

IIA moduli stabilization with badly broken supersymmetry.

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Abstract

Scherk-Schwarz compactification in string theory can be defined as orbifolding by an R symmetry, a symmetry that acts differently on bosons and fermions. Such a symmetry can arise in many situations, including toroidal and orbifold compactifications, as well as smooth Calabi-Yau spaces. If the symmetry acts freely then for large radius there are no tachyons in the spectrum. We focus mainly on stabilization by fluxes, and give examples with all moduli stabilized where the coupling is small and the internal manifold is large. Such models appear to be perturbatively stable with supersymmetry broken at the Kaluza-Klein scale. These are interesting laboratories for a variety of theoretical questions and provide models of a non-supersymmetric landscape.

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1 Introduction

One of the simplest ways to think about supersymmetry breaking in higher dimensions is Scherk-Schwarz compactification [1]. Here one usually considers compactification on a torus, and imposes periodic boundary conditions for bosons and anti-periodic boundary conditions for fermions. As a result, in the four dimensional theory, supersymmetry is broken at the scale of compactification, $1/R$; below this scale, there is no light gravitino, and the breaking appears explicit. These theories have other interesting features. Typically, at small radius, they have tachyons and are T-dual to compactifications of Type 0 theories [2]. In addition, even in the tachyon-free regime, these compactifications suffer from the Witten Kaluza-Klein instability [3]. The significance of these features is hard to assess, however, due to the perturbative instabilities which arise already at one loop. Generally, it is not clear whether any of these classical solutions correspond to stable or metastable states of the quantum theory.

The term “Scherk-Schwarz compactification” is most often used for these toroidal compactifications, but, the concept is more general[4, 5, 6]. As we will review and elaborate in this paper, it should be applied to a broader array of models. This more general set of constructions is obtained by modding out a string solution by a freely acting symmetry operation that acts differently on bosons and fermions, namely, a freely acting R symmetry^{1,2}. These symmetries can include rotations in compact or in non-compact directions as well as abstract symmetries of conformal field theories. Modding out by these symmetries can be thought of as including Wilson lines for the spin-connection, which break supersymmetry. From the point of view of the resulting string models, there is no fundamental difference between freely interacting symmetries and those with fixed points. The reason we confine our attention to symmetries that are freely acting is to ensure that at large radius there are no tachyons or new light states in twisted sectors. Even then, there are often tachyons at small radius, as one would predict from the dualities alluded to above. Even though the effective theory below the compactification scale has no symmetry, the moduli space, classically, consists of the entire subspace invariant under the symmetry.

In Scherk-Schwarz Models, the vacuum energy receives corrections already at one loop[7]. In a geometrical compactification, the effective potential will typically drive the radius towards smaller values, where tachyons start to appear in the spectrum. Recently, however, there has been much progress in constructing stable or metastable string ground states in supersymmetric (and nearly supersymmetric) models [8]-[17]. Crucial to these constructions is the role of fluxes in stabilizing moduli. Much attention has been focussed on supersymmetric compactifications of (orientifolded) IIB theories, where fluxes fix complex structure moduli already at the level of a classical analysis. Kahler moduli then are often fixed by non-perturbative effects[16]. The resulting spaces can be dS or AdS.

As we will explain, one can repeat the IIB construction with a Scherk-Schwarz projection for many Calabi-Yau manifolds. It is only necessary that the original Calabi-Yau possess a suitable R symmetry on some subspace of its moduli space, and to set to zero all fluxes which transform non-trivially under the symmetry. At the classical level, the potential for

¹As we explain below, in practice these R symmetries are Z_2 's times ordinary symmetries.

²One can also generalize the Scherk-Schwarz projection in other directions. For example one can consider a combination of a U-duality transformation and a translation [13], [14].

the moduli which survive the projection is the same as in the model before the projection; the absence of fixed points insures the absence of new massless particles. So again, one can find examples where all of the complex structure moduli can be fixed by fluxes. However, the Kahler moduli will receive corrections already in perturbation theory, and any minimum of their potential is likely to reside at small radii (radii of order the string scale), so one can at best speculate about the possible existence of metastable states (much as in [18]).

An alternative possibility for fixing moduli is provided by a recent analysis of IIA compactifications with fluxes[19]. In this case, it has been argued that, for suitable fluxes, one can stabilize all of the moduli classically. More dramatically, it appears that there are an infinite series of such states, with arbitrarily small string coupling and curvature. Moreover, the geometry of the solutions turns out to be a product of 4 dimensional AdS space times a compact CY manifold with an arbitrarily large hierarchy between the AdS curvature radius and the KK scale, so that unlike Freund-Rubin compactifications[20], the physics is well captured by a 4 dimensional effective description. A subset of these states are supersymmetric. The rest are *approximately supersymmetric*, in the sense that the *gravitino mass is parameterically small compared to the Kaluza-Klein (KK) scale*. In this paper we will show that one can consistently apply a generalized Scherk-Schwarz projection to these models, thus constructing, with the same level of reliability, an infinite sequence of states with *badly broken supersymmetry*, namely, where supersymmetry is broken at the KK scale. From a four dimensional viewpoint, such generalized Scherk-Schwarz models are non-supersymmetric. There is no scale at which the theory appears four dimensional and is (even approximately) supersymmetric. These states provide an interesting laboratory in which to study a variety of questions, including perturbative and non-perturbative stability and statistics of non-supersymmetric states.

