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A note on Gauge Theories Coupled to Gravity

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Abstract

We analyze the bound on gauge couplings $e \geq m/m_p$, suggested by Arkani-Hamed et.al. We show this bound can be derived from simple semi-classical considerations and holds in spacetime dimensions greater than or equal to four. Non abelian gauge symmetries seem to satisfy the bound in a trivial manner. We comment on the case of discrete symmetries and close by performing some checks for the bound in higher dimensions in the context of string theory.

1 Introduction

In the context of string theory there are by now several examples of consistent models of quantum gravity, but we are far from understanding the full catalog of such models. It is therefore of interest to formulate *general lessons* coming from the marriage of gravity and quantum mechanics. An archetypal example is Hawking's robust calculation of black hole (BH) evaporation [1]. This calculation can be formulated in purely semiclassical terms, and there are also cases where it can be derived as a rigorous consequence of string theory. Another, somewhat less rigorous, idea is the holographic principle [2, 3, 4].

On a more solid footing is the general result that the only allowed continuous global symmetries in quantum gravity are asymptotic gauge transformations including asymptotic diffeomorphisms of space-times with symmetric asymptotics. This statement can be derived from a general semiclassical argument based on black holes, which will be reviewed below and is supported by arguments from perturbative string theory [5] and from AdS/CFT [6].

Recently, Arkani-Hamed et.al. [7] (AMNV) suggested a general bound governing any consistent theory of p-form gauge fields coupled to gravity (the case $p = 0$ was previously studied by Banks et.al in [8]). Their suggestion, which was motivated in part by holography and in part by experience from string theory, can be rephrased in several ways. Take a model that includes GR in $3+1$ asymptotically flat space-time dimensions, as well as a $U(1)$ gauge field with coupling e. In effective field theory, e and Newton's constant G are independent. In particular, from the traditional point of view of effective field theory, if we assume that e is small enough so that the Landau pole is above the Planck scale, we would generally assume a cutoff of order m_p. Arkani-Hamed et.al. claim that

1. There has to be a light charged particle satisfying

$$m < e \cdot m_p. \tag{1.1}$$

2. The effective gauge theory breaks down at a prematurely[1, 2] low scale $\Lambda < e \cdot m_p$.

We note that if condition 1 is strictly satisfied as an inequality, then extremal black-holes are kinematically able to decay[3]. Indeed, take an extremal BH in 4 dimensions satisfying $GM = \sqrt{G}eQ$ where M, Q are the mass and the *integral* charge and $G \sim l_p^2 = \kappa^2$ is Newton's constant. A necessary condition for such a BH to decay is the existence of a particle in the theory with a smaller mass/charge ratio[4] which if normalized to $q = 1$ gives condition 1.

[1] Condition 2 follows from condition 1 e.g. by considering the magnetic monopole as the light particle for the magnetic interaction

[2] It is perhaps worthwhile mentioning that while this suggestion seems surprising from the perspective of effective field theory, it is expected (at least qualitatively) within string theory. The reason is that both the gravitational and the gauge interactions are governed by the string coupling g_s. In perturbative string theory, the "premature" cutoff is the string scale itself.

[3] If the extremal holes are BPS states then they are marginally stable against decay. When the gauge charge in question is in the SUSY algebra, the lightest BPS state will exactly saturate the AMNV bound.

[4] The particle with smallest mass/charge ratio is exactly stable. Take a particle of charge/mass ration $\gamma = M/Q$ which decays into a bunch of particles of smaller charge $\sum q_i = Q$ and mass $\sum m_i < M$ and assume none of them have smaller mass/charge ratio $\gamma_i = m_i/q_i > \gamma$, then $\sum m_i = \sum \gamma_i q_i > \gamma Q = M$ in contradiction.

As suggested in [7], condition 1 written as

$$e > \frac{m}{m_p} \tag{1.2}$$

bounds the gauge coupling away from zero and therefore is a *generalization* of the statement that there are no continuous global symmetries in a consistent theory of quantum gravity. The logic behind this statement is that if 1.2 is not satisfied then one can draw pathological conclusions such as

- Charged BHs can posses entropies larger than their Bekenstein-Hawking entropy.
- Charged BHs can decay to Planck size remnant with very large entropies. Such particles can violate entropy bounds and potentially dominate the phase space of thermodynamic systems.

