentum from one body to another. (Calculated as force multiplied by time of impact)

Key Equations

- $J = N \cdot m = kg \cdot m^2/s^2$ — Energy is measured in joules
- $K = ½ mv^2$ — Kinetic energy (measured in Joules, J)
- $U_g = mgh$ — Gravitational potential energy (J)
- $U_{sp} = ½k(\Delta \mathbf{x})^2$ — Spring potential energy (J)
- $W = F_X \cdot \Delta x$ — Work given to or taken away from a system by a force (J)
- $\Sigma E_{\text{initial}} = \Sigma E_{\text{, final}}$ — Energy is conserved; the total energy does not change from an initial configuration and the final configuration
- $P = \Delta E/\Delta t$ — Power delivered to or from a system is the rate of change of energy in a certain time (measured in Watts; 1 W = 1 J/s)
- $\mathbf{J} = \Delta \mathbf{p} = \mathbf{F}\Delta t$ — Impulse is the change in momentum of a system (kg-m/s).
- $\mathbf{p}_{\text{tot,initial}} = \mathbf{p}_{\text{tot,final}}$ — Momentum is a vector, so all components are conserved.

Key Concepts

- Impulse is how momentum is transferred from one system to another. You can always determine the impulse by finding the changes in momentum, which are done by forces acting over a period of time. If you graph force vs. time of impact the area under the curve is the impulse.
- Work is simply how much energy was transferred from one system to another system. You can always find the work done *on* an object (or done *by* an object) by determining how much energy has been transferred into or out of the object through forces. If you graph force vs. distance the area under the curve is work. (The semantics take some getting used to: if *you* do work on *me*, then you have lost energy and I have gained energy.)

Key Applications

- When working a problem that asks for *height* or *speed*, energy conservation is almost always the easiest approach.
- Potential energy of gravity, U_g, is always measured with respect to some arbitrary 'zero' height defined to be where the gravitational potential energy is zero. You can set this height equal to zero at any altitude you like. Be consistent with your choice throughout the problem. Often it is easiest to set it to zero at the lowest point in the problem.
- When using the equation $E_{tot,initial} = E_{tot,final}$ to solve a problem, it is important to know which side of the equation the kinetic and potential energy, work, heat, etc., go on. To figure this out, think about what kind of energy the person or object had in the beginning (*initial)* and what kind it had in the end *(final)*.
- Some problems require you to use both energy conservation *and* momentum conservation. Remember, in every collision, momentum is conserved. Kinetic energy, on the other hand, is not always conserved, since some kinetic energy may be lost to heat.
- If a system involves no energy losses due to heat or sound, no change in potential energy and no work is done by anybody to anybody else, then kinetic energy is conserved. Collisions where this occurs are called *elastic*. In elastic collisions, both kinetic energy and momentum are conserved. In *inelastic* collisions kinetic energy is not conserved; only momentum is conserved.
- Sometimes energy is "lost" when crushing an object. For instance, if you throw silly putty against a wall, much of the energy goes into flattening the silly putty (changing intermolecular bonds). Treat this as lost energy, similar to sound, chemical changes, or heat. In an inelastic collision, things stick, energy is lost, and so kinetic energy is not conserved.
- When calculating work, use the component of the force that is in the same direction as the motion. Components of force perpendicular to the direction of the motion don't do work. (Note that centripetal forces never do work, since they are always perpendicular to the direction of motion.)
- When calculating impulse the time to use is when the force is in contact with the body.

Energy and Force Problem Set

1. At 6:37 PM, a bomb exploded in mid-air. Which of the following is true of the pieces of the bomb after explosion? (Select all that apply.)

   a. The vector sum of the momenta of all the pieces is zero.
   b. The total kinetic energy of all the pieces is zero.
   c. The chemical potential energy of the bomb has been converted entirely to the kinetic energy of the pieces.
   d. Energy is lost from the system to sound, heat, and a pressure wave.

