



## The Big Idea

The universe has many remarkable qualities, among them a rather beautiful symmetry: the total amount of motion in the universe balances out ... always. This law only makes sense if we measure “motion” in a specific way: as the product of mass and velocity. This product, called *momentum*, can be exchanged from one object to another in a collision. The rapidity with which momentum is exchanged over time is determined by the forces involved in the collision. This is the second of the five fundamental conservation laws in physics. The other four are conservation of energy, angular momentum, charge and CPT. (See Chapter 22 for an explanation of CPT.)

## Key Equations

- $\mathbf{p} = m\mathbf{v}$

The momentum vector points in the same direction as the velocity vector, but its length depends on both the mass and the speed of the object. To get the magnitude, just multiple the mass by the speed.

- $\mathbf{F} = \Delta\mathbf{p}/\Delta t$

Note that  $\Delta\mathbf{p}/\Delta t = m\Delta\mathbf{v}/\Delta t = m\mathbf{a}$ . This is just Newton’s 2<sup>nd</sup> Law!

- $\sum \mathbf{p}_i = \sum \mathbf{p}_f$

The sum of all the momenta before a collision equals the sum of all the momenta after the collision. When adding up momenta, be sure to add them as vectors, and to include *all* the objects in the system.

## Key Concepts

- Momentum is a vector that points in the direction of the velocity vector. The magnitude of this vector is the product of mass and speed.
- The total momentum of the universe is always the same and is equal to zero. The total momentum of an isolated system never changes.
- Momentum can be transferred from one body to another. In an isolated system in which momentum is transferred internally, the total initial momentum is the same as the total final momentum.
- Momentum conservation is especially important in collisions, where the total momentum just before the collision is the same as the total momentum after the collision.
- The force imparted on an object is equal to the change in momentum divided by the time interval over which the objects are in contact.
- *Internal forces* are forces for which both Newton’s 3<sup>rd</sup> Law force pairs are contained within the system. For example, consider a two-car head-on collision. Define the *system*

as just the two cars. In this case, internal forces include that of the fenders pushing on each other, the contact forces between the bolts, washers, and nuts in the engines, etc.

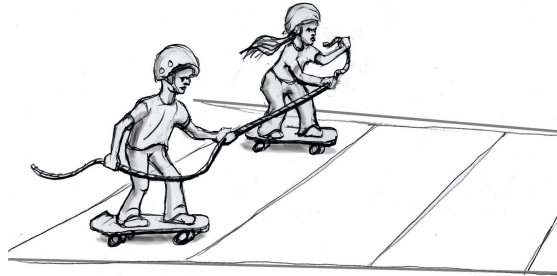
- *External forces* are forces that act on the system from outside. In our previous example, external forces include the force of gravity acting on both cars (because the other part of the force pair, the pull of gravity the Earth experiences coming from the cars, is not included in the system) and the forces of friction between the tires and the road.
- If there are no external forces acting on a system of objects, the initial momentum of the system will be the same as the final momentum of the system. Otherwise, the final momentum will change by  $\Delta \mathbf{p} = \mathbf{F}\Delta t$ . We call a change in momentum,  $\Delta \mathbf{p}$ , an *impulse*.