The purpose of this note is to fill in arguments left implicit by AMNV and provide a semiclassical derivation of the AMNV bound from the covariant entropy bound. The paper is structured as follows. In section 2 we review semiclassical arguments against having global symmetries[5] in a consistent theory of quantum gravity. In section 3 we review the Reissner-Nordstrom solution of Einstein-Maxwell theory and explain our strategy for extending the argument of the previous section to the case of gauge symmetries. In section 4 we discuss the semi-classical loss of charge from a RN black-hole in a way that generalizes easily to higher dimensions. In section 5 we present a semi-classical derivation of the AMNV bound in any space-time dimension bigger than or equal to 4. In section 6 we make some checks of the validity of the AMNV bound in higher dimensions, within string theory. In section 7 we make some comments on the case of non-abelian gauge symmetries and discrete symmetries. Our conclusions are in section 8.

2 Global Symmetries in QG.

Let us first remind the reader of the black hole based argument that there can be no global symmetries in a consistent theory of quantum gravity.

2.1 No-Hair

A first indication of what is going on comes from the *no hair* theorem. Local $U(1)$ symmetries obey a form of Gauss's law which enables any observer *outside* the BH horizon to determine its charge. On the other hand, if there were also strictly Global symmetries then when a charged particle is thrown into the BH, there is no way to determine this fact from the outside. It thus looks like the charge was "deleted" in contradiction to its conservation. However, at this level of analysis, one can simply assign a charge to the BH and avoid this difficulty.

[5] From now on, unless explicitly stated, we only discuss continuous symmetries.

2.2 Hawking evaporation

The real issue with global symmetries has to do with Hawking evaporation and the problem of remnants. Since for a global symmetry there is no associated gauge interaction, we can throw as many charged particles as we want into a BH, and increase its charge Q to any value. The BH clearly can not radiate any appreciable amount of charge in the Hawking radiation before $T_{Hawking} \geq m$, where m is the mass of the lightest charged particle. Demanding that at the time when $M \leq m_p^2/m$ the BH still has enough mass to be able to get rid of all its charge, we find

$$Q \leq (\frac{m_p}{m})^2 \tag{2.1}$$

This bound can be violated by taking Q large enough.

In fact, the problem is worse. The Hawking radiation is thermal and particles of opposite charge will be produced in equal numbers. The BH will thus not get rid of any charge before becoming a Planck size remnant. Such a situation leads to the following pathologies:

One can have a black hole of fixed mass M and arbitrarily[6] high charge Q. Since an external observer cannot discern the global charge, his micro-canonical ensemble of all states, regardless of charge, with fixed energy inside a fixed area, would lead us to to count microstates of all charge, and assign an entropy at least of order $\log Q$ to the BH. This can be taken arbitrarily high and contradict the Bekenstein-Hawking formula. If however, as suggested by exact degeneracy counting in string theory, we accept that the Bekenstein-Hawking entropy is a count of the number of states of a black hole, such objects are ruled out, and global continuous symmetries with them. Conversely, the assumption that the BH entropy alone is a measure of the number of states, can be seen as the requirement that all black holes can discharge (which in turn implies the AMNV bound).

The problem is made worse by Hawking evaporation, which in a theory with an unbounded global charge would lead to an infinite number of Planck mass remnants. The charged Planck size remnants are particles that can be confined to a small box and their entropy bounded by something much smaller than the Bekenstein Hawking entropy of the initial black hole. Since these remnants can have any charge Q, the entropy associated with the remnant in the box should be infinite.

Alternatively, we can view the infinite degeneracy associated with the global charge Q as a variant on the species problem. Such particles may dominate the computation of any low energy matrix element of a product of two operators. This is not necessarily the case if there is an additional form factor, analogous to the form factor suppression of the production of monopoles in gauge theory. Susskind [9] appears to sidestep this objection by considering the production of remnants in the thermal bath of Rindler space, where the production of a species is fixed by its mass alone. Said differently, one cannot make the existence of an infinite number of stable low mass remnants consistent with the assumption that Rindler or black hole horizons are thermal systems.

A crucial feature of all these arguments is the fact that the hypothetical global charge is unobservable from the exterior, so that measurements are made on ensembles of states with uncertain charge q. In some sense, this feature changes discontinuously when we turn on a

[6] If the charge is not high enough, throw another charged particle in and wait for the BH to evaporate the excess mass.

coupling between the conserved charge and a gauge field. However there is a clear sense in which there is not in fact a discontinuity. Measurements of charge are done by performing scattering experiments and are ultimately limited by the energy and momentum resolution of detectors. As the gauge coupling is taken to zero for fixed resolution, larger and larger integer charges will become unobservable. Roughly speaking, a change of charge will have to be $o(1/e)$ in order to be detectable. (Recall that for electromagnetism in the real world, $\frac{1}{e} \sim 3$). In our discussion of entropy below, we will be referring to ensembles with charge uncertainty of this order.