This paper is organized as follows. In section 2 we present a brief review of the standard Scherk-Schwarz projection [1] its application in string theory [7] and briefly explain the duality to type 0 strings. We describe in particular how the usual Scherk-Schwarz projection can be phrased as modding out by a freely acting discrete R-symmetry. This is particularly transparent in the Green-Schwarz formulation; a more detailed analysis in the RNS formalism appears in an appendix. In section 3 we consider a variety of generalizations of Scherk-Schwarz. In section 4 we present a generalized Scherk-Schwarz projection on the type IIA T^6/\mathbb{Z}_3^2 CY orientifold of DeWolfe et.al [19]. We show how the classical stability analysis carried in [19] remains intact even with broken supersymmetry, and give a formal argument that quantum correction are parameterically small. ³ In the concluding section, we remark on some possible applications of these ideas. These include developing a more general understanding of Witten's bubble of nothing[3], and the use of non-supersymmetric models with stabilized moduli to study questions about the landscape. Two appendices present an RNS formulation of the standard Scherk-Schwarz projection and an analysis of a T^6/\mathbb{Z}_4^3 CY orientifold.

There are other approaches to constructing models of broken supersymmetry in string theory which lead to models with some similar features, such as supersymmetry broken at the Kaluza-Klein scale (see, for example, [22], where compactification on products of Riemann surfaces leads to models with badly broken supersymmetry). The reason for our

³Recent work calls into question the possibility of a systematic weak coupling analysis[21].

focus on Scherk-Schwarz models lies in their simple realizations, classically, as critical string theories, and in some cases quantum mechanically as small distortions of such theories.

2 Scherk-Schwarz models in field theory and string theory

Consider a string theory compactified on one periodic dimension:

$$X \sim X + 2\pi R \tag{2.1}$$

with left and right moving momenta given by

$$p_L = \frac{m}{R} + \frac{wR}{2}, \quad p_R = \frac{m}{R} - \frac{wR}{2} \tag{2.2}$$

where m, w are integral momentum and winding quantum numbers, and we set $\alpha' = 2$. E.g, the energy of a superstring excitation on $R^{1,8} \times S^1_X$ in the sector with winding number w and with m units of momentum and with left (right) oscillator numbers N (\tilde{N}) is

$$m^2 = 2(N - \frac{1}{2}) + p_L^2 = 2(\tilde{N} - \frac{1}{2}) + p_R^2 = (N + \tilde{N} - 1) + (\frac{m}{R})^2 + (\frac{wR}{2})^2 \tag{2.3}$$

subject to the level matching condition

$$N - \tilde{N} + mw = 0. \tag{2.4}$$

In a Scherk-Schwartz compactification [1] one imposes anti-periodic boundary conditions for the spacetime fermions along the periodic coordinate X . Consequently, fermions have half integral momenta along the circle parameterized by X . Supersymmetry is spontaneously broken as the bosons and fermions have different mode expansions along the circle which translate to different energies in the dimensionally reduced model. This fact is true already in field theory.

In string theory there are additional features [7]. Imposing anti-periodic boundary conditions for fermions around the circle is equivalent to modding out by the \mathbb{Z}_2 R-symmetry $e^{2\pi i R P^X} (-1)^F$. In order to obtain a modular invariant partition function one needs to add the twisted sectors. These are particularly easy to understand in the Green-Schwarz formulation⁴ where the space-time fermion number operator reverses the sign of the two dimensional fields S_a .

$$\begin{aligned} X(\sigma + \pi) &= X(\sigma) + w \cdot 2\pi R \\ S_a(\sigma + \pi) &= (-1)^w S_a(\sigma). \end{aligned} \tag{2.5}$$

The ground state energy in odd winding sectors is thus

$$m^2 = (\frac{wR}{2})^2 - 1 \tag{2.6}$$

⁴A useful trick is to think of this as a geometrical orbifold on a circle with a doubled radius $2R$. The twisted states wind half-way around that circle and acquire anti-periodic boundary conditions.

which becomes tachyonic for small enough radius. Also, this projection effectively reverses the familiar type *II* GSO projection between odd and even winding sectors⁵. Thus, by imposing Scherk-Schwarz boundary conditions in type *IIA/B*, the bosonic spectrum is given by (using the notations of Polchinski [23])

- $w = 2w' : (NS+, NS+), (R+, R\pm)$
- $w = 2w' + 1 : (NS-, NS-), (R-, R\mp)$.

2.1 Duality with type 0

Scherk-Schwarz compactifications of type II superstrings obey a certain T-duality relation with type 0 string theory. In the limit that $R \rightarrow 0$ all the spacetime fermions become very massive because they can not have zero modes along the circle. It can be shown⁶ that type *IIA/B* compactified on a circle of radius R with Scherk-Schwarz boundary conditions, in the limit $R \rightarrow 0$, is equivalent to type *0B/A* in uncompactified spacetime.

Calabi-Yau spaces are classical solutions of the Type 0 theories. One can see this in two ways, which will be useful for us in what follows. First, at the classical level, the field equations for the bosonic fields are the same as in the Type II theories. Therefore, classically, solutions of the two theories coincide. Alternatively, a compactification of Type II on a Calabi-Yau space is described by two dimensional left and right $\mathcal{N} = 2$ SCFT with $c = 9$. This CFT can always be used also as a sensible background of Type 0. Thus, there are large classes of smooth manifolds which solve the classical equations of the Type 0 theory. One can even introduce fluxes, and classically, the potential for these will be as in the Type II theories. But these constructions do not seem terribly interesting, since at large radius they possess tachyons (at least at weak coupling), and quantum corrections will generally destabilize them. In the next section, we will consider generalized Scherk-Schwarz constructions with the potential to avoid these difficulties.