2. A rock with mass *m* is dropped from a cliff of height *h*. What is its speed when it gets to the bottom of the cliff?

   a. $\sqrt{mg}$
   b. $\sqrt{\frac{2g}{h}}$
   c. $\sqrt{2gh}$
   d. gh
   e. None of the above

3. Two cats, Felix and Meekwad, collide. Felix has a mass of 2 kg and an initial velocity of 10 m/s to the west. Meekwad has a mass of 1 kg and is initially at rest. After the collision, Felix has a velocity of 4 m/s to the west and Meekwad has a velocity of 12 m/s to the west. Verify that momentum was conserved. Then, determine the kinetic energies of the system before and after the collision. What happened?! (All numbers are exact.)

4. You are at rest on your bicycle at the top of a hill that is 20 m tall. You start rolling down the hill. At the bottom of the hill you have a speed of 22 m/s. Your mass is 80 kg. Assuming no energy is gained by or lost to any other source, which of the following must be true?

   a. The wind must be doing work on you.
   b. You must be doing work on the wind.
   c. No work has been done on either you or the wind.
   d. Not enough information to choose from the first three.

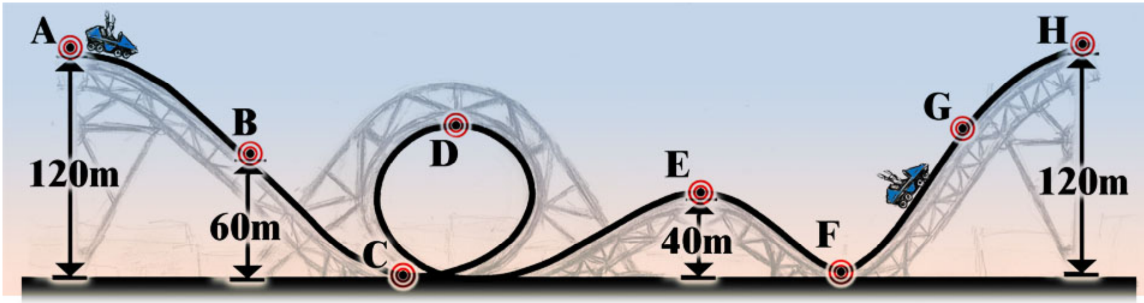
5. A snowboarder, starting at rest at the top of a mountain, flies down the slope, goes off a jump and crashes through a second-story window at the ski lodge. Retell this story, but describe it using the language of energy. Be sure to describe both how and when the skier gained and lost energy during her journey.

6. An airplane with mass 200,000 kg is traveling with a speed of 268 m/s.

   a. What is the kinetic energy of the plane at this speed?

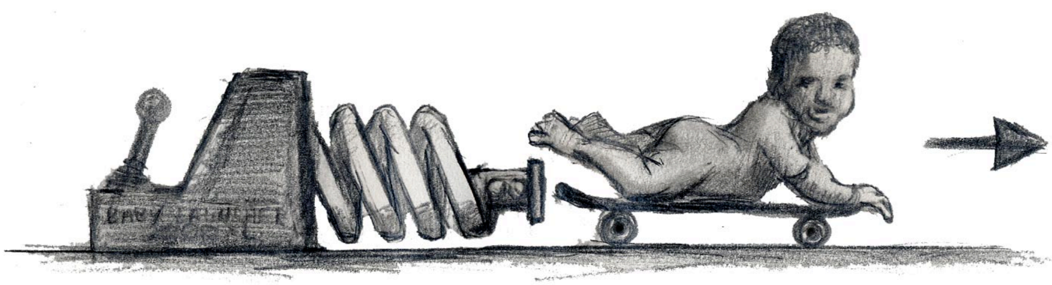
   A wind picks up, which causes the plane to lose $1.20 \text{ X } 10^8$ J per second.

   b. How fast is the plane going after 25.0 seconds?



