

Space-like Singularities and Thermalization

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ABSTRACT: We conjecture that space-like singularities are simply regions in which all available degrees of freedom are excited, and the system cycles randomly through generic quantum states in its Hilbert space. There is no simple geometric description of the interior of such a region, but if it is embedded in a semi-classical space-time an external observer sees it as a black hole. Big Bang and Crunch singularities, for which there is no such embedding, must be described in purely quantum terms. We present several possible descriptions of such cosmologies.

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String Theory has provided a resolution of a variety of time-like singularities[1]. The case of space-like singularities, relevant for the interior of black holes, and for cosmology, has proven far less tractable. In this essay we want to suggest a general physical picture of space-like singularities, which may provide a basis for more rigorous understanding.

The basic claim we would like to make is that a generic future space-like classical singularity, corresponds in the quantum theory to a situation in which all of the available physical degrees of freedom associated with the region of space-time in which the singularity occurs, are maximally excited, and brought into a state resembling thermal equilibrium.

Let us begin by examining the case of a black hole in asymptotically flat or AdS space-time. From the point of view of an external observer, this is a thermal system, with finite entropy. The internal geometry of the black hole is singular, and has the form

$$ds^2 = -\frac{dt^2}{(\frac{t}{R_S} - 1 + \frac{t^2}{R^2})} + dr^2 \left(\frac{t}{R_S} - 1 + \frac{t^2}{R^2} \right) + r^2 d\Omega^2.$$

Small fluctuation analysis leads to the conclusion that this geometry is unstable near the singularity. We will consider two ways of analyzing the non-linear evolution of the instability. In the first, we model a fluctuation by the behavior of two mass points following geodesics of the unperturbed metric. Define the center of mass energy, \sqrt{s} , of these two mass points by the norm of the sum of the momentum of the first, with the parallel transport of the momentum of the second, to the position of the first. The Schwarzschild radius corresponding to this energy always exceeds the impact parameter

of the two geodesics, as the singularity is approached. Thus, the internal observer witnesses multiple black hole formation, which it interprets as thermalization of the degrees of freedom that it can observe. It is not hard to imagine that, *from the interior point of view*, all the excitations are swept up into black holes as the singularity is approached.

The other analysis, which leads to similar conclusions, is that of [2]. These authors showed that solutions of Einstein's equations near a generic space-like singularity are chaotic. The general principles of statistical mechanics again lead us to suppose that the entire interior is thermalized. Indeed, a more general argument that suggests this conclusion, is that with any definition of energy far from the singularity, the time dependence of the singular solutions should excite states of arbitrarily high energy. Most systems we know of in quantum mechanics have a huge degeneracy of high energy states, and no quantum numbers which prevent the system from exploring this entire degenerate subspace, starting from generic initial conditions with a large expectation value of the energy. Our basic hypothesis in this paper is that any internal description of the states of a black hole will have this property.

Indeed, for many future space-like singularities, like the interior of a black hole formed through collapse, or the Coleman-De Luccia Crunch for negative c.c. , the covariant entropy bound tells us that observers which eventually encounter the singular part of space-time, can only detect a finite entropy. Let us interpret this as a bound on the total number of states necessary to describe the quantum mechanics of such an observer. Then the interior dynamics is described by a quantum system with time dependent Hamiltonian $H(t)$ which becomes singular at some t we choose to call $t = 0$. Let $N \gg 1$ be the number of states of the system. Consider the Lie algebra generated by the collection of $H(t)$ at different times. If the Hilbert space is in a reducible representation of this algebra, then the system has conservation laws. We do not expect this to be true near a space-like singularity.

If the algebra is $su(N)$, the time evolution operator $U(t)$ must cycle randomly through $SU(N)$ as $t \rightarrow 0$, because the $SU(N)$ group is compact. The state of the system thus explores the entire Hilbert space as $t \rightarrow 0$. If it is some proper subalgebra of $su(N)$, introduce the coherent states

$$|\omega^a\rangle = e^{i\omega^a T_a} |\psi\rangle,$$

where $|\psi\rangle$ is any state in the representation. Let H be the stability subgroup of $|\psi|^1$. The inequivalent values of the parameters ω^a are coordinates on the compact coset manifold G/H . For any choice of $|\psi\rangle$ the coherent states are an overcomplete

¹ H is a proper subgroup of G , because the representation is irreducible.

basis. of the Hilbert space. Our singular time evolution operator corresponds to a space-filling trajectory on the compact manifold G/H . So again, we conclude that the system explores its entire Hilbert space as the singularity is approached.