3 Generalizing Scherk-Schwarz

Modding out a string theory by an R symmetry will yield a string configuration with less supersymmetry. Supersymmetry will be broken at the KK scale associated with the compact manifold.

In this section we provide several examples of this kind of generalized Scherk-Schwarz projections. We start with toroidal and toroidal orbifold models. These are familiar models described by free two dimensional fields. Then we note that large classes of Calabi-Yau compactifications, both of Type II and heterotic strings, admit such projections. These are distinctly less trivial. In the next section, we will show that fluxes can stabilize some or all of the moduli of such compactifications.

⁵A simple derivation of this fact appears in appendix A

⁶More details appear in appendix B.

3.1 Scherk-Schwarz as an orbifold.

The usual Scherk-Schwarz projection is a special case of the general toroidal orbifold construction. As explained in [24] a class of orbifolds can be constructed as a quotient of the Euclidean space \mathbb{R}^d by a subgroup of rotations and translations $g = (\theta, v)$ called the *space-group* S . The orbifold is the quotient $\Omega = \mathbb{R}^d/S$ where the elements of S act on a vector $x \in \mathbb{R}^d$ as $gx = \theta x + v$. The subgroup Λ of translations $(1, v) \in S$ is called *the lattice of* S . The subgroup of $O(d)$ of rotations θ such that $(\theta, v) \in S$ is called the *point-group*. It can be shown that each element of the point group is associated with a unique vector v (up to lattice translation). The point-group is also the holonomy group of the orbifold. An orbifold with a space-group that has a non-trivial point-group can break supersymmetry because bosons and fermions transform differently under rotations.

Scherk-Schwarz compactifications have a natural description in this language as modding out by the space group generated by two elements that include a 2π rotation and a translation in an orthogonal direction \vec{e} , namely, $g = \{(e^{2\pi i J_{ab}}, 0) ; (1, 2\pi R \cdot \vec{e})\}$. More generally [4, 5, ?], take any singular limit of a CY n-fold described by an orbifold of n products $T^2 \times \dots \times T^2$. Since the tori are smooth, the holonomy matrix is just the identification matrix of the orbifold $z_i \sim A_i^{\bar{j}} z_j$. Demanding that $\det A = 1$ gives a CY. However, if we demand instead that $\det A = e^{2\pi i}$ then the geometry does not change, but fermions will pick up a minus sign under the action of the orbifold. This will break supersymmetry. In general there will be tachyons in twisted sectors located at fixed points of the orbifold. However, if we add a translation to the space group so that the action is free, we can avoid tachyonic instabilities at large radius. However, the radius modulus will generally tend to decrease in size until it enters the tachyonic regime. This can be avoided in a flux compactification. We give a detailed example of this procedure in section 4.

3.2 Smooth Calabi-Yau Spaces in Type II and Heterotic Theories

We can perform a Scherk-Schwarz projection in any Calabi-Yau space which admits a freely acting symmetry.⁷ This is already possible for the familiar quintic in CP^5 . Take, for example, the quintic polynomial to be

$$P = z_1^5 + z_2^5 + z_3^5 + z_4^5 + z_5^5 = 0. \quad (3.1)$$

Then the transformation

$$z_i \rightarrow \alpha^i z_i \quad \text{with} \quad \alpha = e^{\frac{2\pi i}{5}} \quad (3.2)$$

is a freely acting symmetry of the theory [28]. As for the orbifold case, we can define the action of this symmetry to flip the sign of the supercharges (covariantly constant spinor), so that supersymmetry is broken. In this case, 21 complex structure moduli and the one Kahler modulus survive the projection.

If one examines, say, lists of Calabi-Yau manifolds defined in weighted projective spaces, one can find many suitable symmetries. In all of these cases, there remains a large moduli

⁷In an earlier version of this paper, we spoke of freely acting R symmetries. However, Volker Braun has pointed out to us that in the case of Calabi-Yau three-folds, freely acting symmetries always leave the holomorphic three-form invariant (this follows from results stated in [28]). As a result, any freely-acting R symmetry must be at most a Z_2 , times some ordinary symmetry.

space. Again, it should be stressed that these are solutions of the classical equations, but quantum corrections, already in string perturbation theory, will generate a potential for the moduli.

It should be noted that these manipulations are valid in Type II and heterotic theories. Heterotic theories actually yield a richer set of models, since in addition to (2, 2) theories, one has (2, 0) theories. Many of the latter admit suitable R symmetries as well[26].

4 Scherk-Schwarz version of IIA models.

In this section we present a Scherk-Schwarz version of the type IIA superstring model analyzed by DeWolfe et.al in [19], resulting in a sequence of non-supersymmetric, tachyon free models. We first review the supersymmetric construction, and then modify it so as to implement a Scherk-Schwarz projection. In an appendix, we study a different orbifold.

