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A Silicon Telescope for Applications in Nanodosimetry

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1 Abstract

Blah!!!!
2 Introduction

Cancer tumors are made up of cells which replicate out of control. The obvious and trivial solution to curing cancer is to stop the cancer cells from replicating—the problem is in not destroying the patient in the process. Cells replicate using information stored in DNA. If the DNA in a cancer cell is destroyed or severely damaged, the cell will be unable to replicate. One way to destroy DNA is to ionize it using radiation. Several types of ionizing radiation are being used in cancer therapy today, including x-rays, gamma-rays, electrons, neutrons, protons, and heavy ions. Protons are currently being used by Loma Linda University Medical Center’s Proton Treatment Center (LLUMC). One major problem with using radiation to destroy cancer cells is that it damages healthy tissue as well. It is relatively easy to cause cancer in normal cells while treating a patient with radiation. Scientists there want to know exactly how protons interact with DNA in the body, so that they can minimize the amount of radiation needed to safely treat a tumor. Since one easily observe these interactions between DNA and protons, one must develop a model that approximates the interaction. Researchers at LLUMC and at the Weizmann Institute of Science in Rehovat, Israel, have attempted to do just this. The technique that they have invented is called Nanodosimetry. Nano- for the nanometer scale, and -dosimetry, for studying the effects of radiation on tissue. They accomplish this by simulating DNA using a wall-less gas volume at .001 atm. To accomplish this in a gas, ions are collected by an aperture, which limits the acceptance. This aperture creates a very small cylindrical "sensitive volume", about a millimeter in diameter and 6 centimeters high. This sensitive volume is then impinged with fast protons. Ions created by the protons in the sensitive volume are collected by an ion counter, and a scintillator gives the number of the impinging particles (see Fig. 2).

One should ask, "Why should this work? How do you know the scaling between ions of gas and ions of DNA?" The answer to this is in the findings of Bethe and Bloch. Bethe and Bloch related how much energy is lost in a medium to how fast the proton is moving. This measurement, sometimes called "LET", is given in units of \( \frac{MeV}{g/cm^2} \). The denominator, \( \frac{g}{cm^2} \), is the amount of mass seen by an incoming proton per unit area. This can be a non-intuitive way to think about it, so it can also be thought of as the density, \( \rho \), times the distance traveled through, \( l \). When the energy deposited by the proton
Figure 1: Diagram of original Nanodosimetric method. An ion counter collects ions created in the sensitive volume. Protons are detected using a scintillator downstream of the sensitive volume.

is negligible compared to the energy itself, $\frac{dE}{dx}$ is effectively constant. This is the case with nanodosimetry. Nanodosimetry is concerned with energy deposition, not with energy. To find deposited energy, nanodosimetry needs a measurement of $\frac{dE}{dx}$, sometimes called Linear Energy Transfer (LET).

$$E_{dep} = \frac{dE}{dx} \cdot \rho \cdot \Delta l$$

(1)

If $\rho$ is lowered by a factor of $10^3$ by going from a solid to a gas at 1 atm, and again lowered by a factor of $10^3$ by going from 1 atm to 0.001 atm, then $l$ will increase by a factor of $10^6$.

$$\rho_{DNA} \cdot l_{DNA} = \frac{\rho_{DNA}}{10^6} \cdot (l_{DNA} \cdot 10^6)$$

(2)

This means that a proton traveling through 1 mm of low pressure gas will deposit the same amount of energy as it would going through 1 nm of DNA.
The LET, the way that protons deposit energy in matter, allows us to scale densities of the same material. A shrewd observer might ask how one makes DNA gas. The answer is that one does not. If we look at the makeup of DNA, it bears a very close resemblance to propane, both in organic makeup and in ionization energies.