3 Black-holes and $U(1)$ gauge symmetries.

The Classical Einstein-Maxwell theory

$$\mathcal{S} = \frac{1}{16\pi G}\int \sqrt{g}\mathcal{R} - \frac{1}{4e^2}\int \sqrt{g}\mathcal{F}^2 \tag{3.1}$$

has the Reissner-Nordstrom (RN) solution of an electrically charged black hole in 4 dimensions

$$\begin{aligned} ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_2^2, \qquad A = \frac{e^2 Q}{r} \\ \text{with} \quad \mathrm{f(r)} = 1 - \frac{2GM}{r} + \frac{Ge^2Q^2}{r^2} \quad \text{and} \quad \mathcal{F} = \mathrm{dA}. \end{aligned} \tag{3.2}$$

Here M is the ADM mass and Q is the integral charge quantized in units of e. The BH (outer) horizon lies at

$$r_+ = GM + \sqrt{(GM)^2 - Ge^2Q^2}. \tag{3.3}$$

This solution is generally believed to make sense physically (i.e. obey Cosmic Censorship) only when

$$(GM)^2 \geq Ge^2Q^2 \quad \Leftrightarrow \quad \boxed{M \geq eQm_p} \tag{3.4}$$

where we have been cavalier about order one coefficients in denoting $m_p \sim 1/\sqrt{G}$. Notice that the horizon is always of order M, namely

$$GM \leq r_+ \leq 2GM. \tag{3.5}$$

The extremal RN BH saturates the inequality in Eq. 3.4. This extremality bound can be understood physically by neglecting the effects of gravity, and finding the energy stored in the electric field resulting from bringing a charge Q in from infinity to $r = r_+ \sim GM$

$$E_Q = \frac{1}{e^2}\int_0^{r_+} \frac{(\frac{e^2Q}{\frac{4}{3}\pi r_+^3})\frac{4}{3}\pi r^3 \cdot (\frac{e^2Q}{\frac{4}{3}\pi r_+^3})4\pi r^2 dr}{r} \sim \frac{e^2Q^2}{GM}. \tag{3.6}$$

We now require that the actual BHs mass should at least account for this energy

$$M \geq \frac{e^2Q^2}{GM} \quad \Rightarrow \quad M \geq eQm_p, \tag{3.7}$$

and thus recover Eq. 3.4.

The extremality bound Eq. 3.4 applies for any non-zero gauge coupling e and reflects the existence of repulsive gauge forces between particles with the same sign of charge. The existence of this bound clearly goes in the "right direction" of avoiding the problems associated with global symmetries because one can not increase the charge of the BH without also increasing its mass.

3.1 The basic strategy

To begin the argument for the AMNV bound, we assume the BH starts with some mass and charge obeying the extremality bound Eq. 3.4, and follow the spontaneous loss of charge by the BH. We make no special assumptions about how this black hole was created or the absolute value of its mass and charge. The discharge of the black hole is exponentially suppressed below a certain threshold that depends on the parameters of the BH. Below this threshold the BH can always lose mass via Hawking radiation of massless particles but will discharge very little. We then demand that at the threshold the BH still has enough mass to be able to radiate away all of its charge, namely, to account at least for the rest mass of Q of the lightest charged particles:

$$M_{discharge} \geq Q \cdot m_{light} \tag{3.8}$$

Even though we can track the evolution of the black hole reliably only down to some mass scale of order m_p, the point here is that energy and charge conservation allow us no conceivable way to evaporate the black hole if Eq. 3.8 is not satisfied. This is a rephrasing of the criterion used in [7] that we want to allow even extremal BHs to decay.

Otherwise, we run into essentially the same set of problems listed in section 2 for global symmetries.

- Thinking in a grandcanonical ensemble, the energy band between $(M, M + \delta M)$ with $\delta M \ll M$ contains a huge number of black holes with the integral charge allowed by Eq. 3.4 to be any number $Q = 1, \ldots, \frac{1}{e}$ leading to an entropy of order $\mathcal{S} \sim -\log e$. By taking the gauge coupling e to be too small (e.g. take $e \sim 1/Q$) this entropy can be much bigger than the Bekenstein-Hawking entropy which is bounded (using Eq. 3.4) by $\log(eQ)$.

