The Big Ideas

Heat is a form of energy transfer. It can change the kinetic energy of a substance. For example, the average molecular kinetic energy of gas molecules is related to temperature. A heat engine provides heat to an engine in order to turn a portion of the input heat into mechanical work. A second portion of the input heat must be exhausted in order for the engine to have repetitive motion. Therefore in a practical engine it is impossible for all the input heat to be converted to work.

Entropy is a measure of disorder, or the variety of ways in which a system can organize itself with the same total energy. The entropy of any isolated system always tends to disorder (i.e. entropy is always increasing). In the universe, the entropy of a subset (like evolution on Earth) can decrease (i.e. more order) but the total entropy of the universe is decreasing (i.e. more disorder).

Thermodynamics is the study of heat engines. Any engine or power plant obeys the laws of thermodynamics. The *first law of thermodynamics* is a statement of conservation of energy. Total energy, including heat, is conserved in any process and in the complete cycle of a heat engine. The *second law of thermodynamics* as it applies to heat engines gives an absolute limit on the efficiency of any heat engine that goes through repetitious cycles.

Key Concepts

- The temperature of a gas is a measure of the amount of average kinetic energy that the atoms in the gas possess.
- The pressure of a gas is the force the gas exerts on a certain area. For a gas in a container, the amount of pressure is directly related to the number and intensity of atomic collisions on a container wall.
- An *ideal* gas is a gas for which interactions between molecules are negligible, and for which the gas atoms or molecules themselves store no potential energy. For an "ideal" gas, the pressure, temperature, and volume are simply related by the ideal gas law.
- Atmospheric pressure (1 atm = 101,000 Pascals) is the pressure we feel at sea level due to the weight of the atmosphere above us. As we rise in elevation, there is less of an atmosphere to push down on us and thus less pressure.
- When gas pressure-forces are used to move an object then work is done on the object by the expanding gas. Work can be done on the gas in order to compress it.

- Adiabatic process: a process where no heat enters or leaves the heat engine.
- Isothermal: a process where the temperature does not change.
- Isobaric: a process where the pressure does not change.
- Isochoric: a process where the volume of the container does not change.
- If you plot pressure on the vertical axis and volume on the horizontal axis the work done in any complete cycle is the area enclosed by the graph. For a partial process work is the area underneath the curve, or $P\Delta V$.
- In a practical heat engine the change in internal energy must be zero over a complete cycle. Therefore over a complete cycle $W = \Delta Q$
- The work done by a gas during a portion of a cycle = $P\Delta V$, note ΔV can be positive or negative.
- The efficiency of any heat engine : $\eta = W/_{Qin}$
- An ideal engine, i.e.: the most efficient even theoretically possible, is called a Carnot Engine. Its efficiency, $\eta = 1 - T_{cold}/T_{hot}$ The temperatures are in Kelvins and are respectively the temperature of the exhaust environment and the temperature of the heat input. In a Carnot engine heat is inputted and exhausted in isothermal cycles.

Key Equations

- $\langle {}^1\!/_2\, mv^2 \rangle_{AVG} = {}^3\!/_2\, kT$

  The average kinetic energy of atoms (each of mass *m* and average speed *v*) in a gas is related to the temperature *T* of the gas, measured in Kelvin. The Bolztmann constant *k* is a constant of nature, equal to 1.38×10^{-23} J/K.

- $P = F / A$

  The pressure on an object is equal to the force pushing on the object divided by the area over which the force is exerted. Unit for pressure are N/m^2 (called Pascals)

- $PV = NkT$

  An ideal gas is a gas where the atoms are treated as point-particles and assumed to never collide or interact with each other. If you have *N* molecules of such a gas at temperature *T* and volume *V*, the pressure can be calculated from this formula. Note that $k = 1.38\times10^{-23}$ J/K; this is the ideal gas law.

- $PV = nRT$

  V is the volume, n is the number of moles; R is the universal gas constant = 8.315 J/K-n; this is the most useful form of the gas law for thermodynamics.

- $Q_{in} = Q_{out} + W + \Delta U$

  U is the internal energy of the gas. (This is the first law of Thermodynamics and applies to all heat engines.)