The LET and track of a specific proton are critical pieces of information for researchers to have when trying to understand how radiation affects DNA. It takes longer for a molecule ionized 5 cm away from the aperture to be collected than for a molecule ionized 1 cm away. This function, called “time of arrival”, is well known. If various proton tracks are known, the different ionization clusters can be attributed to specific tracks by using the temporal separation between them, caused by the differences in time of arrival. In addition, ions created farther from the aperture stand a higher chance of not being collected than ones created very near to the aperture. If the location of the ionizations is known from the proton track, the number of ionizations collected can be normalized to the number produced, which is obviously the more meaningful piece of information.

If two very fine “grids” could be made to identify the particle track, one in front and one in back of the sensitive volume, one of the two missing pieces of information would be given. But LET of each particle is also needed. This can be determined by measuring the energy deposited in the “grid“. If the LET is known, the amount of energy deposited in the propane is known, and the relationship between LET and ionization can be mapped. These “grids” are actually devices known as silicon strip detectors(SSD’s), and have been incorporated into a “silicon telescope”, developed by the Santa Cruz Institute for Particle Physics(SCIPP), based on technology developed for the Gamma Ray Large Area Space Telescope(GLAST).
Figure 2: A schematic diagram of the nanodosimeter based on single-ion counting. Only ions formed in the sensitive volume (shaded yellow) are collected by the ion counter.

Shown in Fig. 2 is a schematic of the Nanodosimeter with the Silicon Telescope. Impinging protons, shown as a red arrow, travel first through the upstream module of the telescope, then through the sensitive volume, shown in yellow, and finally through the downstream module of the Telescope. Ions are collected through a 1 mm aperture into an ion counter. A sample readout of six ions is shown in the lower right-hand corner. The Silicon Telescope gives the number of particles, their trajectory, and their LET. This information about the particle track is used to correlate ionizations to specific proton tracks.
Figure 3: Mechanical Drawing of the Nanodosimeter Vessel. Protons travel through the horizontal tube, impinging first the upstream ST module, then the sensitive volume, and finally the downstream ST module as they exit. Ions created by the protons in the sensitive volume are electrically drawn through a 1 mm. aperture in the vertical tube.

Silicon strip detectors consist of hundreds of very thin and long diodes. When the diodes are back-biased and a particle impinges one of them, a small amount of current flows across the diode junction. The amount of current depends on the energy of the proton. If this current is measured, one can deduce the energy of the impinging particle. One also knows which strip the particle passed through, since that is the one which had current flowing in it. By orienting two SSD’s at 90 degrees (with respect to the normal of the SSD), one can get a measurement of position in a plane, and two measurements of the particle’s energy (one from each SSD). The silicon telescope, designed and constructed by SCIPP, consists of two modules of two detectors each (Fig. 4).

After construction, the modules needed to be tested to see if they could detect differences in LET and position. One way to perform this test is to impinge mono-energetic protons on a target, and detect the protons downstream with one module. The target used for the test was an aluminum annulus, 5 cm deep, 1.5 cm outer radius, and .125 ” inner radius. The degree to which the module can reconstruct the target will measure its ability to detect proton energy and position. This thesis will demonstrate the theory behind the modules, and explain how well they perform under testing.
Figure 4: A completed module. Two modules are used in the ND vessel, each of which give x, y, and two measurements of the proton’s energy. Only the X detector is visible—the Y detector is behind it, rotated 90 degrees. The front end chips are on the left edge of the circuit board. They are issued commands and read out data to the controller chip, located in the lower central part of the circuit board.
3 Methods of Energy Measurement in SSD’s

3.1 LET

The way in which protons lose energy in a material as a function of energy is given by the formula of Bethe and Bloch, shown below in Fig. 5. In the region shown, \( \frac{dE}{dX} \propto E^{-0.758} \). The units of \( X \), on the y-axis, appear strange, but they are for a good reason. \((g/cm^2) = \langle pl \rangle\), where \( p \) is density and \( l \) is the distance traveled through. \( X \) can be thought of as the amount of mass per unit area that an incoming particle “sees” as it impinges on a target. The more energy a proton has, the less energy it deposits, and as it slows down, it loses energy more and more rapidly until it stops. This is the trait that makes protons so attractive for use in cancer therapy. One can send a high energy proton into a patient’s head and do little damage to the surrounding tissue while depositing much of the energy (as the proton slows down) in a tumor in the center.