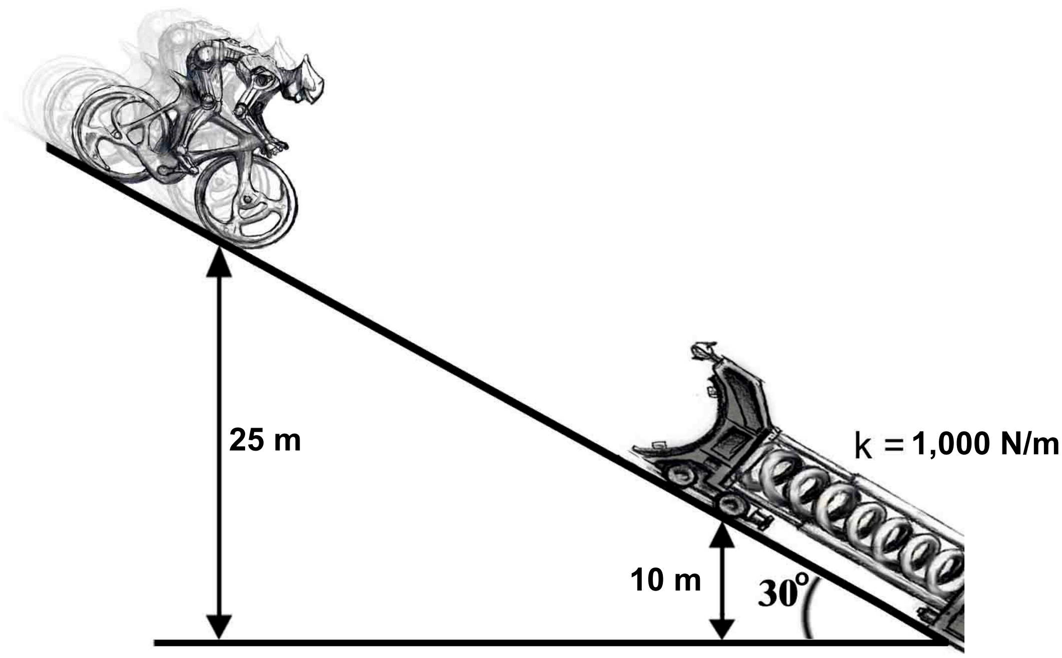
7. A roller coaster begins at rest 120 m above the ground, as shown. Assume no friction from the wheels and air, and that no energy is lost to heat, sound, and so on. The radius of the loop is 40 m.

    a. Find the speed of the roller coaster at points B, C, D, E, F, and H.
    b. Assume that 25% of the initial potential energy of the coaster is lost due to heat, sound, and air resistance along its route. How far short of point H will the coaster stop?
    c. Does the coaster actually make it through the loop without falling? (Hint: you might review the material from Chapter 6 to answer this part.)



8. In the picture above, a 9.0 kg baby on a skateboard is about to be launched horizontally. The spring constant is 300 N/m and the spring is compressed 0.4 m. For the following questions, ignore the small energy loss due to the friction in the wheels of the skateboard and the rotational energy used up to make the wheels spin.

    a. What is the speed of the baby after the spring has reached its uncompressed length?
    b. After being launched, the baby encounters a hill 7 m high. Will the baby make it to the top? If so, what is his speed at the top? If not, how high does he make it?
    c. Are you finally convinced that your authors have lost their minds? Look at that picture!



9. When the biker is at the top of the ramp shown above, he has a speed of 10 m/s and is at a height of 25 m. The bike and person have a total mass of 100 kg. He speeds into the contraption at the end of the ramp which slows him to a stop.

   a. What is his initial total energy? (Hint: set $U_g = 0$ at the very bottom of the ramp.)
   b. What is the length of the spring when it is maximally compressed by the biker? (Hint: the spring does *not* compress all the way to the ground so there is still some gravitational potential energy. It will help to draw some triangles.)

10. An elevator in an old apartment building in Switzerland has four huge springs at the bottom of the shaft to cushion its fall in case the cable breaks. The springs have an uncompressed height of about 1 meter. Estimate the spring constant necessary to stop this elevator, following these steps:

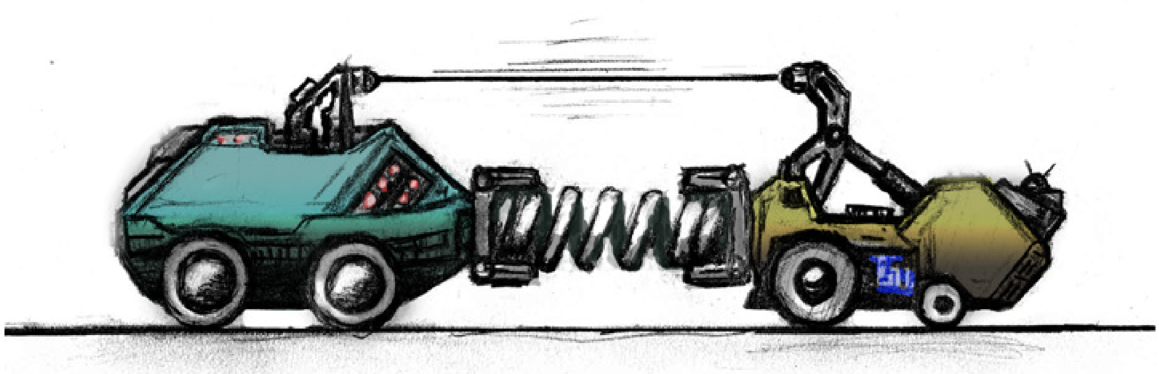
    a. First, guesstimate the mass of the elevator with a few passengers inside.
    b. Now, estimate the height of a five-story building.
    c. Lastly, use conservation of energy to estimate the spring constant.

11. You are driving your buddy to class in a car of mass 900 kg at a speed of 50 m/s. You and your passenger each have 80 kg of mass. Suddenly, a deer runs out in front of your car. The coefficient of friction between the tires and the freeway cement is $\mu_k = 0.9$. In addition there is an average force of friction of 6000 N exerted by air resistance, friction of the wheels and axles, etc. in the time it takes the car to stop.

    a. What is your stopping distance if you skid to a stop?
    b. What is your stopping distance if you roll to a stop (*i.e.*, if the brakes don't lock)?

12. You are skiing down a hill. You start at rest at a height 120 m above the bottom. The slope has a 10.0° grade. Assume the total mass of skier and equipment is 75.0 kg.

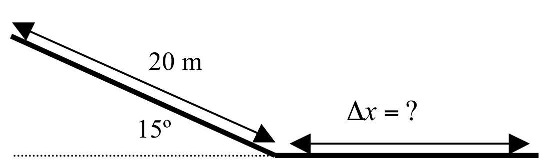
    a. Ignore all energy losses due to friction. What is your speed at the bottom?
    b. If, however, you just make it to the bottom with zero speed what would be the average force of friction, including air resistance?



13. Two horrific contraptions on frictionless wheels are compressing a spring (k = 400 N/m) by 0.5 m compared to its uncompressed (equilibrium) length. Each of the 500 kg vehicles is stationary and they are connected by a string. The string is cut! Find the speeds of the masses once they lose contact with the spring.

14. You slide down a hill on top of a big ice block as shown in the diagram. Your speed at the top of the hill is zero. The coefficient of kinetic friction on the slide down the hill is zero ($\mu_k = 0$). The coefficient of kinetic friction on the level part just beneath the hill is 0.1 ($\mu_k = 0.1$).

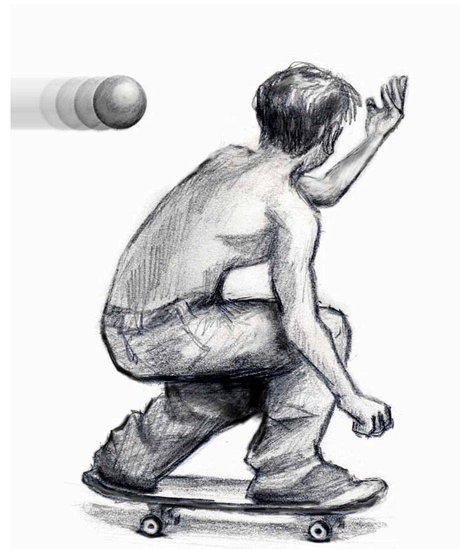
    a. What is your speed just as you reach the bottom of the hill?
    b. How far will you slide before you come to a stop?



15. A 70 kg woman falls from a height of 2.0 m and lands on a springy mattress.

    a. If the springs compress by 0.5 m, what is the spring constant of the mattress?
    b. If no energy is lost from the system, what height will she bounce back up to?