Thermodynamics and Heat Engines Problem Set

1. Consider a molecule in a closed box. If the molecule collides with the side of the box, how is the force exerted by the molecule on the box related to the momentum of the molecule? Explain conceptually, in words rather than with equations.

2. If the number of molecules is increased, how is the pressure on a particular area of the box affected? Explain conceptually, in words rather than with equations.

3. The temperature of the box is related to the average speed of the molecules. Use momentum principles to relate temperature to pressure. Explain conceptually, in words rather than with equations.

4. What would happen to the number of collisions if temperature and the number of molecules remained fixed, but the volume of the box increased? Explain conceptually, in words rather than with equations.

5. Use the reasoning in the previous four questions to qualitatively derive the ideal gas law.

6. Typical room temperature is about 300 K. As you know, the air in the room contains both O_2 and N_2 gases, with nitrogen the lower mass of the two. If the average kinetic energies of the oxygen and nitrogen gases are the same (since they are at the same temperature), which gas has a higher average speed?

7. Use the formula $P = F / A$ to argue why it is easier to pop a balloon with a needle than with a finger (pretend you don't have long fingernails).

8. Take an empty plastic water bottle and suck all the air out of it with your mouth. The bottle crumples. Why, exactly, does it do this?

9. You will notice that if you buy a large drink in a plastic cup, there will often be a small hole in the top of the cup, in addition to the hole that your straw fits through. Why is this small hole necessary for drinking?

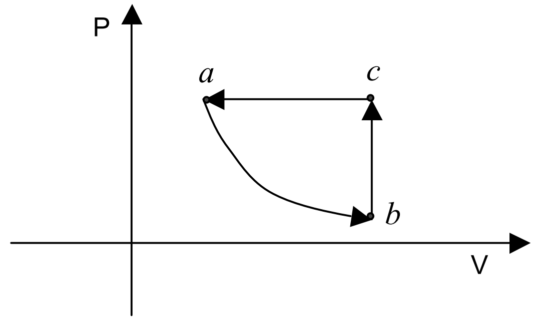
10. Suppose you were swimming in a lake of liquid water on a planet with a lower gravitational constant g than Earth. Would the pressure 10 meters under the surface be the same, higher, or lower, than for the equivalent depth under water on Earth? (You may assume that the density of the water is the same as for Earth.)

11. Why is it a good idea for Noreen to open her bag of chips before she drives to the top of a high mountain?

12. Explain, using basic physics conservation laws, why the following conditions would cause the ideal gas law to be violated:

    a. There are strong intermolecular forces in the gas.
    b. The collisions between molecules in the gas are inelastic.
    c. The molecules are not spherical and can spin about their axes.
    d. The molecules have non-zero volume.

To the right is a graph of the pressure and volume of a gas in a container that has an adjustable volume. The lid of the container can be raised or lowered, and various manipulations of the container change the properties of the gas within. The points *a*, *b*, and *c* represent different stages of the gas as the container undergoes changes (for instance, the lid is raised or lowered, heat is added or taken away, etc.) The arrows represent the flow of time. Use the graph to answer the following questions.



13. Consider the change the gas undergoes as it transitions from point *b* to point *c*. What type of process is this?

    a. adiabatic
    b. isothermal
    c. isobaric
    d. isochoric
    e. entropic

14. Consider the change the gas undergoes as it transitions from point *c* to point *a*. What type of process is this?

    a. adiabatic
    b. isothermal
    c. isobaric
    d. isochoric
    e. none of the above

15. Consider the change the gas undergoes as it transitions from point *a* to point *b*. Which of the following *best* describes the type of process shown?

    a. isothermal
    b. isobaric
    c. isochoric

16. How would an isothermal process be graphed on a P-V diagram?

17. Write a scenario for what you would do to the container to make the gas within undergo the cycle described above.

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18. Calculate the average speed of N_2 molecules at room temperature (300 K). (You remember from your chemistry class how to calculate the mass (in kg) of an N_2 molecule, right?)

19. How high would the temperature of a sample of O_2 gas molecules have to be so that the average speed of the molecules would be 10% the speed of light?

20. How much pressure are you exerting on the floor when you stand on one foot? (You will need to estimate the area of your foot in square meters.)

21. Calculate the amount of force exerted on a 2 cm × 2 cm patch of your skin due to atmospheric pressure (P_0 = 101,000 Pa). Why doesn't your skin burst under this force?