Figure 5: NIST \( \frac{dE}{dX} \) for Silicon.
3.2 Predicting Energy Loss in a Material

One cannot look up the function of energy as a function of distance, E(x), because it depends on the material, the thickness, and the initial energy. However, LET is known, and can be used to numerically compute E(x):

$$E(x) = E_0 - \sum_{i=1}^{i=N} \left( \frac{dE}{dX} (E_{i-1}) \cdot \rho \cdot \Delta x_i \right)$$

$x_{tot}$ is the thickness of the material, and $\Delta x$ is a small fraction of that thickness. Thus, a prediction of how much energy protons will lose in the Al can be made. For example, the annulus to be imaged is 5 cm thick, and the proton beam is 111 MeV. Using this method, the outgoing energy should be 50 MeV.

3.3 Using LET to Measure Proton Energy

As stated above, a proton deposits energy as it passes through matter. In a back-biased semiconductor junction, this energy liberates electron-hole pairs. The amount of energy needed to liberate a pair in a depleted (no free electrons or holes) SSD is 3.4 eV [6]. These liberated charges are collected at the detector terminals, which translate into a pulse of current. The amount of energy deposited is given by:

$$E_{dep} = Q_{meas} \cdot \frac{3.4 eV}{q}$$

and Edep is also

$$\Delta E = \frac{dE}{dX} \cdot \rho \cdot \Delta l$$

the LET of the impinging particle is

$$\frac{dE}{dX_{meas}} = \frac{Q_{lib} \cdot 3.4}{\rho l}$$
Figure 6: Impinging particles liberate e-h pairs in depleted silicon. Liberated electrons and holes are collected at the detector terminals, creating current.

Since we know that \( \frac{dE}{dX} \) in silicon is (Fig. 5)

\[
\frac{dE}{dX} = 193 \cdot E^{-0.78}
\]

We can measure the energy of the proton:

\[
E_{\text{meas}} = \left( \frac{193}{dE/dX} \right)^{0.78}
\]

One has to be careful when using LET to find energy. The error on energy, \( \sigma_E \), diverges at high energy, because

\[
\sigma_E = \sigma \frac{dE}{dX} \cdot \frac{dE}{dX} \quad \text{and} \quad \frac{dE}{dX} \to 0 \text{ as } E \to \infty
\]

Normally one cannot make the assumption, as above, that \( \frac{dE}{dX} \) is constant, but in the case of SSD’s, which are low density and very thin, the proton energy is in a range of nearly constant \( \frac{dE}{dX} \), the approximation is quite accurate. For example, a 150 MeV proton (\( \frac{dE}{dX} = 4.526 \text{--see} [9] \)) would deposit

\[
4.526 \frac{MeV \cdot cm^2}{gm} \cdot (2.32) \frac{gm}{cm^3} \cdot (.03)cm = .351 MeV
\]
in a 300 um thick detector, creating a

\[
351000\text{eV} \cdot \frac{1\text{electron}}{3.4\text{eV}} \cdot \frac{1\text{fC}}{6242\text{electrons}} = 16.5\text{fC}
\]

(11)
pulse of charge. The energy lost (0.35 MeV) is negligible compared with the energy of the proton (150 MeV); therefore SSD's are a very transparent way of measuring proton energy in this range.