4.1 Review of supersymmetric IIA Constructions

DeWolfe et al construct an infinite set of AdS states where the string coupling and curvature can be made arbitrarily small by choice of fluxes. They start with compactification of Type II theory on the \mathbb{Z}_3 orbifold [25]. This orbifold is a singular limit of a CY with $\chi = 72$ obtained by starting with a T^6 defined by the identifications

$$z_i \sim z_i + 1 \sim z_i + \alpha, \quad i = 1, 2, 3; \quad \alpha = e^{\frac{\pi i}{3}} \quad (4.1)$$

and then modding out by a \mathbb{Z}_3 symmetry of this lattice defined by

$$z_i \rightarrow T_i^j z_j \quad T = \begin{pmatrix} \alpha^2 & 0 & 0 \\ 0 & \alpha^2 & 0 \\ 0 & 0 & \alpha^{-4} \end{pmatrix} \quad (4.2)$$

The choice of the phases in 4.2 ensures that the orbifold has $SU(3)$ holonomy. This leaves, in the case of Type II theories, $N = 2$ supersymmetry in 4 dimensions. As was noted in [25] one can mod out by a further freely acting \mathbb{Z}_3

$$z_i \rightarrow Q_i^j z_j + a_i \quad Q = \begin{pmatrix} \alpha^2 & 0 & 0 \\ 0 & \alpha^{-2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad a = \frac{1 + \alpha}{3} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad (4.3)$$

The last step reduces the supersymmetry down to $N = 1$.

The orientifold projection involves the symmetry

$$\mathcal{O} = \Omega_P(-1)^{FL} \sigma \quad (4.4)$$

with Ω_P world-sheet parity. σ is the reflection

$$\sigma : z_i \rightarrow -\bar{z}_i \quad (4.5)$$

The effect of these transformations is to reduce the supersymmetry to $N = 1$ and to eliminate many of the moduli of the original toroidal compactification. In the untwisted sector, only the diagonal moduli,

$$g_{i\bar{i}} = v_i, \quad i = 1, 2, 3. \quad (4.6)$$

survive; they are each part of a chiral multiplet, whose scalar components have the form

$$t_i = b_i + iv_i. \quad (4.7)$$

There are nine twisted moduli, $t_A = b_A + iv_A$.

Now one introduces a number of fluxes. There is a zero form flux. There are two-form fluxes, three and four-form fluxes. A basis of two-form fluxes, odd under the reflection σ , is provided by:

$$\omega_i = (\kappa\sqrt{3})^{1/3} idz^i \wedge d\bar{z}^i, \quad i = 1, 2, 3. \quad (4.8)$$

There is a corresponding set of four-cycles

$$\tilde{w}^i = \left(\frac{3}{\kappa}\right)^{1/3} (idz^j \wedge d\bar{z}^j) \wedge (idz^k \wedge d\bar{z}^k). \quad (4.9)$$

The three-form invariant under T and Q is:

$$\Omega = e^{1/4} idz_1 dz_2 dz_3 = \frac{1}{\sqrt{2}}(\alpha_0 + i\beta_0). \quad (4.10)$$

DeWolfe et al turn on the fluxes:

$$H_3 = -p\beta_0 \quad F_r = e_i \tilde{w}^i; \quad F_0 = m_0. \quad (4.11)$$

A simple analysis leads to a potential for the moduli:

$$V = \frac{p^2}{4} \frac{e^{2\phi}}{\text{vol}^2} + \frac{1}{2} \left(\sum_i^3 e_i^2 v_i^2 \right) \frac{e^{4\phi}}{\text{vol}^3} + \frac{m_0^2}{2} \frac{e^{4\phi}}{\text{vol}} + \sqrt{2} m_0 p \frac{e^{3\phi}}{\text{vol}^{3/2}}. \quad (4.12)$$

Here $\text{vol} = \kappa v_1 v_2 v_3$. The potential has a local minimum:

$$v_i = \frac{1}{|e_i|} \sqrt{\frac{5}{3} \frac{|e_1 e_2 e_3|}{\kappa m_0}} \quad e^\phi = \frac{3}{4} |p| \left(\frac{5}{12} \frac{\kappa}{|m_0 e_1 e_2 e_3|} \right)^{1/4}. \quad (4.13)$$

DeWolfe et al also show that the moduli associated with the fixed points can be stabilized as well. They work out explicitly the geometry of the smooth manifold, and show that, introducing suitable four-form fluxes associated with the fixed points, the corresponding moduli have minima where the manifold is smooth.

Ref. [19] then gave a purely four dimensional description of all of this, in terms of a theory of light chiral fields, whose lagrangian is described by a Kahler potential and a superpotential. In this analysis, they included two-form fluxes, m_i . These can be used to tune the axions (if the two form fluxes vanish, so do the axion fields). We won't reproduce

their full equations for the superpotential, but will focus on the piece involving the t_a fields, and only in the special case that the m_a 's all vanish:

$$W^K(t_a) = e_0 + e_a t^a - \frac{m_0}{6} \kappa_{abc} t_a t_b t_c. \quad (4.14)$$

The Kahler potential for these fields is:

$$K^K(t_a) = -\log\left(\frac{4}{3} \kappa_{abc} v_a v_b v_c\right). \quad (4.15)$$