22. Use the ideal gas law to estimate the number of gas molecules that fit in a typical classroom.

23. Assuming that the pressure of the atmosphere decreases exponentially as you rise in elevation according to the formula $P = P_o e^{\frac{-h}{a}}$, where P_0 is the atmospheric pressure at sea level (101,000 Pa), h is the altitude in km and a is the *scale height* of the atmosphere ($a \approx 8.4$ km).

    a. Use this formula to determine the change in pressure as you go from San Francisco to Lake Tahoe, which is at an elevation approximately 2 km above sea level.
    b. If you rise to half the scale height of Earth's atmosphere, by how much does the pressure decrease?
    c. If the pressure is half as much as on sea level, what is your elevation?

24. At Noah's Ark University the following experiment was conducted by a professor of Intelligent Design (formerly Creation Science). A rock was dropped from the roof of the Creation Science lab and, with expensive equipment, was observed to gain 100 J of internal energy. Dr. Dumm explained to his students that the law of conservation of energy required that if he put 100 J of heat into the rock, the rock would then rise to the top of the building. When this did *not* occur, the professor declared the law of conservation of energy invalid.

    a. Was the law of conservation of energy violated in this experiment, as was suggested? Explain.
    b. If the law wasn't violated, then why didn't the rock rise?

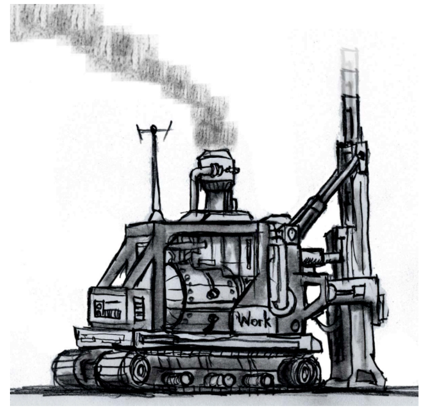
25. An instructor has an ideal monatomic helium gas sample in a closed container with a volume of 0.01 m^3, a temperature of 412 K, and a pressure of 474 kPa.

    a. Approximately how many gas atoms are there in the container?
    b. Calculate the mass of the individual gas atoms.
    c. Calculate the speed of a typical gas atom in the container.
    d. The container is heated to 647 K. What is the new gas pressure?
    e. While keeping the sample at constant temperature, enough gas is allowed to escape to decrease the pressure by half. How many gas atoms are there now?
    f. Is this number half the number from part (a)? Why or why not?
    g. The closed container is now compressed isothermally so that the pressure rises to its original pressure. What is the new volume of the container?
    h. Sketch this process on a P-V diagram.
    i. Sketch cubes with volumes corresponding to the old and new volumes.

26. A famous and picturesque dam, 80 m high, releases 24,000 kg of water a second. The water turns a turbine that generates electricity.

    a. What is the dam's maximum power output? Assume that all the gravitational potential energy of the water is converted into electrical energy.
    b. If the turbine only operates at 30% efficiency, what is the power output?
    c. How many Joules of heat are exhausted into the atmosphere due to the plant's inefficiency?

27. A heat engine operates at a temperature of 650 K. The work output is used to drive a pile driver, which is a machine that picks things up and drops them. Heat is then exhausted into the atmosphere, which has a temperature of 300 K.



   a. What is the ideal efficiency of this engine?
   b. The engine drives a 1200 kg weight by lifting it 50 m in 2.5 sec. What is the engine's power output?
   c. If the engine is operating at 50% of ideal efficiency, how much power is being consumed?
   d. How much power is exhausted?
   e. The fuel the engine uses is rated at 2.7×10^6 J/kg. How many kg of fuel are used in one hour?

28. Calculate the ideal efficiencies of the following sci-fi heat engines:

   a. A nuclear power plant on the moon. The ambient temperature on the moon is 15 K. Heat input from radioactive decay heats the working steam to a temperature of 975 K.
   b. A heat exchanger in a secret underground lake. The exchanger operates between the bottom of a lake, where the temperature is 4 C, and the top, where the temperature is 13 C.
   c. A refrigerator in your dorm room at Mars University. The interior temperature is 282 K; the back of the fridge heats up to 320 K.

