

The Big Ideas

Einstein believed that the laws of physics do not depend on the how fast you are moving through space, that every *reference frame* sees the same world of physics. In other words, if you are on a moving train and drop a ball or if you are standing on a farm and drop a ball, the physics that describe the motion of that ball will be the same. Einstein realized that the speed of light, c , should depend only on the laws of physics that describe light as electromagnetic radiation. Therefore, Einstein made the bold assertion that light always travels at the same speed, *no matter how fast you are moving with respect to the source of light*. Consider for a moment how counterintuitive this concept really is. This is the theoretical underpinning of Einstein's theory of Special Relativity, one of the most successfully predictive theories of physics every formulated.

The most important consequence of this new understanding is that our intuition that time moves at the same rate for everyone (whether standing still or moving at a fast speed) is WRONG. In fact, the rate at which time passes depends on your speed. Since Einstein's work in the early part of the 20th century, this fact has been demonstrated many times by experiments in particle accelerators and through the use of atomic clocks aboard fast moving jet airplanes. The effect is only noticeable at extremely fast speeds, thus the normal laws of motion apply in all but the most extreme cases.

Einstein was finally led to believe that the very fabric of space and time must have a more active and influential role in the laws of physics than had previously been believed. Eventually, Einstein became convinced that gravity itself amounted to no more than a curvature in *spacetime*. This theory is called General Relativity.

Key Concepts

- The speed of light will always be measured to be the same (about 3×10^8 m/s) *regardless* of your motion towards or away from the source of light.
- In order for this bizarre fact to be true, we must reconsider what we mean by 'space,' 'time,' and related concepts, such as the concept of 'simultaneous' events. (Events which are seen as simultaneous by one observer might appear to occur at different times to an observer moving with a different velocity. Note that both observers see the same laws of physics, just a different sequence of events.)

- Clocks moving towards or away from you run more slowly, and objects moving towards or away from you shrink in length. These are known as Lorentz time dilation and length contraction; both are real, measured properties of the universe we live in.
- If matter is compressed highly enough, the curvature of spacetime becomes so intense that a black hole forms. Within a certain distance of a black hole, called an *event horizon*, nothing can escape the intense curvature, not even light. No events which occur within the horizon can have any effect, ever, on events which occur outside the horizon.

Key Equations

- $\beta = v / c$

An object moving with speed v has a dimensionless speed β calculated by dividing the speed v by the speed of light ($c = 3 \times 10^8$ m/s). $0 \leq \beta \leq 1$.

- $\gamma = 1 / \sqrt{1 - \beta^2}$

The dimensionless Lorentz “gamma” factor γ can be calculated from the speed, and tells you how much time dilation or length contraction there is. $1 \leq \gamma \leq \infty$.

Object	Speed (km/sec)	$\beta = v / c$	Lorentz γ Factor
Commercial Airplane	0.25	8×10^{-7}	1.00000000000
Space Shuttle	7.8	3×10^{-5}	1.00000000034
UFO ☺	150,000	0.5	1.15
Electron at the Stanford Linear Accelerator	~300,000	0.9999999995	~100,000

- $L' = L / \gamma$

If you see an object of length L moving towards you at a Lorentz gamma factor γ , it will appear shortened (contracted) in the direction of motion to new length L' .

- $T' = \gamma T$

If a moving object experiences some event which takes a period of time T (say, the amount of time between two heart beats), and the object is moving towards or away from you with Lorentz gamma factor γ , the period of time T' measured by you will appear longer.

- $R_s = 2GM/c^2$

The radius of the spherical event horizon of a black hole is determined by the mass of the black hole and fundamental constants. A typical black hole radius is about 3 km.

- $m = m_0 \gamma$

The mass of an object moving at relativistic speeds increases by a factor of γ .

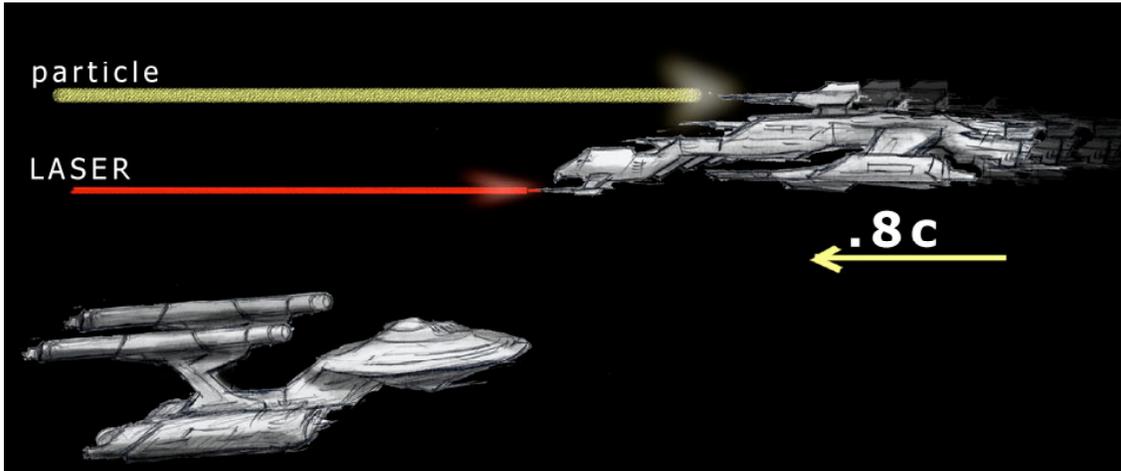
- $E = (m - m_0)c^2$

The potential energy of mass is equal to mass times the speed of light squared.