In conventional statistical physics, which deals primarily with systems which have an energy conservation law, thermal systems are those which explore their entire Hilbert space, subject to the constraint of fixed energy. Time averages in a thermal system are the same as averages over the micro-canonical ensemble. Similarly, quantum systems with a finite number of states and a singular time dependent Hamiltonian of the type described above will have time averages equivalent to ensemble averages over the maximally uncertain density matrix. We will continue to use the words thermal and thermalization for such systems, even though they have no energy conservation law.

Notice that quantum mechanics, and the assumption of a finite number of states, regularizes singularities without removing them. As the singularity is approached, the rapidity with which the system cycles through all of its states increases without bound. Thus, if we define time averages with some fixed scale, we will find that they become time independent and equal to ensemble averages in the maximally uncertain density matrix. This is not a particularly interesting system, but after time averaging it seems to give a perfectly well defined limit.

In many situations, we can argue that the number of states of the system is finite at a space-like singularity by using our refined version of the covariant entropy bound[3] in which the area of causal diamonds bounds the actual number of states rather than the entropy of some unspecified density matrix. This is the only version of the bound which would seem to make sense in a general time dependent situation.

These heuristic arguments lead to a resolution of the black hole information paradox. If they are correct, then both internal and external observations lead to the expectation of complete thermalization of the degrees of freedom of the black hole. The heuristic picture of black holes formed by collisions inside the black hole suggests that the internal observer does not really experience the stretching and shrinking of the interior Schwarzschild geometry. Rather it experiences a chaotic excitation of all of its degrees of freedom. We believe that semi-classical space-time pictures are not an appropriate description of the interior, which is better described by the random quantum mechanics of a finite system.

For large AdS black holes, this thermalization conjecture is enough to make the internal and external observers' descriptions compatible with each other. The internal observer cannot survive long enough to see a small black hole decay², but it can see

²This is a coordinate dependent statement, since there exist *nice slice* coordinates on which the local curvature is everywhere small, and much of the Hawing radiation has already passed observers

the thermalization which leads to the decay.

1. Thermalization in the Big Crunch

Horowitz and Polchinski[5] were the first to study black holes formed by excitations of a model singular universe: the null orbifold. Banks and Fischler [6] extended these arguments to infinite flat FRW crunches, and suggested that in this case the black hole formation evolved to a *dense black hole fluid*, a state of matter/space-time with equation of state $p = \rho$ and maximal entropy in any causal diamond. These arguments were heuristic, but suggestive.

A much more detailed picture of the relation between Big Crunch space-times and thermalization, was achieved in recent work of Hertog and Horowitz[7] (HH). These authors analyzed the Big Crunch that appears in the Lorentzian continuation of the Coleman De Lucia[8] (CDL) bubble describing the decay of an AdS space. There are many peculiar aspects to this situation, not least of which is that the decaying AdS space is supersymmetric and one would have thought it was stable. HH explain this by noting that the CDL solution does not satisfy the normalizability condition for solutions which are associated with states in the Hilbert space of the stable supersymmetric AdS theory. Rather, it corresponds to a change in the Hamiltonian of the boundary field theory. The addition to the Hamiltonian is the integral of a marginal triple trace operator which is unbounded from below. The CDL solution, with no additional perturbations, does not correspond to a sensible quantum theory.