### 3.4 Position Measurement Using SSD's

When a proton impinges the center of a strip, nearly all of the liberated charge is collected by that strip. When a proton impinges between two strips, they share the charge depending on the location of the proton's track. In the current implementation of the hardware, it is very difficult to calculate the energy of a particle if two strips (sometimes three) share the charge. Since it is easy to generate many protons, we are able to select only those protons which deposit most of their energy in one strip. Assuming only single strip events are selected, and due to the geometry of the detector, the resolution of position is \( \frac{194\mu\text{m}}{\sqrt{12}} \) [8]. As stated previously, when two detectors are placed at 90 degrees to one another, the x-y position is known, as long as only single particle, single strip events, are selected.
4 Experiment

The completed Silicon Telescope modules need to be tested to confirm that they can detect both position and energy of protons with great precision. One way to test these capabilities is by imaging an object using one module. The module is placed 1 cm downstream from an aluminum target in a beam of mono-energetic protons (Fig. 4). Protons that interact with the aluminum target lose energy due to LET. Protons that do not interact with the aluminum should lose no energy, and are used to measure the average beam energy. The energies of the interacting and non-interacting protons can be used to find the thickness of the aluminum (see Section 3.2). This seems like a fairly simple task, with the exception of one factor. When the protons interact with the aluminum, they are continually being scattered off of the Al atoms. Due to this random scattering, they do not all exit the aluminum still going in the direction that they entered with. Instead, they leave in a random direction. To understand the macroscopic effect that multiple scattering has on the image, Monte-Carlo simulations of the experiment must be performed. Thus, it is beyond the scope of this paper to discuss multiple scattering at any quantitative level. Nonetheless, provided the distance between the target and the detector is small, this effect should be small (see [6]). If the detector is not very close to the annulus, the energy information will still exist, but the image will be diffuse, reducing the position resolution.
4.1 Readout Electronics

For the readout of the silicon detector signals, we are using a low-noise, low-power front-end Application Specific Integrated Circuit (ASIC) Glast Tracker Front End (GTFE) developed for the GLAST mission. The GTFE has 64 channels, one for each strip (there are 5 chips on one detector). The amplified and shaped analog signal from the detector is compared with a threshold voltage, which then constitutes the output of the individual strip’s electronics. The GTFE is a binary chip, with a threshold able to be set for every channel, and has a fast output of the time over threshold (TOT), used for LET measurement. The threshold is set such that noise and minor charge sharing between strips are not mis-interpreted as events. The TOT is an OR of all the channel outputs. The five GTFEs on one silicon detector are read out via a digital controller ASIC Glast Tracker Readout Controller (GTRC) into a computer.

![Diagram of the readout electronics](image)

Figure 9: The charge liberated by the proton creates a voltage on the capacitor. If the output voltage of the shaper is greater than \( V_{\text{thresh}} \), the OR is TRUE.

Self-triggering is accomplished through an OR of the output of all channels on one detector—as soon as one channel’s signal goes above the threshold the system records the length of the TOT and the address of the hit strip. The GTRC also allows digitization of the TOT, yielding a measurement of the input charge through the pulse width, i.e. the TOT signal, over a large dynamic range. One of the major strengths of this system is that it can accept a huge pulse, causing the strip signal to be pinned to the supply voltage, but can still accurately measure the charge, because TOT depends only on pulse width, not height. One weakness of this system is that all
events with significant charge sharing have to be rejected, because the TOT is no longer representative of the charge deposited—two strips that share $Q$ charge might generate the same TOT as one strip with $\frac{Q}{2}$.

![Diagram of TOT output](image)

Figure 10: Each strip in the detector has its own preamplifier and shaper, the output of which is compared with a threshold voltage. The outputs of the channels are ORed to form the TOT signal. The length of time that the OR is TRUE for is the TOT(Time Over Threshold). The TOT can be used to determine the energy of the proton.

### 4.2 Data Acquisition

The user programs an AND of coincidences that determine an event. For instance, if the first, second, and third detector all have FAST-OR’s that are TRUE, it is very likely that a particle has made a track through them. If the first and third go true, something anomalous is occurring, because the supposed particle didn’t interact with the middle detector. Therefore the AND of the FAST-OR’s determines the validity of the event.