- We can have Planck size ($M \simeq m_p$) remnants with charge given by any number $Q = 1, \ldots, \frac{1}{e}$, Again, the entropy one would associate with a system made up of putting such a particle in a box can be dialed to contradict any entropy bound by taking e too small.

- The species argument continues to work, where instead of an infinite number of species we only have a finite but very large $\sim 1/e$ number of species.

Notice that for very massive objects, where we can safely use the classical Lagrangian 3.1, we can recover 1.1 by insisting, as suggested in [7], that extremal BHs can decay, so the extremal mass should obey 3.8, namely

$$M_{ext} = Ge^2Q^2 \geq G^2(Qm)^2 \quad \Rightarrow e > \frac{m}{m_p}. \tag{3.9}$$

Following the strategy outlined in this section, we now show that when we take into account the actual quantum effects of Schwinger pair production and Hawking evaporation, responsible for any BH's discharge (not necessarily an extremal one), we obtain the same bound.

4 Semi-classical discharge of Black Holes

Gibbons [10] gave an exact analysis, using Bogoliubov transformations, for the spontaneous loss of charge by 4-dimensional Reissner-Nordstrom black holes. In this section we summarize the relevant results in a manner amenable to generalization to higher spacetime dimensions.

Assume, as we do here, that there is a lightest charged particle of mass m. The physical process responsible for discharging the hole can be heuristically understood as follows. A pair is produced outside the BH horizon and the member whose charge is opposite to the BH is attracted and falls into the BH, while the other member of the pair escapes to infinity.

It is useful to discuss two opposite regions of parameter space, that of small/hot black holes where the discharge is dominated by Hawking evaporation of charged particles and that of large/cold black holes where the discharge is dominated by Schwinger pair production [7].

4.1 Small/Hot BHs - The Hawking process

This regime is defined by

$$GM \ll 1/m \quad \Leftrightarrow \quad T_{Hawking} \gg m \tag{4.1}$$

where the Hawking temperature is larger than the mass, m, of the lightest charged particle so the latter will be thermally produced. In contrast to the global case, where the thermal nature of the radiation did not allow the hole to discharge, here the electric field outside the horizon gives a "chemical potential" term in the Boltzman factor which is asymmetric between the charges

$$\mathcal{P}(m, \pm 1) \sim e^{-\frac{1}{T}(m \mp \frac{e^2 Q}{r_+})}. \tag{4.2}$$

This favors the emission of a particle with the same sign of charge as the BH. This is crucial for the discharge of the hole, and kicks in when the contribution from the electric potential becomes of order the rest mass m, namely

$$\frac{e^2 Q}{r_+} \geq m. \tag{4.3}$$

4.2 Large/Cold BHs - The Schwinger process

This regime is defined by

$$GM \gg 1/m \quad \Leftrightarrow \quad T_{Hawking} \ll m \tag{4.4}$$

[7]The physical process of discharging the hole is really the same in both cases in the sense that both correspond to the pair production of particles of opposite charge.

where the BH temperature is not large enough to produce the lightest charged particle by the Hawking process discussed in the previous section. Instead, the dominant mechanism for discharge is Schwinger pair-production in a constant electric field[8]. While the field is not constant everywhere, we can safely approximate the electric field outside the horizon as almost constant for Large BHs. Gibbons' result in this regime predicts the following rate for charge loss due to pair-production

$$\frac{dQ}{dt} \sim e^{-\frac{\pi m^2 r_+^2}{eQ}} \tag{4.5}$$

which is exactly the dependence expected from the Schwinger process [11]. The hole starts to discharge appreciably when

$$\frac{m^2 r_+^2}{eQ} \leq 1. \tag{4.6}$$

In fact, one can guess this dependence. Ask when the electric field right outside the horizon has enough energy *in a volume of size the Compton wavelength of the lightest charged particle* to account for the creation of two of those particles from the vacuum. Since the energy density in the electric field is given by

$$\epsilon = \frac{1}{e^2} \int d^4x \vec{E}^2 \tag{4.7}$$

and in this case $|\vec{E}| = \frac{e^2 Q}{r^2}$ we can demand that

$$\frac{1}{e^2}\vec{E}^2 \cdot \lambda_c^3 \sim \frac{e^2 Q^2}{r_+^4} \cdot \frac{1}{m^3} \geq m \tag{4.8}$$

resulting in Eq. 4.6.

In the next section, we use these results to derive the AMNV bound in arbitrary spacetime dimensions.