## Key Applications

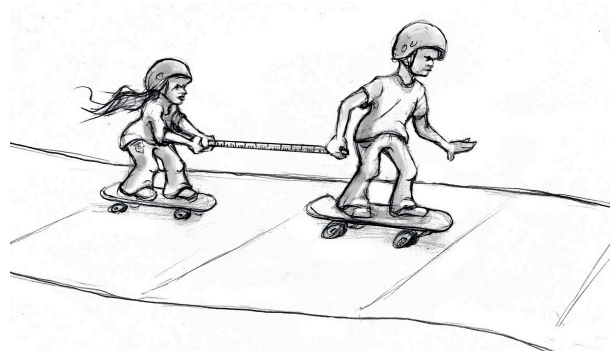
- Two cars collide head-on ... two subatomic particles collide in an accelerator ... a bird slams into a glass office building: all of these are examples of one-dimensional (straight line) collisions. For these, pay extra attention to direction: define one direction as positive and the other as negative, and be sure everybody gets the right sign.
- A firecracker in mid-air explodes ... two children push off each other on roller skates ... an atomic nucleus breaks apart during a radioactive decay: all of these are examples of *disintegration* problems. The initial momentum beforehand is zero, so the final momentum afterwards must also be zero.
- A spacecraft burns off momentum by colliding with air molecules as it descends ... hail stones pummel the top of your car ... a wet rag is thrown at and sticks to the wall: all of these are examples of *impulse* problems, where the change in momentum of one object and the reaction to the applied force are considered. What is important here is the rate: you need to come up with an average time  $\Delta t$  that the collision(s) last so that you can figure out the force  $\mathbf{F} = \Delta \mathbf{p}/\Delta t$ . Remember as well that if a particle has momentum  $\mathbf{p}$ , and it experiences an impulse that turns it around completely, with new momentum  $-\mathbf{p}$ , then the total change in momentum has magnitude  $2\mathbf{p}$ . It is harder to turn something totally around than just to stop it!
- A car going south collides with a second car going east ... an inflatable ball is thrown into the flow of a waterfall ... a billiard ball strikes two others, sending all three off in new directions: these are all examples of two-dimensional (planar) collisions. For these, you get a break: the sum of all the momenta in the  $x$  direction have to remain unchanged before and after the collision — *independent* of any  $y$  momenta, and vice-versa. This is a similar concept to the one we used in projectile motion. Motions in different directions are independent of each other.
- Momenta vectors add just like any other vectors. Refer to the addition of vectors material in Chapter 1.

## Momentum Conservation Problem Set

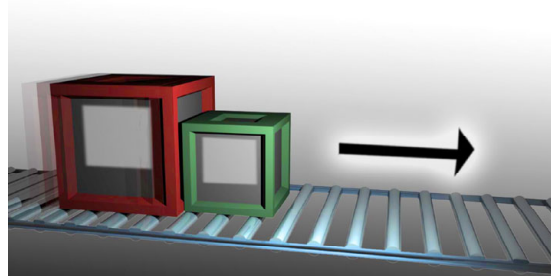
1. You find yourself in the middle of a frozen lake. There is no friction between your feet and the ice of the lake. You need to get home for dinner. Which strategy will work best?
  - a. Press down harder with your shoes as you walk to shore.
  - b. Take off your jacket. Then, throw it in the direction opposite to the shore.
  - c. Wiggle your butt until you start to move in the direction of the shore.
  - d. Call for help from the great Greek god Poseidon.
2. You jump off of the top of your house and hope to land on a wooden deck below. Consider the following possible outcomes:
  - I. You hit the deck, but it isn't wood! A camouflaged trampoline slows you down over a time period of 0.2 seconds and sends you flying back up into the air.
  - II. You hit the deck with your knees locked in a straight-legged position. The collision time is 0.01 seconds.
  - III. You hit the deck and bend your legs, lengthening the collision time to 0.2 seconds.
  - IV. You hit the deck, but it isn't wood! It is simply a piece of paper painted to look like a deck. Below is an infinite void and you continue to fall, forever.
  - a. Which method will involve the greatest force acting on you?
  - b. Which method will involve the least force acting on you?
  - c. Which method will land you on the deck in the least pain?
  - d. Which method involves the least impulse delivered to you?
  - e. Which method involves the greatest impulse delivered to you?
3. You and your sister are riding skateboards side by side at the same speed. You are holding one end of a rope and she is holding the other. Assume there is no friction between the wheels and the ground. If your sister lets go of the rope, how does your speed change?
  - a. It stays the same.
  - b. It doubles.
  - c. It reduces by half.



4. You and your sister are riding skateboards (see Problem 3), but now she is riding behind you. You are holding one end of a meter stick and she is holding the other. At an agreed time, you push back on the stick hard enough to get her to stop. What happens to your speed? Choose one. (For the purposes of this problem pretend you and your sister weigh the same amount.)
  - a. It stays the same.
  - b. It doubles.
  - c. It reduces by half.