If the event is recognized as being valid, the DAQ sends a command to the controller chip to freeze the data taking on all of the front end chips. The controller chip reads out which channels have registered hits. It then waits for the DAQ to send a TOKEN, a special bit pattern which signals the controller to read out its data. After having received the TOKEN, the controller sends the channel numbers that have been hit on its board. It then passes the TOKEN to the backside controller, which has a nearly identical
configuration, except that its strips are perpendicular to the front side (with respect to the normal of the SSD’s plane). When the back side controller receives the token from the front side, it reads out its data serially to the front side controller. The front side controller then sends the data to the DAQ. Data is written to a binary file in the PC. The data consists of the number of hits, the strip I.D. and the TOT for each of the planes.
5 Energy Calibration Results and Analysis

In order to calibrate the energy measurement component of the ST, we exposed a prototype module to the beam of the proton synchrotron at Loma Linda University Medical Center. Mono-energetic beam energies of 250 MeV and 40 MeV were selected. Beams of lower proton energies were produced by degrading the proton beam with polystyrene absorbers. In addition, we have analyzed the data taken in a 13.5 GeV proton beam during the GLAST beam test in January 2000, using identical Si detectors and front-end ASIC’s. Figure 11 shows TOT spectra for single hits in the LLUMC data.

![TOT Spectra for low-energy Protons](image)

Figure 11: TOT spectra for low energy protons.

These distributions allow us to find the mean and FWHM of the deposited charge for different incident proton energies[6, 7]. Table 1 shows the data derived from the LLUMC runs and the SLAC beam test, as well as the expected input charge for 400 μm silicon, derived using GEANT 4. The data agreement is excellent, with all measured quantities being within error of the expected.

The mean measured and calculated TOT values vs. the energy of the primary protons are shown in Fig. 12. The RMS of the TOT spectra are shown as error bars. The agreement between measured and predicted TOT values is excellent over the entire energy range from 20 -250 MeV.
<table>
<thead>
<tr>
<th>Proton Energy MeV</th>
<th>TOT Expected us</th>
<th>Mean TOT Measured us</th>
<th>RMS TOT us</th>
<th>$Q_{\text{dep}}$ (400 $\mu$m Si) fC</th>
</tr>
</thead>
<tbody>
<tr>
<td>13.500</td>
<td>6.5</td>
<td>7</td>
<td>1.4</td>
<td>5.3</td>
</tr>
<tr>
<td>250</td>
<td>13.7</td>
<td>12.3</td>
<td>2.6</td>
<td>13.5</td>
</tr>
<tr>
<td>39</td>
<td>55</td>
<td>53.4</td>
<td>6.4</td>
<td>54</td>
</tr>
<tr>
<td>27</td>
<td>69</td>
<td>70.4</td>
<td>7.5</td>
<td>67.5</td>
</tr>
<tr>
<td>24</td>
<td>78</td>
<td>78.3</td>
<td>8.5</td>
<td>76.5</td>
</tr>
<tr>
<td>22</td>
<td>82</td>
<td>84.4</td>
<td>9.8</td>
<td>81</td>
</tr>
<tr>
<td>17.6</td>
<td>101</td>
<td>105</td>
<td>11.5</td>
<td>99</td>
</tr>
<tr>
<td>9.5</td>
<td>105</td>
<td>108</td>
<td>15</td>
<td>189</td>
</tr>
<tr>
<td>7.4</td>
<td>105</td>
<td>109</td>
<td>21</td>
<td>243</td>
</tr>
</tbody>
</table>

Table 1: Measured and expected values of TOT for several energies of protons.

Figure 12: Mean TOT as a function of Energy. The RMS of the TOT distributions are shown as error bars.

The ASIC voltages Vanalog and threshold settings Vthr were slightly different for the two set-ups (LLUMC and SLAC), resulting in slightly different TOT calibration curves, which were taken into consideration in the data analysis (see Fig. ??). (LLUMC data: Vanalog = 1.92V, Vthr = 5fC; SLAC
data $V_{\text{analog}} = 2.3\,\text{V}$, $V_{\text{thr}} = 1.4\,\text{fC}$).

Figure 13: The slope of the calibration curve varies with analog supply voltage. Two different voltages were used by LLUMC and SLAC, resulting in two slightly different gain curves.