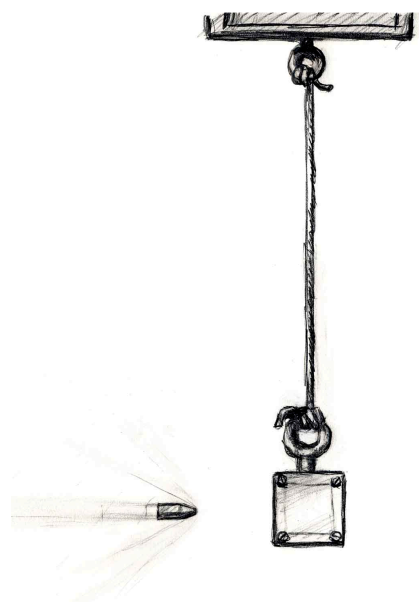
16. Marciel is at rest on his skateboard (total mass 50 kg) until he catches a ball traveling with a speed of 50 m/s. The baseball has a mass of 2 kg. What percent of the original kinetic energy is transferred into heat, sound, deformation of the baseball, and other non-mechanical forms when the collision occurs?



17. You throw a 0.5 kg lump of clay with a speed of 5 m/s at a 15 kg bowling ball hanging from a vertical rope. The bowling ball swings up to a height of 0.01 m compared to its initial height. Was this an elastic collision? Justify your answer.

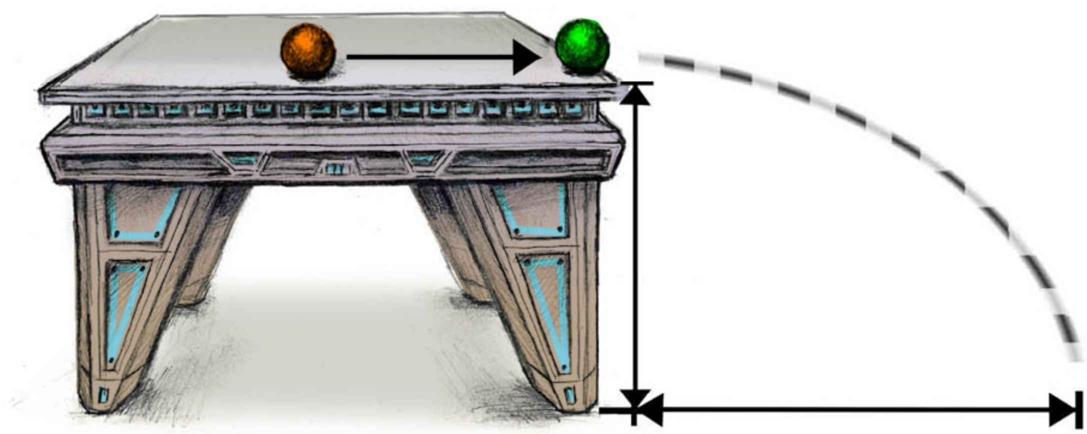
18. The 20 g bullet shown below is traveling to the right with a speed of 20 m/s. A 1.0 kg block is hanging from the ceiling from a rope 2.0 m in length.

    a. What is the maximum height that the bullet-block system will reach, if the bullet embeds itself in the block?
    b. What is the maximum angle the rope makes with the vertical after the collision?



19. You are playing pool and you hit the cue ball with a speed of 2 m/s at the 8-ball (which is stationary). Assume an elastic collision and that both balls are the same mass. Find the speed and direction of both balls after the collision, assuming neither flies off at any angle.

20. A 0.045 kg golf ball with a speed of 42.0 m/s collides elastically head-on with a 0.17 kg pool ball at rest. Find the speed and direction of both balls after the collision.



21. Ball A is traveling along a flat table with a speed of 5.0 m/s, as shown. Ball B, which has the same mass, is initially at rest, but is knocked off the table (1.5m high) in an elastic collision with Ball A. The Find the horizontal distance that Ball B travels before hitting the floor.

22. Manrico (80.0 kg) and Leonora (60.0 kg) are figure skaters. They are moving toward each other. Manrico's speed is 2.00 m/s; Leonora's speed is 4.00 m/s. When they meet, Leonora flies into Manrico's arms.

    a. With what speed does the entwined couple move?
    b. In which direction are they moving?
    c. How much kinetic energy is lost in the collision?