Here the quantity κ_{abc} is the triple intersection,

$$\kappa_{abc} = \int \omega_a \wedge \omega_b \wedge \omega_c. \quad (4.16)$$

The resulting equations for the v_a 's are:

$$3m_0^2 \kappa_{abc} v_b v_c + 10m_0 e_a = 0. \quad (4.17)$$

In the case of the \mathbb{Z}_3 orbifold, these equations are simple to analyze. This is because κ_{abc} breaks up into distinct pieces involving untwisted and twisted moduli. In [19], an argument is given for this based on the geometry, but there is another way to understand this selection rule.. The model has a ‘‘quantum symmetry’’ [27], a \mathbb{Z}_3 , under which the fields in the sector twisted once by T transform with a phase $e^{2\pi i/3}$, while there antiparticles in the doubly-twisted sector transform by $e^{-2\pi i/3}$. κ determines the prepotential of the $N = 2$ theory (before the orbifold projection). This potential is holomorphic in the fields, and must be invariant under the \mathbb{Z}_3 . As a result, only terms with zero or three twisted fields are non-vanishing, and the equations for the untwisted and twisted moduli decouple. DeWolfe et al denote these by the indices i and A , respectively, so they are able to take

$$\kappa_{123} = \kappa; \quad \kappa_{AAA} = \beta. \quad (4.18)$$

In this way, they are able to find supersymmetric solutions, corresponding to the solutions above, for certain choices of the signs of the e_i 's, and f_A 's (the four-form fluxes at the fixed points):

$$v_i = \frac{1}{|e_i|} \sqrt{\frac{-5e_1 e_2 e_3}{3m_0 \kappa}} \quad v_A = \sqrt{\frac{-10f_A}{3\beta m_0}}. \quad (4.19)$$

The potential of eqn. 4.12 is quadratic in the e 's and f 's, so the location of the minima is independent of the signs of the fluxes. But only for some choices of signs are there solutions, as in eqn. 4.19; the minima for other choices of flux are not supersymmetric. But they are approximately so. The gravitino mass is of order:

$$m_{3/2} = e^{K/2} W \approx |e|^{-3/4} |e|^{-3/2} |e|^{3/2} \sim |e|^{-3/4}. \quad (4.20)$$

Here the first $|e|^{-3/2}$ factor arises from the Kahler potential for the dilaton. This is to be compared to the Kaluza-Klein scale, which is $1/\sqrt{v} \sim |e|^{-1/4}$. So the gravitino is isolated from the Kaluza-Klein tower, i.e. there is a range of energy scales with a single, light gravitino, and the theory has an approximate supersymmetry at low energies.

4.2 A T^6/\mathbb{Z}_3^2 orbifold with a generalized Scherk-Schwarz projection.

The model of [19] admits an immediate generalization. These authors performed two orbifold projections, both to reduce the supersymmetry to $N = 1$, and to obtain a comparatively small number of moduli. We will also perform two projections, the first one is the same as in [19] and the second is a slight variation which implements the Scherk-Schwarz projection. This will allow us to eliminate all supersymmetry, while obtaining essentially the same set of moduli.

We thus start with the standard Z_3 orbifold described in the previous section in equations 4.1-4.3. Our generalized Scherk-Schwarz projection involves taking, in the second step, a slightly different matrix:

$$z_i \rightarrow \tilde{Q}_i^j z_j + a_i \quad \tilde{Q} = \begin{pmatrix} \alpha^2 & 0 & 0 \\ 0 & \alpha^4 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad a = \frac{1+\alpha}{3} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad (4.21)$$

Gravitinos with zero momentum will pick up a minus sign, due to the overall 2π rotation in the internal tori, under this transformation. (Equivalently, under parallel transport around the non-contractible loop which in the original T^6/\mathbb{Z}_3 orbifold connected the two fixed points at $(0,0,0)$ and $((1+\alpha)/3, (1+\alpha)/3, (1+\alpha)/3)$, the gravitino picks up a phase, -1 .) So the last orbifolding breaks supersymmetry. The \mathbb{Z}_3 orbifold 4.1-4.2 is supersymmetric and so has no tachyons in the spectrum. Only the last orbifold by 4.21 breaks supersymmetry, but this transformation acts freely due to the translation in the third T^2 . So the mass terms in the twisted sectors associated with 4.21 will not have tachyons at large radius.

4.3 Moduli stabilization

The projectors in this model are almost the same as those of DeWolfe et al. They differ only in their action on space-time fermions. As a result, the moduli are the same as those of [19]. Moreover, the fluxes are the same as well. The moduli include the diagonal components of the metric

$$g_{i\bar{i}} \equiv \gamma_i. \quad (4.22)$$

The forms include a set of 2-forms

$$\omega_i \propto dz^i \wedge dz^{\bar{i}} \quad (4.23)$$

invariant under 4.2, 4.21 and odd under the spacetime reflection 4.5. There are 3 NSNS 2-form moduli b_i with $B_2 = b_i \omega^i$ that combine with 4.22 to form 3 complex moduli. The holomorphic 3-form invariant under 4.2, 4.21 is again 4.10 with the $O6$ plane wrapping the even cycle α_0 . Finally, the dilaton joins the RR 3-form axion $\xi \alpha_0$ to form a fourth complex scalar. As a result, the potential for the untwisted moduli is again given by eqn. 4.12 with minima given by eqn. 4.13. Also the stability analysis for the twisted moduli carries through in the exact same way as in [19].

To recapitulate, we used a flux compactification to stabilize all the moduli, so that by taking a large enough CY orientifold we can ensure that there are no tachyons in the spectrum, and this non-supersymmetric compactification is perturbatively stable. An alternative model based on a Z_4 orbifold is described in appendix A.

4.4 Quantum corrections

One might worry that quantum corrections in such non-supersymmetric models would be large due to the lack of supersymmetry, either destabilizing the would-be AdS vacua, or simply making any analysis impossible. After all, there are no longer non-renormalization theorems, and the effective cutoff for loop diagrams is at least as large as the Kaluza-Klein scale. But, as in the supersymmetric case, higher order corrections appear to be suppressed by powers of the four form flux, as we now show.