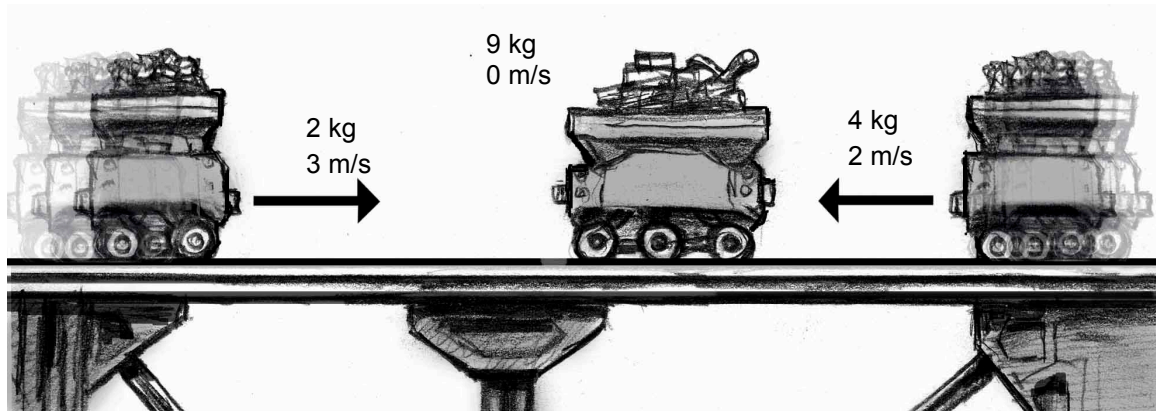


5. You punch the wall with your fist. Clearly your fist has momentum before it hits the wall. It is equally clear that after hitting the wall, your fist has no momentum. But momentum is always conserved! Explain.
6. An astronaut is using a drill to fix the gyroscopes on the Hubble telescope. Suddenly, she loses her footing and floats away from the telescope. What should she do to save herself?
7. You look up one morning and see that a 30 kg chunk of asbestos from your ceiling is falling on you! Would you be better off if the chunk hit you and stuck to your forehead, or if it hit you and bounced upward? Explain your answer.
8. A 5.00 kg firecracker explodes into two parts: one part has a mass of 3.00 kg and moves at a velocity of 25.0 m/s towards the west. The other part has a mass of 2.00 kg. What is the velocity of the second piece as a result of the explosion?
9. A firecracker lying on the ground explodes, breaking into two pieces. One piece has twice the mass of the other. What is the ratio of their speeds?
10. You throw your 6.0 kg skateboard down the street, giving it a speed of 4.0 m/s. Your friend, the Frog, jumps on your skateboard from rest as it passes by. Frog has a mass of 60 kg.
  - a. What is the momentum of the skateboard before Frog jumps on it?
  - b. Find Frog's speed after he jumps on the skateboard.
  - c. What impulse did Frog deliver to the skateboard?
  - d. If the impulse was delivered over 0.2 seconds, what was the average force imparted to the skateboard?
  - e. What was the average force imparted to the Frog? Explain.
11. Two blocks collide on a frictionless surface, as shown. Afterwards, they have a combined mass of 10 kg and a speed of 2.5 m/s. Before the collision, one of the blocks was at rest. This block had a mass of 8.0 kg. What was the mass and initial speed of the second block?
12. While driving in your pickup truck down Highway 280 between San Francisco and Palo Alto, an asteroid lands in your truck bed! Despite its 220 kg mass, the asteroid does not destroy your 1200 kg truck. In fact, it landed perfectly vertically. Before the asteroid hit, you were going 25 m/s. After it hit, how fast were you going?
13. A baseball player faces a 80.0 m/s pitch. The bat is in contact with the baseball for 0.020 seconds and the batter hits a 50.0 m/s line drive back at the pitcher. Calculate the force on the bat while in contact with the ball (you'll need to look something up to finish this problem).





14. An astronaut is 100 m away from her spaceship doing repairs with a 10.0 kg wrench. The astronaut's total mass is 90.0 kg and the ship has a mass of  $1.00 \times 10^4$  kg. If she throws the wrench in the opposite direction of the spaceship at 10.0 m/s how long would it take for her to reach the ship?
15. A place kicker applies an average force of 240 N to a football of .040 kg. The force is applied at an angle of 20.0 degrees from the horizontal. Contact time is .010 sec.
- Find the velocity the ball upon leaving the foot.
  - Assuming no air resistance find the time to reach the goal posts 40.0 m away.
  - The posts are 4.00 m high. Is the kick good? by how much?