5 Deriving the AMNV bound in $N+1$ spacetime dimensions.

Following Meyers and Perry [12], the generalization of Reissner-Nordstrom BHs to higher dimensions is given by

$$\begin{aligned} ds^2 &= -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_{N-1}^2, \\ f(r) &= 1 - \frac{16\pi GM}{(N-1)A_{N-1}r^{N-2}} + \frac{8\pi G e^2 Q^2}{A_{N-1}(N-1)(N-2)r^{2(N-2)}}, \\ A^0 &= \frac{e^2 Q}{(N-2)r^{N-2}} \quad \Rightarrow \quad F^{0r} = \frac{e^2 Q}{r^{N-1}} \end{aligned} \tag{5.1}$$

Here M, Q, e, G are as before and A_N is the area of a unit S^N.

[8] This is the regime of parameters relevant for the decay of extremal BHs which have zero temperature.

The BH (outer) horizon lies at

$$r_+^{N-2} = \frac{8\pi GM}{(N-1)A_{N-1}} + \sqrt{(\frac{8\pi GM}{(N-1)A_{N-1}})^2 - \frac{8\pi Ge^2Q^2}{A_{N-1}(N-1)(N-2)}}. \tag{5.2}$$

Neglecting order 1 coefficients in the subsequent analysis, we write

$$G \sim m_p^{1-N}. \tag{5.3}$$

Since the extremality bound is of the same form in arbitrary spacetime dimensions, the conditions for the discharge of an extremal black hole in 3 + 1 dimensions presented in Eq. 3.9 suggests that Eq. 1.1 generalizes to higher dimensional spacetimes if instead of the gauge coupling e we use the dimensionless gauge coupling

$$\tilde{e^2} \equiv e^2 m_p^{N-3}. \tag{5.4}$$

In the following sections, we will show that this is indeed the conclusion reached by considering semi-classical discharge processes[9] resulting in the generalized bound, suggested in [7]

$$\boxed{\tilde{e} \geq \frac{m}{m_p}}. \tag{5.5}$$

This discussion is restricted to space-times with $N \geq 3$ because for $N \leq 2$ there are no BHs in asymptotically flat space.

5.1 Hot BHs in $N + 1$ dimensions

Using the physical criterion of section 4.1 the BH will start to discharge thermally in an appreciable manner when the electric potential at the horizon is of order the rest mass of the lightest charged particle, $A^0_{|Horizon} \geq m$. We then obtain the condition that the horizon radius *at this time* (denoted by a tilde) is bounded by

$$\tilde{r}_+^{N-2} \leq \frac{e^2 Q}{(N-2)m}. \tag{5.6}$$

Demanding the BHs mass *at that time* be large enough to allow for a complete discharge $M_{discharge} \geq Q \cdot m_{light}$ we get

$$Qm \leq M_{discharge} \leq \frac{e^2 Q}{Gm} \quad \Rightarrow \quad \boxed{m \leq \tilde{e} m_p} \tag{5.7}$$

where we have used the relations Eq. 5.4 and Eq. 5.3 and the fact that r_+^{N-2} is of order GM as seen from Eq. 5.2. This is the bound 5.5.

[9]In principal, it could have been the case that in $N + 1$ spacetime dimensions 1.1 becomes $\tilde{e} > (\frac{m}{m_p})^{f(N)}$ where $\tilde{e}$ is the *dimensionless* coupling and $f(N)$ is some function of the space dimensions that happened to obey $f(3) = 1$.

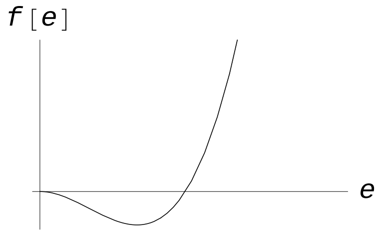

Figure 1: The qualitative behavior of the bound.

5.2 Cold BHs in $N+1$ dimensions

Here, generalizing the argument of section 4.2, the BH will start to discharge appreciably when $\frac{1}{e^2}\vec{E}^2 \cdot \lambda_c^N \geq m$ yielding the inequality

$$\frac{e^2 Q^2}{r_+^{2(N-1)}} \cdot \frac{1}{m^N} \geq m \quad \Rightarrow \quad (r_+ m_p)^{N-1} \leq \tilde{e}Q(\frac{m_p}{m})^{\frac{N+1}{2}}. \tag{5.8}$$

Rewriting 5.2 in dimensionless units and ignoring all order 1 numbers gives

$$(r_+ m_p)^{N-2} = \frac{M}{m_p} + \sqrt{(\frac{M}{m_p})^2 - \tilde{e}^2 Q^2}. \tag{5.9}$$