However, HH speculate that one can stabilize the boundary field theory by adding what are known technically as *dangerous irrelevant operators*. That is, the superficially renormalizable, but unbounded from below, theory is obtained as the infra-red limit of a stable ultraviolet theory. If Φ is the dimension one, single trace operator whose cube couples to the CDL solution, then HH propose a boundary effective potential of the form

$$V(\Phi) = R^{-2}\Phi^2 + a\Phi^3 + \frac{b}{M}(\Phi^4) + \dots$$

The mass scale $M \gg R^{-1}$ is the scale of the stable UV theory which provides the definition of this system. It has a stable vacuum with massive excitations of scale M , at a non-zero value of Φ . The claim of HH is that this same theory, when compactified on a sphere of radius R has a metastable state, with energy at some scale of order M ,

located large but finite distances from the horizon. However, as pointed out by some of the authors of [4], the internal time resolution on such slices is super-Planckian and such an observer must use the full non-local apparatus of quantum gravity to describe the system.

above the ground state, whose low-lying excitations at energy $\ll M$ (above the meta-stable ground state), are described by the low lying excitations of the superconformal field theory. The meta-stable state decays, with a rate approximately given by the CDL calculation, into a highly excited state of the massive field theory. Quite generically, we expect the energy produced in this decay to thermalize. From the boundary field theory point of view, the endpoint of the decay is a thermal state, at a temperature of order M , of the massive field theory. Note that this state has a finite entropy, of order $(MR)^3$, because the boundary field theory lives on a sphere of radius R .

This seems to be a completely sensible and general description of the meaning of an AdS decay into a Big Crunch, from the point of view of boundary field theory. The final state may not have a good bulk space-time description, but it is a sensible state in a well-defined boundary field theory. From our present point of view, what is important about this result is that the Big Crunch represents a complete thermalization of the degrees of freedom previously associated with the AdS space-time.

From the bulk point of view, there is one peculiarity of the boundary description. In the regime where the CDL calculation is valid, the decay rate is exponentially small (as a function of the AdS radius in Planck units). Thus, before the decay occurs, an observer in the meta-stable vacuum can bounce photons off the boundary of AdS space and verify that it is living in an infinite space-time. However, most of the high energy black hole states that one would associate with this infinite space-time, do not exist. When the black hole energy becomes of order the effective potential barrier, black holes can decay in times of order R^{-1} . Thus, the meaning of the phrase "asymptotically AdS space-time" is ambiguous for this system. Gentle exploration of the geometry by test particles, over times short compared to the decay time, reveal what appears to be a space-time described by this phrase, but does not guarantee the existence of the full set of CFT states that we have come to associate with such a geometry. Most of the high energy states do not exist. This is obvious from the boundary field theory point of view, and could presumably be reproduced from the bulk point of view, by computing the CDL transition rate for AdS black holes as a function of the black hole mass. Most of the true high energy states of this system are those of the massive boundary field theory and probably do not have any useful bulk space-time description³.

One may even wonder about the degree of universality of the HH proposal for the UV completion of the field theory corresponding to the CDL space-time. General

³General arguments suggest that the UV entropy of the fixed point theory defining the massive boundary correlations should be larger than that of the SUSY theory. This would imply that if it had an AdS dual, that dual would have smaller c.c. than the SUSY theory. However, it would then be hard to understand the CDL instanton as a space-time process. It seems more likely that, like free field theory, this UV fixed point does not have an AdS dual.

renormalization group arguments would suggest that it is not very unique. Many UV theories have the same IR behavior. Before coming to this conclusion, it would be well to understand better the one peculiar feature of the HH proposal from the boundary field theory point of view. The HH proposal is not the conventional story of RG flow from a UV to an IR fixed point. The low energy excitations whose behavior is governed by the CFT dual to the original AdS space-time, are excitations of a highly excited meta-stable state of the massive theory. Furthermore this state only exists when the theory is compactified on a sphere. If we study the massive boundary theory in flat Euclidean space, it has a unique vacuum with a correlation length of order M^{-1} . RG flow goes from the UV fixed point (possibly Gaussian) and a trivial fixed point. One must ask whether a generic UV theory which flows to the same IR behavior in flat space will have a meta-stable state on the sphere with the required properties.

The answer to this question is quite unclear. To answer it, it would be good to have a soluble example of a boundary field theory with such a compactification induced meta-stable conformal state.

The HH proposal is the most rigorous argument we know that the correct description of (at least certain) Big Crunches is in terms of thermalization of available degrees of freedom.