### 5.1 Calculation of Energy Resolution

The energy resolution of the system $\sigma_E$ can be derived from the observed TOT RMS $\sigma_{TOT}$ and the derivative of the TOT vs. energy curve (Fig. 12):

$$\sigma_E = \frac{dE}{dTOT} \cdot \sigma_{TOT} \tag{12}$$

Fig. 14 shows the relative width of the measured TOT spectra $\sigma_{TOT}$ and the energy resolution $\sigma_E$. Below 40 MeV, the energy resolution of the detector is on the order of 15%, and increases to 25% at 250 MeV. While $\sigma_E$ worsens above 250 MeV, $\sigma_{\text{LET}}$ stays approximately constant. Therefore, we still have very accurate information about energy deposition.
Figure 14: Energy resolution, $\sigma_E$, and TOT RMS, $\sigma_{TOT}$ as a function of energy.

5.2 Spatial Calibration: Analysis of Charge Sharing

As stated previously, neighboring strips “share” charge, meaning that the charge liberated by an impinging proton does not generally go only to one strip or another, but rather is distributed between strips. For an event to be used, a large enough percentage of the charge must go to one strip such that no other strips receive enough charge to generate a signal above $V_{thresh}$.

Fig. 15 shows the TOT spectrum from the SLAC beam test for all events, for those with one hit, two adjacent hits and three and more adjacent hits, respectively.
6  IMAGE ANALYSIS

![Image of a graph showing charge sharing for 13.5 GeV protons in the GLAST Beam Test.]

Figure 15: Charge sharing for 13.5 GeV protons in the GLAST Beam Test.

6  Image Analysis

Using the data from LLU and SLAC, the function relating proton energy to tot was computed. Next the intensity and profile of the beam were computed. Next, a histogram of the energy as a function of x and y was made. From this, an area of the image was cut on, and projected into a 1d histogram, showing the cross-section of the annulus. Using this cross-section, measurements of the ST’s position and energy resolution were made. In an effort to improve this resolution, the energy was plotted as a function of distance from the center of the annulus. This method succeeded in improving the resolution.

6.1  Image of the Annulus

Now that energy and spatial calibration have been completed, and a suitable target defined, imaging data can be collected and analyzed. A curve was fit to the earlier energy calibration data, yielding (Fig. 16):

\[
E_{\text{meas}} = \left( \frac{965}{\text{TOT}} \right)^{1.26}
\]  

(13)
The analysis package ROOT was used to analyze the data. Fiducial cuts were made around the area of interest in order to improve contrast. A strip hit cut was also made, requiring single hits on both planes. The image was created by filling two histograms in ROOT. The first histogram recorded the beam intensity. Fig. 17 shows the first histogram. It yields information about both the beam profile and the effects of multiple scattering (note the focusing effect of the hole, and the depletion effect of protons scattering out of the outside edge).

The second histogram recorded the sum of the energy measurements for each bin. This histogram is clearly not a good measure of anything on its own, since the magnitude is subject to the intensity of the beam. But when we divide the second histogram by the first, we compute the average energy for each pixel. Fig. 6.1 shows the average \( \frac{E_1 + E_2}{2} \) energy in MeV measured by the ST module. The areas in white are places with no hits, and correspond to the bench that the annulus was resting on.

Figure 16: A fit of energy calibration data. This function is used to convert TOT to energy.
Figure 17: Intensity of proton beam. Each pixel is 2 x 2 strips, corresponding to an area of 0.150 mm$^2$, since each strip is 194 μm wide.