Therefore 5.8 reads

$$\frac{M}{m_p} + \sqrt{(\frac{M}{m_p})^2 - \tilde{e}^2 Q^2} \leq (\tilde{e}Q)^{\frac{N-2}{N-1}} (\frac{m_p}{m})^{\frac{(N+1)(N-2)}{2(N-1)}}. \tag{5.10}$$

Rearranging and squaring we get

$$(\tilde{e}Q)^{2\frac{N-2}{N-1}} (\frac{m_p}{m})^{\frac{(N+1)(N-2)}{(N-1)}} - 2(\tilde{e}Q)^{\frac{N-2}{N-1}} (\frac{m_p}{m})^{\frac{(N+1)(N-2)}{2(N-1)}} \cdot \frac{M}{m_p} + (\tilde{e}Q)^2 \geq 0. \tag{5.11}$$

Demanding again that $M \geq Qm$, and dividing the inequality by Q^2 we get

$$\tilde{e}^{2\frac{N-2}{N-1}} Q^{-\frac{2}{N-1}} (\frac{m_p}{m})^{\frac{(N+1)(N-2)}{(N-1)}} - 2\tilde{e}^{\frac{N-2}{N-1}} Q^{-\frac{1}{N-1}} (\frac{m_p}{m})^{\frac{N(N-3)}{2(N-1)}} + \tilde{e}^2 \geq 0. \tag{5.12}$$

Now note that for $N \geq 3$ the fraction $1/2 \leq \frac{N-2}{N-1} \leq 1$ and therefore the inequality has the same qualitative features as the condition $f(\tilde{e}) \geq 0$, where

$$f(\tilde{e}) = \tilde{e}^{3/2} - \tilde{e}^{3/4} + \tilde{e}^2. \tag{5.13}$$

$f(\tilde{e})$ is graphed in Figure 1. Therefore, we can get a bound by finding the non trivial zero $\tilde{e}_0$ of f, and demanding that $\tilde{e} \geq \tilde{e}_0$. It is useful to look for the strictest bound as a function of Q. We thus choose the equal sign in 5.12, which defines $\tilde{e}_0$. We then extremize with respect to Q. Because only two terms depend on Q we get the following relation for the charge $\hat{Q}$ that gives the strictest bound

$$(\widehat{Q})^{-\frac{1}{N-1}} = \tilde{e}_0^{-\frac{N-2}{N-1}} (\frac{m_p}{m})^{-\frac{N^2+N-4}{2(N-1)}}. \tag{5.14}$$

Plugged back into 5.12 this gives

$$\boxed{\tilde{e}_0^2 \geq (\frac{m_p}{m})^2}, \tag{5.15}$$

which is exactly the expected bound 5.5.

We conclude that to avoid the problems discussed in section 3, in arbitrary spacetime dimensions, one is universally faced with 5.5 as a lower bound on gauge interactions in the presence of gravity.

6 String theory consistency checks for $N > 3$

Having established the bound 5.5 in all dimensions $N \geq 3$, by a semiclassical gravitational argument, it is interesting to perform some checks of it in the context of string theory. We need to look for situations where a $U(1)$ gauge field and gravity coexist[10]. We will show agreement with the bound in the following 3 cases

- Type-I superstring compactifications.
- $D0$ branes in type IIA superstring theory.
- Dp branes in type II with a compactified worldvolume.

6.1 Type I String Theory on $\mathbb{R}^{1,9-d} \times \mathcal{M}^{(d)}$

Consider type-I superstrings compactified down to $10 - d$ spacetime dimensions on a d dimensional compact manifold of volume $V^{(d)} \sim R^d$. Suppose the compactification breaks $SO(32)$ down to a $U(1) \times \ldots$. For $R > l_s$, the lightest charged particles under this $U(1)$ subgroup are the off diagonal $SO(32)$ gauge bosons[11] whose mass is set by the KK scale R. As long as $R \gg l_s$ the mass of the lightest charged particle is $m_{light} \sim \frac{1}{R}$. When $R < l_s$ there are stringy excitations that are charged under this $U(1)$ and are lighter than the KK modes. Therefore, it is true in this case that

$$m_{light} \leq m_s. \tag{6.1}$$

Having said that we can now use the following relations

$$\begin{aligned}
m_s^4 &= g_s m_{(10)}^4 \\
m_{(10)}^8 R^d &= m_{(10-d)}^{8-d} \\
\frac{1}{g_{(10-d)}^2} &= \frac{V^{(d)}}{g_{(10)}} \\
[g_{(10-d)}]_m &= \frac{d-6}{2} \quad \Rightarrow \quad \tilde{g}_{(10-d)} = g_{(10-d)} \cdot m_{(10-d)}^{\frac{6-d}{2}} \\
g_{(10)} &\sim \sqrt{g_s} m_s^{-3}
\end{aligned} \tag{6.2}$$

[10]Note that (except for $D9$ branes) the worldvolume theory of Dp branes in superstring theory is made up only of open string modes and thus does not contain gravity.