16. In the above picture, the carts are moving on a level, frictionless track. After the collision all three carts stick together. Determine the direction and speed of the combined carts after the collision. (Assume 3-significant digit accuracy.)
17. Your author's Italian cousin crashed into a tree. He was originally going 35 km/hr. Assume it took 0.4 seconds for the tree to bring him to a stop. The mass of the cousin and the car is 450 kg.
- What average force did he experience? Include a direction in your answer.
  - What average force did the tree experience? Include a direction in your answer.
  - Express this force in pounds.
  - How many g's of acceleration did he experience?



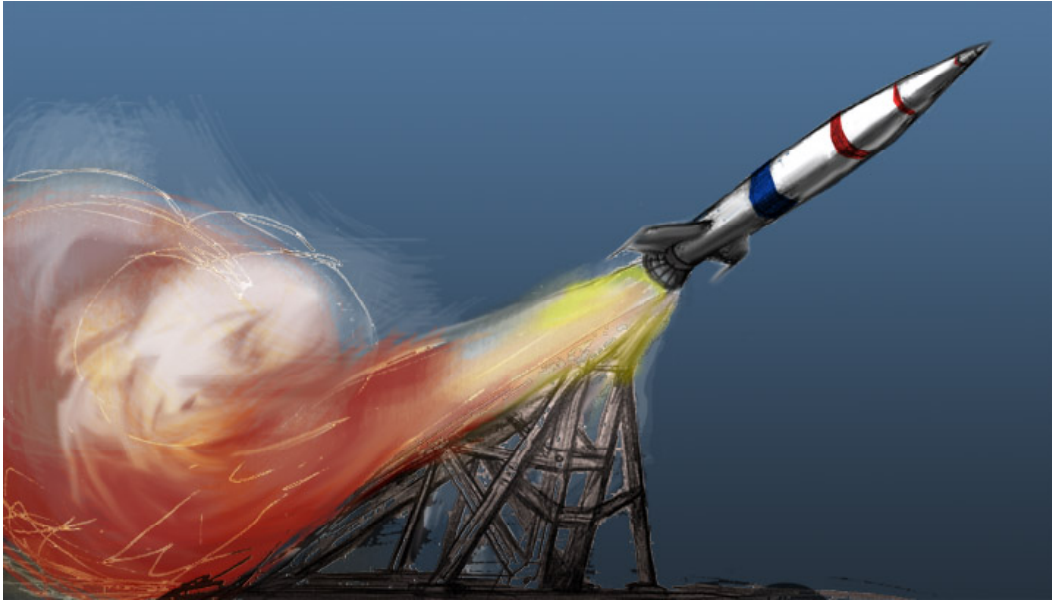


18. The train engine and its four boxcars are coasting at 40 m/s. The engine train has mass of 5,500 kg and the boxcars have masses, from left to right, of 1,000 kg, 1,500 kg, 2,000 kg, and 3,000 kg. (For this problem, you may neglect the small external forces of friction and air resistance.)
- What happens to the speed of the train when it releases the last boxcar? (*Hint: think before you blindly calculate.*)
  - If the train can shoot boxcars backwards at 30 m/s relative to the train's speed, how many boxcars does the train need to shoot out in order to obtain a speed of 58.75 m/s?

19. Serena Williams volleys a tennis ball hit to her at 30 m/s. She blasts it back to the other court at 50 m/s. A standard tennis ball has mass of 0.057 kg.
- If Serena applied an average force of 500 N to the ball while it was in contact with the racket, how long did the contact last?
  - The impact of the ball causes Serena's racket to decrease velocity by 12.1 m/s. What is the mass of her racket?