2. The Big Bang

The space-like singularities discussed above all occur in the future. They are described in terms of an organization of the states of the theory in terms of some Hamiltonian, with the assumption that the initial state has fairly low entropy. This cannot be the description of the Big Bang singularity if we want a theory which contains the concept of a particle horizon, and/or explains how the universe began in a state of low entropy, in such a way that the cosmological and thermodynamic arrows of time point in the same direction.

The authors of [9]constructed a general formalism for constructing quantum cosmologies consistent with the existence of a particle horizon and the holographic entropy bound. Then, using the BKL results on chaotic behavior and the Problem of Time as motivation, they argued that a given observer near the Big Bang should be described by a random sequence of Hamiltonians. The variables of the system are fermionic oscillators, which were interpreted as pixels of the holographic screen of the observer's causal diamond. If the probability distribution from which the Hamiltonian is chosen, is concentrated in the neighborhood⁴ of Hamiltonians quadratic in the fermions, then

⁴Here neighborhood is understood in the sense of the renormalization group. The large fermion

the system obeyed all the rules of quantum cosmology, and exhibited the scaling laws of the flat FRW universe with equation of state $p = \rho$.

The $p = \rho$ universe is thus a generic initial state for a quantum system obeying the rules of [9]. In this sense it is a generalization of the completely thermalized states we encountered in our description of future space-like singularities. Only those degrees of freedom within the particle horizon are thermalized at any given time. There cannot be any real observers in a $p = \rho$ universe⁵, because all of the degrees of freedom describing any causal diamond are collapsed into a large black hole. The idea of holographic cosmology is that one should choose the initial state in as generic a way as possible, consistent with the existence of real observers. The earlier papers of [9] give a heuristic picture of what this maximally entropic but *observaphilic* universe looks like, designed to convince the reader that it resembles what we see around us. In particular, the flatness, homogeneity and horizon problems are explained in terms of properties of the $p = \rho$ universe, and the requirement that the observaphilic portion of the universe be stable against “decay” back to the more entropic $p = \rho$ state. In order to explain the existence of CMB correlations on our current horizon scale, as well as the fact that fluctuations of all scales begin their sub-horizon oscillations with zero velocity[10], one must invoke a period of inflation.

Holographic cosmology provides the only known clues to the question of why the observable part of the universe began in a low entropy initial state. It identifies the generic initial condition as one in which all parts of the universe are in continuous interaction, and no isolated observers are possible. The initial conditions for the observable universe are supposed to be those of maximal entropy, consistent with the existence of observers. The authors of [9] claim that this implies a flat FRW universe, whose energy density is initially dominated by an almost uniform distribution of black holes. Given these initial conditions, and the existence of scalar fields dynamically capable of giving slow roll inflation, inflation is a reasonably probable outcome. There is no other comparably robust explanation of why inflation began.

In particular, the oft-repeated statement that inflation only needs special initial conditions in a small patch of the universe, which then expands to become all we see, begs the question. In quantum field theory, the expanded patch contains many more

limit of random quadratic fermion systems is the massless $1 + 1$ dimensional Dirac equation. For large N the probability distribution must be concentrated in the RG basin of attraction of this fixed point theory.

⁵We use the word observer in a very general sense. It is a quantum system with many semiclassical observables (like a large volume , cutoff, quantum field theory), which can be isolated from the rest of the universe, and interact with it in a controlled manner. Observers need have neither gender, nor consciousness.

degrees of freedom than the original one. At the initial time, these degrees of freedom are not well described by effective field theory, so one cannot make a reliable estimate of the probability of inflationary initial conditions without a better understanding of quantum gravity⁶. The claim of holographic cosmology is that inflation is not generic, but that the most generic observaphilic initial conditions (which already explain the homogeneity and flatness problems, without inflation) also predict a high probability for inflation to begin if the dynamical equations permit it.

From the point of view of the present paper, holographic cosmology gives a prescription for past space like singularities which similar to, but different than our description of future space like singularities. In constructing holographic cosmology we insisted on incorporating the concept of particle horizon, which allows degrees of freedom to interact only after “they have come into causal contact”.