Classically, the potential for the volume moduli v_i behaves as

$$V_0 = \frac{e^{2\phi}}{v^6} \tag{4.24}$$

at the minimum. It is important to note that this result is expressed in Planck units, i.e. it is multiplied by M_p^4 . If we translate this into the mass of the scalar, we need to recall that the scalar kinetic term is proportional to $1/v^2$, so the tree level mass behaves as

$$m_0^2 = \frac{e^{2\phi}}{v^6} M_p^2 = \frac{1}{v^3} M_s^2. \tag{4.25}$$

Let's compare with what we might expect at one loop. We can estimate these effects in two equivalent ways. First, consider a Casimir computation. In string frame, this will give a result of the form

$$\delta V \sim \frac{1}{R^4} \sim \frac{1}{v^2} M_s^4. \tag{4.26}$$

so

$$\delta m^2 \sim e^{2\phi} \frac{1}{v^5} M_s^2. \tag{4.27}$$

The, the loop correction is suppressed by $1/v^3 \sim |e_i|^{-3/2}$ and for large four-form flux, the corrections are under control⁸.

This analysis is arguably naive. It is not known how to perform a string perturbation expansion for these configurations, and the work of [21] suggests that this might not be possible. We simply note here that the low energy theory, viewed as a cutoff field theory, appears to be under control.

This example shows that there are models with badly broken supersymmetry but where classically all the moduli are stabilized. Our basic strategy was to note that the Scherk-Schwarz projection does not change the classical equations for the bosonic fields, and these can be shown in some cases to be subject to small quantum corrections. In appendix 5.2 we present a closely related $\mathbb{Z}_4 \times \mathbb{Z}_4$ CY orientifold, where again we are able to break susy at the KK scale while retaining perturbative stability.

⁸Alternatively, we can consider a direct computation of the mass. For this we can work with canonically normalized fields. There is a one loop correction to the mass involving emission of a graviton. This correction is quadratically divergent; we expect that it is cut off at $1/R$. So the size of the mass correction is $\delta m^2 \approx G_N \frac{1}{R^4} = e^{2\phi} \frac{1}{v^5} M_s^2$ as above.

5 Conclusions: Potential Applications

We have seen that it is possible to generate a wide array of non-supersymmetric string configurations using a generalized notion of Scherk-Schwarz projection. Models of this type should be useful theoretical laboratories for studying a number of questions. To conclude, we mention some areas for further study.

5.1 Non-Perturbative Instabilities with Stabilized Moduli

Kaluza-Klein spaces with standard Scherk-Schwarz boundary conditions suffer from a non-perturbative instability to decay into a “bubble of nothing”, even in the regime where the size of the orbifold is big and there are no winding tachyons. The instability was derived by Witten in [3] using an analytical continuation of the Schwarzschild solution. In most applications, the significance of this solution is obscure for a variety of reasons, perhaps the most important being that the classical, non-supersymmetric string or field theory Kaluza-Klein solutions are already unstable in perturbation theory.

The IIA models offer promise of studying this question, as all of the moduli are stable. Presumably, the Witten bounce in this case is associated with the non-contractible loops introduced by the Scherk-Schwarz projection. Efforts to construct such solutions in states with all moduli fixed will be reported elsewhere.

5.2 The Landscape

Non-supersymmetric models with fluxes of the type described here provide an arena for examining landscape statistics. There are a variety of questions which one might try to address. These include raw counting, e.g. trying to compare numbers of supersymmetric and non-supersymmetric states with various properties. In the IIA case, for large fluxes, the states, classically, are AdS, and small quantum effects will not change this. We have noted that non-supersymmetric IIB theories are more challenging to study than their supersymmetric counterparts. The complex structure moduli are fixed, classically, as in the supersymmetric case, but the Kahler moduli will have potentials already in perturbation theory, and stationary points will typically lie at small radius. One would expect, however, that such states would exist, and that some will be dS, some AdS. More refined questions include counting of states with discrete symmetries and with exponentially large warping (we thank Steve Giddings for a discussion which led us to consider this application). Studies of supersymmetric vacua[17], for example, having indicated that exponential warping occurs in a substantial fraction of states. The models described here with most or all moduli stabilized provide a further laboratory for investigating this question. It seems likely that one will again find that exponential warping is common. This might suggest that non-supersymmetric vacua with warping are more likely solutions of the hierarchy problem than supersymmetric ones. Of course, whether this can be reconciled with other facts of low energy physics is an important question.

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Appendix A -Generalized Scherk-Schwarz in a \mathbb{Z}_4^3 example.

In this section we consider a $\mathbb{Z}_4 \times \mathbb{Z}_4$ orbifold. We start from a T^6 defined by

$$z_j \sim z_j + 1 \sim z_j + i \quad j = 1, 2, 3 \tag{5.1}$$

and mod out by a $\mathbb{Z}_4 \times \mathbb{Z}_4$ symmetry

$$z_i \rightarrow A_i^j z_j \quad \text{with} \quad A = \begin{pmatrix} i & 0 & 0 \\ 0 & -i & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{and} \tag{5.2}$$

$$z_i \rightarrow B_i^j z_j \quad \text{with} \quad B = \begin{pmatrix} i & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -i \end{pmatrix}.$$

There are many fixed points (some of which are only fixed under Z_2 subgroups). This orbifold model is consistent with the same orientifold projection employed in [19] and described here in eqn. 4.4. The $\mathbb{Z}_4 \times \mathbb{Z}_4$ torus orientifold keeps precisely the same untwisted moduli as in [19]. It is fairly straightforward to check that the stabilization analysis of the untwisted moduli for this model precisely follows the analysis carried by DeWolfe et.al in [19]. As a result, the potential for the untwisted moduli has a form identical to that of the \mathbb{Z}_3 case (reproduced here in eqn. 4.12), with similar minima given by eqn. 4.13.