[11]In a T-dual picture, those are string states connecting the separated $D8$ brane to the remaining stack.

where we denote by $m_{(d)}, g_{(d)}$ the Planck mass and the YM coupling in d spacetime dimensions. It is straightforward to conclude from these equations that

$$g_{(10-d)} = g_{(10)} R^{-\frac{d}{2}} = \frac{\sqrt{g_s}}{m_s^3} \cdot \frac{m_{(10)}^4}{m_{(10-d)}^{\frac{8-d}{2}}} = \frac{m_s}{\sqrt{g_s}} \frac{1}{m_{(10-d)}^{\frac{6-d}{2}}} \cdot \frac{1}{m_{(10-d)}} \tag{6.3}$$

Thus,

$$\tilde{g}_{(10-d)} \cdot m_{(10-d)} = \frac{m_s}{\sqrt{g_s}} \geq m_s > m_{light} \tag{6.4}$$

confirming the bound 5.5. These inequalities hold whenever $g_s < 1$. For $g_s > 1$ one would need to go to a dual weakly coupled picture. Note that when $R \to 0$ we have a dual picture of separated D branes. Thus, at weak coupling 5.5 holds in compactifications of type-I superstrings in arbitrary spacetime dimensions, as expected from the general analysis in section 5.

6.2 $D0$ branes in flat $\mathbb{R}^{1,9}$

$D0$ branes are the lightest charged objects under the RR 1-form potential in type IIA superstring theory. In this case

$$\begin{aligned} m_s^4 &= g_s m_{(10)}^4 \\ \frac{1}{g^2} &= \frac{1}{l_s^6} \\ [g]_m &= -3 \quad \Rightarrow \quad \tilde{g} = (\frac{m_{(10)}}{m_s})^3 \end{aligned} \tag{6.5}$$

and it is trivial to check that indeed 5.5 is obeyed

$$m_{D0} = \frac{m_s}{g_s} = m_s \cdot (\frac{m_{(10)}}{m_s})^4 = (\frac{m_{(10)}}{m_s})^3 \cdot m_{(10)} = \tilde{g} \cdot m_{(10)}. \tag{6.6}$$

In fact, this must have been true because the $D0$ branes are $\frac{1}{2}$BPS. This case gives a check of 5.5 in flat $\mathbb{R}^{1,9}$

6.3 Branes and higher dimensional p-form potentials in flat space.

It is tempting to generalize 5.5 also to the case where the $U(1)$ symmetry is mediated by a p-form gauge field with $p > 1$ so that the lightest charged objects are not "particles" (1 dimensional world volume) but "branes". One needs to figure out how exactly to generalize 5.5 to this case because branes have an infinite mass (but a finite tension). In fact, for branes the semi-classical decay arguments we used in this paper are void because, having infinite mass, *they will not be emitted by BHs at all.* One way to get a meaningful set up is to compactify the p spacelike directions along the brane's worldvolume so that it becomes an effective particle in the $8-p$ dimensional transverse space. This particle is the lightest state charged under the RR gauge field.

Let us then compactify the spacelike directions of the brane on a compact manifold of volume $V^{(p)} \sim R^p$. In this case

$$\begin{aligned} \frac{1}{g^2} &= \frac{1}{l_s^{6-p} R^p} \\ [g]_m &= \frac{p}{2} - 3 \quad \Rightarrow \quad \tilde{g} = g \cdot m_{(10-p)}^{3-\frac{p}{2}} \\ m_{(10-p)}^{8-p} &= \frac{R^p}{g_s^2 l_s^8} \\ m_{Dp} &= \frac{R^p}{g_s l_s^{p+1}} \end{aligned} \tag{6.7}$$

Therefore we write

$$\tilde{g} \cdot m_{(10-p)} = g \cdot m_{(10-p)}^{4-\frac{p}{2}} = l_s^{3-\frac{p}{2}} R^{\frac{p}{2}} \cdot \frac{R^{\frac{p}{2}}}{g_s l_s^4} = m_{Dp} \tag{6.8}$$

where again because of the BPS property we get an equality.