23. Aida slides down a 20.0 m high hill on a frictionless sled (combined mass 40.0 kg). At the bottom of the hill, she collides with Radames on his sled (combined mass 50.0 kg). The two children cling together and move along a horizontal plane that has a coefficient of kinetic friction of 0.10.

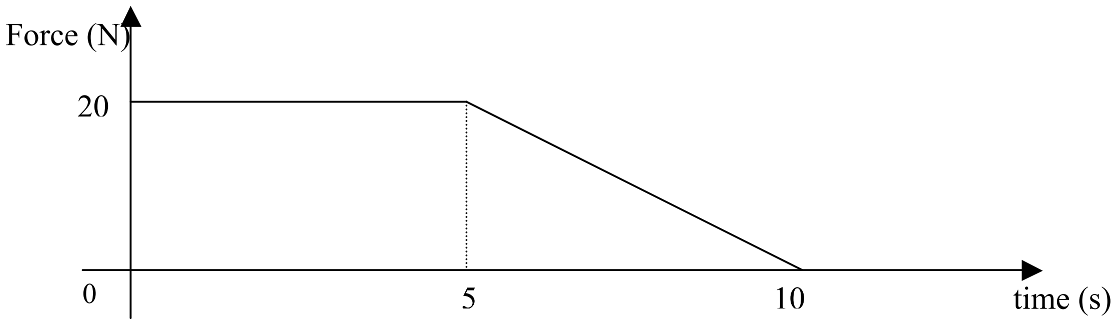
    a. What was Aida's speed before the collision?
    b. What was the combined speed immediately after collision?
    c. How far along the level plane do they move before stopping?

24. A pile driver lifts a 500 kg mass a vertical distance of 20 m in 1.1 sec. It uses 225 kW of supplied power to do this.

    a. How much power was used in actually lifting the mass?
    b. What is the efficiency of the machine? (This is the ratio of power used to power supplied.)
    c. The mass is dropped on a pile and falls 20 m. If it loses 40,000 J on the way down to the ground due to air resistance, what is its speed when it hits the pile?

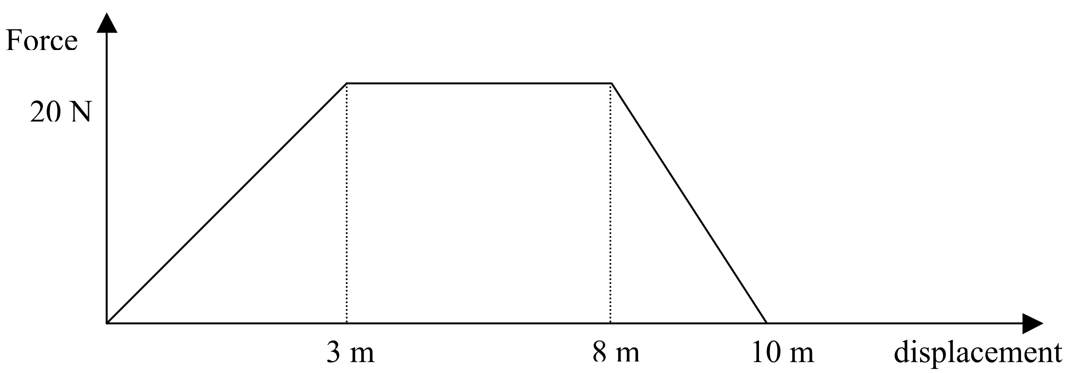
25. Investigating a traffic collision, you determine that a fast-moving car (mass 600 kg) hit and stuck to a second car (mass 800 kg) which was initially at rest. The two cars slid a distance of 30.0 m over rough pavement with a coefficient of friction of 0.60 before coming to a halt. What was the speed of the first car? Was the driver above the posted 60 MPH speed limit?

26. Force is applied in the direction of motion to a 15.0 kg cart on a frictionless surface. The motion is along a straight line and when t = 0, then x = 0 and v = 0. (The displacement and velocity of the cart are initially zero.) Look at the following graph:



a. What is the change in momentum during the first 5 sec?
b. What is the change in velocity during the first 10 sec?
c. What is the acceleration at 4 sec?
d. What is the total work done on the cart by the force from 0 – 10 sec?
e. What is the displacement after 5 sec?