Figure 18: Image in MeV of the Al annulus.
Figure 19: Image of annulus in TOT and MeV

Fig. 6.1 is an image in MeV of the annulus. A comparison of the image in TOT with that in MeV can give a qualitative feel for the relationship between the two (Fig. 6.1):

### 6.2 1D projection of Annulus

While the images above yield a great deal of qualitative information about both the beam and the ST, it is useful to quantify both the energy and spatial resolution of the ST. Since the annulus is radially symmetric, we can take a slice of the image and project it onto a 1 dimensional histogram. There will be some inherent distortion from projecting a chord onto a plane, but if the slice is sufficiently small the distortion from multiple scattering should dominate. In order to ensure that the slice went through the geometric center of the annulus, another slice was taken perpendicular to the first. Lines were then fit to that slice which outlined the energy shape of the annulus. The intersections of these lines mark points of symmetry around the geometric center that are insensitive to the exact location of the cut. Fig. 6.2 shows the slice and the lines fit to it.
Figure 20: Slice of id.2 projected into id.1. The locations of the intersecting lines were used to find the center of the annulus in id.1.

<table>
<thead>
<tr>
<th>Description</th>
<th>x constant</th>
<th>$\sigma_c$</th>
<th>slope</th>
<th>$\sigma_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>left line</td>
<td>-256</td>
<td>NA</td>
<td>2.75</td>
<td>NA</td>
</tr>
<tr>
<td>right line</td>
<td>641</td>
<td>NA</td>
<td>-3.46</td>
<td>NA</td>
</tr>
<tr>
<td>offset</td>
<td>54.42</td>
<td>1.609</td>
<td>NA</td>
<td>NA</td>
</tr>
</tbody>
</table>

Table 2: Data used to find center of annulus in id.1.

Using the data above, the intersections between the two lines and the offset were computed. The error on these intersections is given by:

$$\sigma_x = \frac{\sigma_y}{\text{slope}}$$

(14)

The total error on the center is:

$$\sigma_{x_{\text{tot}}} = \sqrt{\sigma_l^2 + \sigma_r^2}$$

(15)

Fig. [?] shows the slice taken from the 3D image:

This slice was then projected onto id.2 (the x-axis). The size of the pixels was kept constant (4x4).
Table 3: calculation of center of annulus in id_1.

<table>
<thead>
<tr>
<th>Description</th>
<th>Location</th>
<th>σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>left</td>
<td>113.18</td>
<td>.585</td>
</tr>
<tr>
<td>right</td>
<td>169.33</td>
<td>.465</td>
</tr>
<tr>
<td>center</td>
<td>141.25</td>
<td>.75</td>
</tr>
</tbody>
</table>

Figure 21: Black lines indicate the fiducial cut made on the annulus image. This area is collapsed to a 1D histogram for use in determining energy and position resolution.

[?] is a 1D histogram of energy against strip ID. The effect of multiple scattering is shown very clearly here in the blurring of the walls of the annulus.

Using the same fitting technique as used to find the center strip, lines were fit to the new histogram, shown in Fig. ??.

Shown below in table 4 are the calculated points of intersection. Simple algebra gives the intersections of the lines with the low energy platform. The error of the intersection is the error on the platform divided by the slope. Since protons will always scatter away from the aluminum, we can use these intersections to make a measurement of the inner and outer diameter of the annulus:
Figure 22: 1D projection of y strips 139-142 onto the x axis.

Figure 23: fit of 1D projection of y strips 139-142 onto the x axis.
<table>
<thead>
<tr>
<th>Description</th>
<th>Location</th>
<th>( \sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>outer left bottom</td>
<td>99.06</td>
<td>.89</td>
</tr>
<tr>
<td>inner left bottom</td>
<td>130.03</td>
<td>1.10</td>
</tr>
<tr>
<td>near right bottom</td>
<td>200.81</td>
<td>.81</td>
</tr>
<tr>
<td>outer right bottom</td>
<td>233.30</td>
<td>.625</td>
</tr>
</tbody>
</table>

Table 4: calculations of points in annulus.

Using the data collected in table 4, the inner and outer diameter of the annulus can be calculated (table 6.2).