29. How much external work can be done by a gas when it expands from 0.003 m^3 to 0.04 m^3 in volume under a constant pressure of 400 kPa? Can you give a practical example of such work?

30. In the above problem, recalculate the work done if the pressure linearly decreases from 400 kPa to 250 kPa under the same expansion. Hint: use a PV diagram and find the area under the line.

31. One mole ($N = 6.02\times10^{23}$) of an ideal gas is moved through the following states as part of a heat engine. The engine moves from state A to state B to state C, and then back again.

State	*Volume (m^3)*	*Pressure (atm)*	*Temperature (K)*
A	0.01	0.60	
B	0.01	0.25	
C	0.02	0.25	

   a. Draw a P-V diagram.
   b. Determine the temperatures in states A, B, and C and then fill out the table.
   c. Determine the type of process the system undergoes when transitioning from A to B and from B to C. (That is, decide for each if it is isobaric, isochoric, isothermal, or adiabatic.)
   d. During which transitions, if any, is the gas doing work on the outside world? During which transitions, if any, is work being done on the gas?
   e. What is the amount of net work being done by this gas?

32. A sample of gas is used to drive a piston and do work. Here's how it works:

The gas starts out at standard atmospheric pressure and temperature. The lid of the gas container is locked by a pin.

The gas pressure is increased isochorically through a spigot to twice that of atmospheric pressure.

The locking pin is removed and the gas is allowed to expand isobarically to twice its volume, lifting up a weight. The spigot continues to add gas to the cylinder during this process to keep the pressure constant.

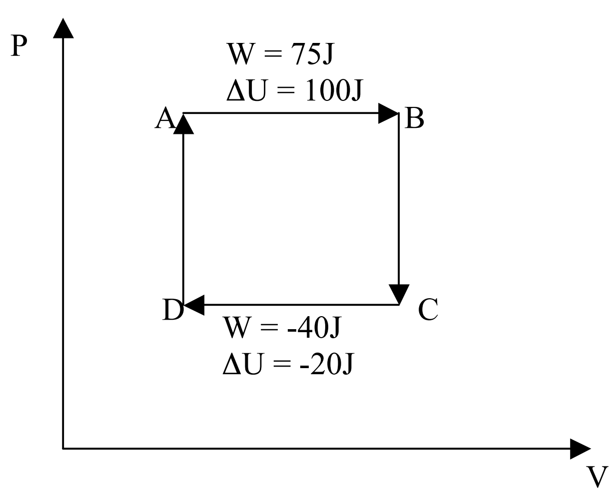
Once the expansion has finished, the spigot is released, the high-pressure gas is allowed to escape, and the sample settles back to 1 *atm*.

Finally, the lid of the container is pushed back down. As the volume decreases, gas is allowed to escape through the spigot, maintaining a pressure of 1 atm. At the end, the pin is locked again and the process restarts.

a. Draw the above steps on a P-V diagram.
b. Calculate the highest and lowest temperatures of the gas.

33. A heat engine operates through 4 cycles according to the PV diagram sketched below. Starting at the top left vertex they are labeled clockwise as follows: a, b, c, and d.

a. From a-b the work is 75 J and the change in internal energy is 100 J; find the net heat.
b. From the a-c the change in internal energy is -20 J. Find the net heat from b-c.
c. From c-d the work is -40 J. Find the net heat from c-d-a.
d. Find the net work over the complete 4 cycles.
e. The change in internal energy from b-c-d is -180 J Find:
    i. the net heat from c-d
    ii. the change in internal energy from d-a
    iii. the net heat from d-a



34. A 0.1 mole of an ideal gas is taken from state A by an isochoric process to state B then to state C by an isobaric process. It goes from state C to D by a process that is linear on a PV diagram then back to A by an isobaric process. The volumes and pressures of the states are given below:

state	Volume in $m^3 \times 10^{-3}$	Pressure in $N/m^2 \times 10^5$
A	1.04	2.50
B	1.04	4.00
C	1.25	4.00
D	1.50	2.50

a. Find the temperature of the 4 states
b. Draw a PV diagram of the process
c. Find the work done in each of the four processes
d. Find the net work of the engine through a complete cycle
e. If 75 J of heat is exhausted in D-A and A-B and C-D are adiabatic, how much heat is inputted in B-C?
f. What is the efficiency of the engine?