27. Force is applied in the direction of motion to a 4.00 kg cart on a frictionless surface. The motion is along a straight line and when t = 0, v = 0 and x = 0. look at the following graph:



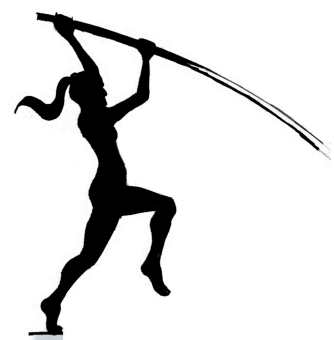
a. What is the acceleration of the cart when the displacement is 4 m?
b. What work was done on the cart between x = 3 m and x = 8 m?
c. What is the total work done on the cart between 0 -10 m?
d. What is the speed of the cart at x = 10 m?
e. What is the impulse given the cart by the force from 1 – 10 m?
f. What is the speed at x = 8 m?
g. How much time elapsed from when the cart was at x = 8 to when it got to x = 10 m?

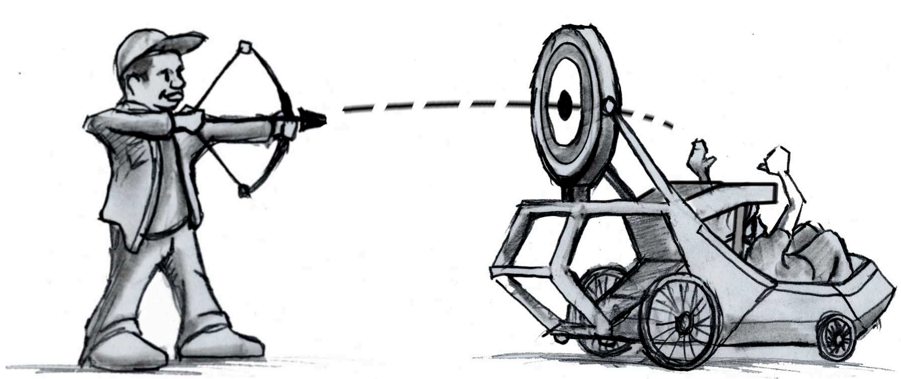
28. You are to design an experiment to measure the average force an archer exerts on the bow as she pulls it back prior to releasing the arrow. The mass of the arrow is known. The only lab equipment you can use is a meter stick.

    a. Give the procedure of the experiment and include a diagram with the quantities to be measured shown.
    b. Give sample calculations using realistic numbers.
    c. What is the single most important inherent error in the experiment?
    d. Explain if this error would tend to make the force that it measured greater or lesser than the actual force and why.

29. Molly eats a 500 kcal (2.09 x 10^6 J) power bar before the big pole vault. The bar's energy content comes from changing chemical bonds from a high to a low state and expelling gases. However, 25.0% of the bar's energy is lost expelling gases and 60.0% is needed by the body for various biological functions.

    a. How much energy is available to Molly for the run?
    b. Energy losses due to air resistance and friction on the run are 200,000 J, Molly's increased heart rate and blood pressure use 55,000 J of the available energy during the run. What top speed can the 50.0 kg Molly expect to attain?
    c. The kinetic energy is transferred to the pole, which is "compressed" like a spring of k = 2720 N/m; air resistance energy loss on the way up is 300 J, and as she crosses the bar she has a horizontal speed of 2.00 m/s. If Molly rises to a height equal to the expansion of the pole what is that height she reaches?
    d. On the way down she encounters another 300 J of air resistance. How much heat in the end is given up when she hits the dirt and comes to a stop?





30. A new fun foam target on wheels for archery students has been invented. The arrow of mass, m, and speed, v_0, goes through the target and emerges at the other end with reduced speed, $v_0/2$. The mass of the target is 7m. Ignore friction and air resistance.

    a. What is the final speed of the target?
    b. What is the kinetic energy of the arrow after it leaves the target?
    c. What is the final kinetic energy of the target?
    d. What percent of the initial energy of the arrow was lost in the shooting?