3. Cyclic Universe?

Various attempts to build cosmologies from the dynamics of string theory moduli[11], have reopened the question of whether the universe could have gone through one or more cycles of Big Crunch followed by Big Bang. If the picture of space-like singularities proposed in this paper is correct then the answer to this question is: *probably not in any useful sense*. The Big Crunch would be described by a thermalization of all degrees of freedom in the universe. There could not be any smooth transition to a low entropy FRW initial condition.

Rather, the only way one could imagine moving from the Big Crunch to such a low entropy state is through a rare thermal fluctuation. This assumes that the entropy of the Big Crunch state is finite. The HH model described above gives a very concrete (though unrealistic) example of how this works. The Big Crunch leads to a finite entropy thermal density matrix of excitations of the true vacuum of the boundary field theory. By the principle of detailed balance, there is a small probability for this state to tunnel thermally to the meta-stable AdS state, after which the whole Big Crunch story can begin again. This does describe a sort of cyclic universe, which spends most of its time in a state which has no bulk space-time description, accompanied by rare and brief sojourns in a smooth space-time.

There is no sense in which the state of the system during the Crunch time gives useful information about initial conditions in AdS space. By their nature, the AdS fluctuations are rare, and therefore singular (in the colloquial rather than mathematical

⁶In situations more controlled than the early Universe, one knows that the attempt to create inflation in a small patch, leads instead to the formation of black holes, with microscopic entropy.

sense). Indeed, the inverse tunneling event can equally well lead to any normalizable low energy excited state of the AdS theory. The energy will thermalize around the AdS vacuum, and the whole system will eventually decay into a thermal state of the true vacuum. Since the meta-stable AdS state is (in cases where semi-classical calculations are valid), highly excited from the point of view of the true vacuum, the finite entropy of the state it decays into will be large. In that case, the initial AdS configuration “the next time around” is effectively unpredictable. This is not the kind of cyclic universe that has been hypothesized in recent string inspired models.

An interesting question that arises is whether our own universe might be describable as a rare fluctuation of a thermal state with some fixed Hamiltonian. Hypotheses like this probably go back to Boltzmann. This hypothesis has recently been explored in the context of a hypothetical quantum theory of stable dS space[12]. The conclusion of that study seems quite general. If we hypothesize that the origin of the universe as a fluctuation then we conclude that the state just after the fluctuation had maximal entropy compatible with the existence of observers. The authors of [12] and most people who have thought about this problem, claim that this principle is in contradiction with observation. For example, increase the entropy in such a way that the current microwave temperature is ten times its observed value. It is then unlikely, but possible, for nuclei to have survived photo-dissociation, in such a way that stars produced a galactic environment conducive to our type of carbon based life. The probability for this to occur is of order 10^{-n} with some relatively small value of n , but because we have increased the entropy of the initial conditions by a factor of 10^4 , there are $e^{10^4} 10^{-n}$ more ways to find life arising in the ensemble of such universes, than in our own. Our universe does not look like a generic fluctuation, constrained only by the existence of observers.

Although this argument sounds persuasive, we feel that the question deserves more study.

4. Conclusions

The hypothesis that the meaning of space-like singularities is maximal thermalization of the available degrees of freedom, helps to resolve the black hole information puzzle. Under this hypothesis, the exterior and interior descriptions of the black hole state coincide for a time: both sorts of observer see a thermalization of all interior degrees of freedom. At this point the internal observer ceases to exist, and the ultimate fate of the black hole is a concept that only applies to the external observer. Thermalization is also a plausible conclusion to a Big Crunch, and the model of [7] provides a very explicit quantum mechanical model of how this can occur.

The correct description of the beginning of the universe is less clear and there seem to be two plausible hypotheses: holographic cosmology and the origin of the universe as a thermal fluctuation. Both could lead to a universe like our own if they could be shown to predict a high probability for a period of slow roll inflation. Holographic cosmology does claim to make such a prediction, if the degrees of freedom of the universe include a scalar field with appropriate Lagrangian. It is likely that a generic fluctuation model does not, but the fluctuation idea is very general and there could be implementations of it, which evade the “Boltzmann’s Brain” paradox of [12] and describe a universe like our own.

The interpretation of future space-like singularities in terms of thermalization does not fit well with models which envision a smooth and predictive transition between Big Bang and Big Crunch. These models require a much more controlled resolution of space-like singularities.

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