7 Other types of symmetries

7.1 Non abelian gauge symmetry

At first sight, the extension of the AMNV bound to the gauge coupling of Type I string theory in ten dimensions seems to fail. If we ignore the gauge supermultiplet, the lightest state charged under the gauge group $SO(32)$ is a perturbative string state. A naive application of the AMNV bound might lead us to expect that

$$m_s \leq \tilde{g}_{YM} m_p. \tag{7.1}$$

The relation between the 10 dimensional string and Planck scales is given by

$$m_s^4 = g_s m_{(10)}^4 \tag{7.2}$$

where $g_s \sim e^{\phi}$ is the closed string coupling[12]. The 10 dimensional YM coupling is irrelevant and is given by

$$g_{YM}^2 \sim g_s m_s^{-6} \quad \Rightarrow \quad \tilde{g}_{YM}^2 = \frac{1}{\sqrt{g_s}}. \tag{7.3}$$

Plugging this back into 7.2 gives

$$m_s = \frac{m_{(10)}}{\tilde{g}_{YM}} \tag{7.4}$$

which is in some sense the "opposite" of 5.5.

In fact, there is no good reason to ignore the gauge multiplet. The essence of non-abelian gauge theory is that gauge bosons are charged. In spacetime dimensions higher than four ($N > 3$ in the notations of this paper) the bound 5.5 is satisfied trivially for any non

[12] A handle costs g_s^2.

abelian gauge group[13]. In those dimensions the gauge coupling is irrelevant, and therefore the IR gauge theory is free. Thus, the massless gluons are the lightest charged particles. In the language of section 1, $m = 0$, and so any gauge coupling is allowed by the bound. Of course, in *e.g.* ten dimensional Type I string theory, there *is* a relation between the gauge and gravitational couplings, related to the low cutoff scale m_S. However, it cannot be phrased in terms of the mass of the lightest charged particle in low energy effective field theory, nor derived from the low energy arguments we have presented here.

In four spacetime dimensions many *asymptotically free gauge theories* confine and there are no global charges. The nearest we can come to an analog of the AMNV bound is the statement

$$\Lambda_{QCD} \leq m_p, \tag{7.5}$$

which we think would be accepted by any effective field theorist. Since there is no surprise here, the non-abelian analog of the AMNV bound again appears to be of little utility.

Similar remarks are valid for the other phases of four dimensional gauge theory. In a non-Abelian Coulomb phase, there are not really any particles, but certainly the mass gap in the charged sectors vanishes. In the Higgs phase there are really no charged particles either. All particles are created from the vacuum by gauge invariant operators. However, if one looks at the broken non-abelian symmetry which acts on gauge invariant states, the gauge bosons themselves have masses

$$m_W \sim e \cdot v \leq e \cdot m_P,$$

so an analog of the AMNV bound is the statement that the Higgs VEV is less than the Planck scale. Again, short of the fact that the mass of the W-boson is much lighter than the Planck mass at weak coupling, the effective field theorist encounters no surprises.

7.2 Discrete symmetries.

The problems with entropy considerations, which form the physical basis for the analysis done in this note, arise because the $U(1)$ charge can assume arbitrarily large values. For a general symmetry group, particle states sit in irreducible representations of the symmetry group. *Discrete finite groups* have only finitely many irreps and the sum of the squares of their dimensions add up to the number of elements in the group. Thus, one can potentially run into trouble with discrete symmetry groups if they have infinite order, or if in some sequence of models their order can be increased without bound.

String theory is full of infinite discrete groups: the duality groups of super Poincare invariant compactifications. However, these groups are effectively spontaneously broken. A generic transformation changes the value of the moduli and *does not act on the particle states* of a given scattering matrix. The stability subgroup of a point in moduli space is, in all known examples, finite. There are simple mathematical explanations of this fact for all known duality groups, but our considerations suggest a general physical reason. Particles of finite mass m can not sit in irreps of arbitrarily large size without violating the covariant entropy bound.

[13] Recall that one cannot Higgs the gauge group in ten dimensional Type I.

8 Conclusions

We have presented a semi-classical derivation of the AMNV bound on the couplings of $U(1)$ gauge fields in arbitrary spacetime dimensions that allow for BH solutions with flat asymptotics. Our derivation is based on the the requirement that charged black hole evaporation not lead to contradictions with entropy bounds. We also investigated an analogous bound for non-Abelian gauge theories, without finding situations that would be shocking to an effective field theorist, and made some remarks about the case of discrete symmetries. Finally, we performed some simple checks of the bounds for systems involving D-branes in various space-time dimensions.

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