To achieve Scherk-Schwarz compactification, one can now orbifold this model by a freely acting \mathbb{Z}_4

$$z_i \rightarrow C_i^j z_j + a_i \quad \text{with} \quad C = \begin{pmatrix} e^{\frac{\pi i}{2}} & 0 & 0 \\ 0 & e^{\frac{3\pi i}{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{and} \quad a = \frac{1+i}{2} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}. \tag{5.3}$$

For this \mathbb{Z}_4^3 CY orientifold, just like in the \mathbb{Z}_3^2 case, susy is broken at the KK scale, and the fluxes can stabilize the geometry to a safe size where the spectrum is free of tachyons⁹. Again, an effective field theory analysis suggests that the quantum corrections are small.

To complete the stabilization analysis one needs to study the stabilization of blow up modes at the various fixed points of this orbifold. The analysis is more complicated in this case, because there are many more sectors to consider, with a more intricate pattern of couplings. We see no reason to believe that one cannot fix the moduli in a controlled fashion, as in the \mathbb{Z}_3^2 case [19]. A complete analysis will be presented elsewhere[30].

⁹Again, modulo possible surprises from blow up modes which were not analyzed.

Appendix B - Type 0 and Scherk-Schwarz in RNS.

1. Projections in operator language

The usual treatment of type 0 string theory [2], is based on modular invariance of the partition function. One of the interesting features of type 0 models is their duality relation with Scherk-Schwarz compactifications [7], or compactification on “twisted circles” as they are sometime called, when the radius of the twisted circle goes to zero. This duality relies on the existence of winding tachyons in Scherk-Schwarz compactifications due to a reversal of the GSO projection in odd winding sectors, as explained e.g. in [29]. In this appendix we present an equivalent treatment in operator language. The reason for doing so is that in this language the modified GSO projection in odd winding sectors as well as the relation to type 0 string theory is particularly transparent. The basic premise we use is that to ensure the consistency of a worldsheet sigma model as a string background one must make sure that

- A) All the operators of the worldsheet sigma model are mutually local.
- B) The operator algebra is closed.
- C) Left-right level matching is imposed.

Projections are carried out by demanding that an operator \mathcal{O} that enforces the projection via its OPE with the rest of the operator algebra, is part of the worldsheet SCFT. Condition A projects on invariant states while condition B will then generate twisted sectors.

For example, the type II GSO projection is arrived at by demanding mutual locality and closure with the spacetime supercharges. Similarly, if we want to describe a compactification of type II on a circle 2.1 we add to the SCFT the two operators

$$R(z, \bar{z}) \equiv e^{i\frac{R}{\alpha'}(X(z) - \tilde{X}(\bar{z}))} \quad (5.4)$$

and

$$M(z, \bar{z}) = e^{i\frac{1}{R}(X(z) + \tilde{X}(\bar{z}))} \quad (5.5)$$

and follow the steps mentioned above. This will project on the correct momentum lattice, via OPE with $e^{ip_L X(z)} e^{ip_R \tilde{X}(\bar{z})}$, and create the winding and momentum sectors.

2. The type 0 projection

Let us examine what happens when we start from a type II superstring on $\mathbb{R}^{1,9}$. The holomorphic spacetime supercharges are built out of the bosonized fermions H_1, \dots, H_5 and the reparameterization superghost φ as

$$Q_\alpha = \oint e^{-\frac{\varphi}{2}} e^{\frac{i}{2}(\epsilon_1 H_1 + \dots + \epsilon_5 H_5)} \quad (5.6)$$

where $\alpha = [\epsilon_1, \dots, \epsilon_5]$ is a spinor index of $SO(1, 9)$ and each H bosonizing a pair of fermions, say $\psi^{1,2}$, according to the familiar formulas

$$\psi^1 = \sqrt{2} \cos H, \quad \psi^2 = \sqrt{2} \sin H, \quad \psi^\pm \equiv \frac{1}{\sqrt{2}}(\psi^1 \pm i\psi^2) = e^{\pm iH} \quad (5.7)$$

Demand now that the worldsheet CFT includes the following operator^{10,11}

$$T(z, \bar{z}) = e^{i(H(z) - \tilde{H}(\bar{z}))}. \quad (5.8)$$

The fact that $T(z, \bar{z})$ is not a chiral operator suggests that the resulting theory will not respect the chiral GSO projection. In fact, we will now show that the resulting theory is the well known type 0 model. Using the standard relation

$$: e^{iaX(z)} :: e^{ibX(w)} := (z - w)^{ab} : e^{i(aX(z) + bX(w))} : \quad (5.9)$$

it is clear that all the operators in the (NS, NS) sector are mutually local with $T(z, \bar{z})$. However, spin fields $S_\alpha \equiv e^{\frac{i}{2}(\epsilon_1 H_1 + \dots + \epsilon_5 H_5)}$ (as well as their anti-holomorphic counterparts) have a square root branch cut in their OPE with $T(z, \bar{z})$. This means that in the resulting theory there are no operators in the (R, NS) , (NS, R) sectors, and hence no spacetime fermions. Moreover, since the spacetime supercharges 5.6 have been projected out, we no longer have the usual chiral GSO projection and in particular the (NS, NS) vacuum corresponding to a tachyon in spacetime survives the projection.