Special and General Relativity Problem Set

1. A subatomic particle moves at a speed close to the speed of light. Do you think the lifetime of this particle would be *longer* or *shorter* than if the particle were at rest?
2. What would be the Lorentz gamma factor γ for a space ship traveling at the speed of light c ? If you were in this space ship, how wide would the universe look to you?
3. Suppose your identical twin blasted into space in a space ship and is traveling at a speed of $0.100c$. Your twin performs an experiment which he clocks at 76.0 minutes. You observe this experiment through a powerful telescope; what duration does the experiment have according to your clock? Now the opposite happens and you do the 76.0 minute experiment. How long does the traveling twin think the experiment lasted?
4. An electron is moving to the east at a speed of 1.800×10^7 m/s. What is its dimensionless speed β ? What is the Lorentz gamma factor γ ?
5. What is the speed v of a particle that has a Lorentz gamma factor $\gamma = 1.05$?
6. How fast would you have to drive in your car in order to make the road 50% shorter through Lorentz contraction?
7. The muon particle (μ^-) has a half-life of 2.20×10^{-6} s. Most of these particles are produced in the atmosphere, a good 5-20 km above Earth, yet we see them all the time in our detectors here on Earth. In this problem you will find out how it is possible that these particles make it all the way to Earth with such a short lifetime?
 - a. Calculate how far muons could travel before half decayed, without using relativity and assuming a speed of $0.999c$ (i.e. 99.9% of the speed of light)
 - b. Now calculate γ , for this muon.
 - c. Calculate its 'relativistic' half-life.
 - d. Now calculate the distance before half decayed using relativistic half-life and express it in kilometers. (This has been observed experimentally. This first experimental verification of time dilation was performed by Bruno Rossi at Mt. Evans, Colorado in 1939.)
8. Calculate the radius of the event horizon of a super-massive black hole (SMBH) with a mass 200,000,000 times the mass of our Sun. (you will have to look up the mass of the Sun in kg.)
9. If an electron were "really" a black hole, what would the radius of its event horizon be? Is this a measurable size?
10. An alien spaceship moves past Earth at a speed of $.15c$ with respect to Earth. The alien clock ticks off 0.30 seconds between two events on the spaceship. What will earthbound observers determine the time interval to be?
11. In 1987 light reached our telescopes from a supernova that occurred in a near-by galaxy 160,000 light years away. A huge burst of neutrinos preceded the light emission and reached Earth almost two hours ahead of the light. It was calculated that the neutrinos in that journey lost only 13 minutes of their lead time over the light.

- a. What was the ratio of the speed of the neutrinos to that of light?
 - b. Calculate how much space was Lorentz-contracted from the point of view of the neutrino.
 - c. Suppose you could travel in a spaceship at that speed to that galaxy and back. If that were to occur the Earth would be 320,000 years older. How much would you have aged?
12. An electron moves in an accelerator at 95% the speed of light. Calculate the relativistic mass of the electron?



13. Enterprise crew members notice that a passing Klingon ship moving $.8c$ with respect to them is engaged in weapons testing on board. At point of closest contact the Klingons are testing two weapons: one is a laser, which in their frame moves at c ; the other is a particle gun, which shoots particles at $.6c$ in the Klingon frame. Both weapons are pointed in the same line as the Klingon ship is moving. Answer the following two questions choosing one of the following options:
- A. $V < .6c$
 - B. $.6c < V < .8c$
 - C. $.8c < V < c$
 - D. $c < V < 1.4c$
 - E. $V > 1.4c$
 - F. $V = c$
 - a. Question 1: What speed, V , does the Enterprise measure the laser gun to be with respect to the Enterprise?
 - b. Question 2: What speed, V , does the Enterprise measure the particle gun to achieve with respect to the Enterprise?
14. How much energy is produced by a $.5$ kilogram softball?
15. The isotope of silicon Si^{31} has an atomic mass of 30.975362 amu. It can go through beta radioactivity, producing P^{31} with a mass of 30.973762 amu.
- a. Calculate the total energy of the beta particle emitted, assuming the P^{31} nucleus remains at rest relative to the Si^{31} nucleus after emission.
 - b. Another possibility for this isotope is the emission of a gamma ray of energy 1.2662 Mev. How much kinetic energy would the P^{31} nucleus gain?
 - c. What is the frequency and wavelength of the gamma ray?
 - d. What is the rebound velocity of the P^{31} nucleus in the case of gamma ray emission?