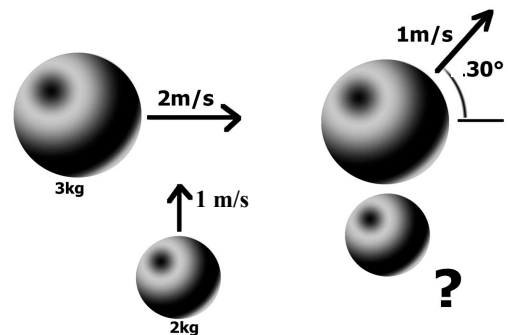


20. Zoran's spacecraft, with mass 12,000 kg, is traveling to space. The structure and capsule of the craft have a mass of 2,000 kg; the rest is fuel. The rocket shoots out 0.10 kg/s of fuel particles with a velocity of 700 m/s with respect to the craft.
- What is the acceleration of the rocket in the first second?
  - What is the average acceleration of the rocket during the first ten minutes have passed?



21. In Sacramento a 4000 kg SUV is traveling 30 m/s south on Truxel crashes into an empty school bus, 7000 kg traveling east on San Juan. The collision is perfectly inelastic.
- Find the velocity of the wreck just after collision
  - Find the direction in which the wreck initially moves

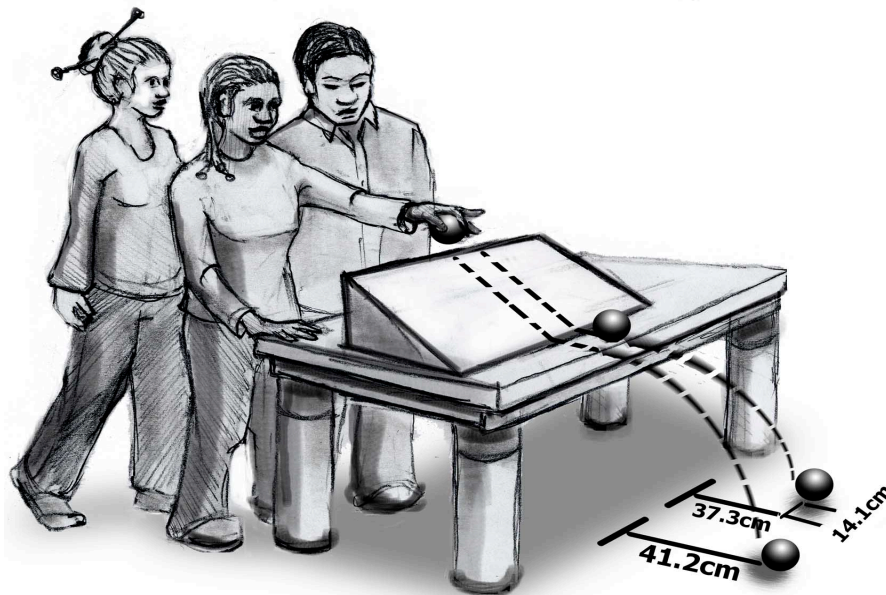
22. A 3 kg ball is moving 2 m/s in the positive x-direction when it is struck dead center by a 2 kg ball moving in the positive y-direction at 1 m/s. After collision the 3 kg ball moves at 1 m/s 30 degrees from the positive x-axis.



- To 3-significant digit accuracy fill out the following table:

	3 kg ball $p_x$	3 kg ball $p_y$	2 kg ball $p_x$	2 kg ball $p_y$
Momentum before				
Momentum after collision				

- Find the velocity and direction of the 2 kg ball.
- Use the table to prove momentum is conserved.
- Prove that kinetic energy is not conserved.



23. Students are doing an experiment on the lab table. A steel ball is rolled down a small ramp and allowed to hit the floor. Its impact point is carefully marked. Next a second ball of the same mass is put upon a set screw and a collision takes place such that both balls go off at an angle and hit the floor. All measurements are taken with a meter stick on the floor with a co-ordinate system such that just below the impact point is the origin. The following data is collected:
- (1) no collision: 41.2 cm
  - (2) target ball: 37.3 cm in the direction of motion and 14.1 cm perpendicular to the direction of motion
- a. From this data predict the impact position of the other ball
  - b. One of the lab groups declares that the data on the floor alone demonstrate to a 2% accuracy that the collision was elastic. Show their reasoning.
  - c. Another lab group says they can't make that determination without knowing the velocity the balls have on impact. They ask for a timer. The instructor says you don't need one; use your meter stick. Explain.
  - d. Design an experiment to prove momentum conservation with balls of different masses, giving apparatus, procedure and design. Give some sample numbers.