<table>
<thead>
<tr>
<th>Location</th>
<th>measured value</th>
<th>( \sigma )</th>
<th>expected value</th>
</tr>
</thead>
<tbody>
<tr>
<td>outer diameter (cm)</td>
<td>2.585</td>
<td>.021</td>
<td>3.00</td>
</tr>
<tr>
<td>inner diameter (cm)</td>
<td>1.37</td>
<td>.004</td>
<td>.635</td>
</tr>
</tbody>
</table>

Table 5: calculations of the annulus dimensions. The error is the quadrature sum of the errors on the left and right sides of the respective diameter.

In an effort to improve on these results, the data was replotted in terms of distance from the center of the annulus. In addition to the cuts used in previous histograms, there was a cut made limiting the radius to the largest value that did not correspond to an arc outside of the active area of the detector. The data from tables 3 and 5 were used to find the x-y center. Fig. 6.2 shows the radial histogram with fitted lines.

Shown below in table 6 are the calculated annulus dimensions using the radial histogram.

<table>
<thead>
<tr>
<th>Description</th>
<th>Location</th>
<th>( \sigma )</th>
<th>expected</th>
</tr>
</thead>
<tbody>
<tr>
<td>inner diameter (cm)</td>
<td>1.17</td>
<td>.018</td>
<td>.635</td>
</tr>
<tr>
<td>outer diameter (cm)</td>
<td>2.80</td>
<td>.016</td>
<td>3.0</td>
</tr>
</tbody>
</table>

Table 6: calculations of points in annulus.

These measurements are closer to the real dimensions of the annulus. \( \sigma_x \) is the error on the fit—not the resolution of the system. To find the resolution,
Figure 24: Energy as a function of distance from the center of the annulus. Note the larger slope, more linear rise, and overall better definition of the image.

The physical limit on the precision of measurement, one needs to calculate the distance in x that is needed to rise from 12% to 88% on a vertical line. The inner and outer diameters of the annulus are to first order perpendicular to the beam, and were used as the reference lines in this measurement. Shown in Table 7 are the data and resolution for the two slopes in the radial histogram.

<table>
<thead>
<tr>
<th>Description</th>
<th>$\Delta (88% - 12%)$</th>
<th>$\sigma_\Delta$</th>
<th>slope</th>
<th>resolution (mm)</th>
<th>$\sigma_{res}$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>inner slope</td>
<td>45.17</td>
<td>2.31</td>
<td>2.78</td>
<td>3.15</td>
<td>.16</td>
</tr>
<tr>
<td>outer slope</td>
<td>45.17</td>
<td>2.31</td>
<td>1.87</td>
<td>2.70</td>
<td>.14</td>
</tr>
</tbody>
</table>

Table 7: calculations of resolution for radial histogram

The total error on the diameters is given by the quadrature sum of all both the resolution and the errors on the fitted corners. Since the resolution is a full order of magnitude greater than the errors on the corners, the resolution is the only significant contributor to the error. Shown below in Table 9 are the measured and expected values of the diameters of the annulus.
<table>
<thead>
<tr>
<th>Description</th>
<th>expected radius (cm)</th>
<th>measured radius (cm)</th>
<th>$\sigma_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>inner slope</td>
<td>.318</td>
<td>.315</td>
<td>.05</td>
</tr>
<tr>
<td>outer slope</td>
<td>1.5</td>
<td>1.39</td>
<td>.14</td>
</tr>
</tbody>
</table>

Table 8: Measured and expected values for annulus radii.

The measured values are within error of the expected. Multiple scattering is clearly the dominant factor in resolving imaged objects.

### 6.3 Thickness Analysis

The two energy levels found by the radial histogram can be used to compute the thickness of the annulus, as discussed in Section 3.2.

<table>
<thead>
<tr>
<th>Description</th>
<th>Energy</th>
<th>$\sigma_E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Energy (MeV)</td>
<td>111.7</td>
<td>1.87</td>
</tr>
<tr>
<td>Final Energy</td>
<td>52.3</td>
<td>1.4</td>
</tr>
<tr>
<td>Thickness (cm)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 9: Computed thickness using change in energy.
7 Conclusion

Blah
References