What about the (R, R) sector? Starting from a type II superstring model we see that any pre-existing (R, R) vertex operator survives the projection

$$T^{(k)}(z, \bar{z}) \cdot S_\alpha(w) \tilde{S}_{\dot{\alpha}}(\bar{w}) \sim (z - w)^{\epsilon_1/2} (\bar{z} - \bar{w})^{-\tilde{\epsilon}_1/2} \dots = |z - w|^{\epsilon_1} (\bar{z} - \bar{w})^{-(\tilde{\epsilon}_1 + \epsilon_1)/2} \dots \quad (5.10)$$

and since $(\tilde{\epsilon}_1 + \epsilon_1)/2 \in \mathbb{Z}$ there is no branch cut. However, The (R, R) spectrum is doubled. The worldsheet fermions act on the spinfields as spacetime Γ matrices and so the “twisted sectors” which one gets on the RHS of 5.10 are (R, R) vertex operators where the spacetime chiralities are flipped *both* for the left and right movers.

To summarize¹², If we started from type IIA/B with the sectors

- IIA: $(NS+, NS+), (R+, NS+), (NS+, R-), (R+, R-)$
- IIB: $(NS+, NS+), (R+, NS+), (NS+, R+), (R+, R+),$

demanding that 5.8 is part of the worldsheet CFT projects the theory onto

- 0A: $(NS+, NS+), (NS-, NS-), (R-, R+), (R+, R-)$
- 0B: $(NS+, NS+), (NS-, NS-), (R-, R-), (R+, R+)$

which is exactly the type 0 projection.

¹⁰Note that this is not a physical operator and should not be confused with a very similar physical operator in the $(-1, -1)$ picture given by $e^{-\varphi - \tilde{\varphi}} \psi^+ \tilde{\psi}^-$.

¹¹In fact, the closure of the OPE algebra shows that it does not really matter if we use any combination of the form $e^{i(H_k(z) \pm \tilde{H}_l(\bar{z}))}$ with $k \neq l$. We thus refer to this operator simply as 5.8 without specifying the labels k, l or the relative sign.

¹²Following the notations used by Polchinski in chapter 10 pages 26-27 of [23]

3. Scherk-Schwarz projection

A worldsheet operator realization of Scherk-Schwartz boundary conditions for type II superstrings can be achieved by adding to the type II worldsheet algebra the *combined* operator

$$P(z, \bar{z}) \equiv [R \cdot T](z, \bar{z}). \quad (5.11)$$

where R, T were defined in 5.4, 5.8. In words, this operator insists that when we create a winding mode we must accompany each winding with the operator that performs the type 0 projection. Let us examine the effect of inserting 5.11. Mutual locality with 5.11 projects out the zero momentum vertex operators 5.6 thus breaking spacetime supersymmetry.

However, this model does have spacetime fermions. Working out the OPE of 5.11 with an (R, NS) vertex operator that carries some momentum along the circle ($p_{L,R}$ where defined in 2.2)

$$\mathcal{V}(z, \bar{z}) \sim e^{\frac{i}{2}(\epsilon_1 H_1 + \dots + \epsilon_5 H_5)} e^{ip_L X} e^{ip_R \tilde{X}} \quad (5.12)$$

we get the following (possible) branch cut in z

$$P(z, \bar{z}) \cdot \mathcal{V}(z, \bar{z}) \sim z^{m+\frac{1}{2}} \dots \quad (5.13)$$

with the conclusion that in order for this vertex operator to be projected in (giving a physical spacetime fermion state) we must restrict

$$m \in \mathbb{Z} + \frac{1}{2}. \quad (5.14)$$

Physically, this is just the familiar Scherk-Schwarz projection where fermions have anti-periodic boundary conditions (and therefore half integral momenta) along the circle.

So far we have checked mutual locality with 5.11 which in the language of orbifolds, is the projection on invariant states. The twisted sectors arise by closing the OPE algebra with 5.11. Let us examine the bosonic spectrum (suppressing the $(R, NS), (NS, R)$ sectors). At the $w = 0$ sector we have the usual type II spectrum $(NS+, NS+), (R+, R\pm)$ depending on whether we are in type IIA/B. For example we have the graviton vertex

$$G^{ij}(z, \bar{z}) \equiv e^{-\varphi - \bar{\varphi}} \psi^i \tilde{\psi}^j. \quad (5.15)$$

Going to the first twisted sector $w = 1$ we have to perform the OPE not with R (eq. 5.4) as if we are doing a “normal” circle but with P (eq. 5.11). The leading singular term will have the T in P contract against the fermions in 5.15 leaving us with the vertex operator of a *winding tachyon*

$$e^{-\varphi - \bar{\varphi}} \cdot \widehat{R} \quad (5.16)$$

In the (R, R) sector, the effect of T is to flip the chiralities on *both* the left and right movers, so we end up in the $w = 1$ sector¹³ with $(NS-, NS-), (R-, R\mp)$. It is straightforward to see that this picture persists to higher winding sectors:

- $w = 2w' : (NS+, NS+), (R+, R\pm)$
- $w = 2w' + 1 : (NS-, NS-), (R-, R\mp)$.

This demonstrates the reversal of the GSO projection in odd winding sectors.

¹³This phenomenon is sometime referred to by saying that the GSO projection is reversed in the odd winding sectors, but one should be careful about this phrasing because in type 0 theories the GSO is not chiral and the reversing happens in each chirality separately.

4. Scherk-Schwarz on a vanishing circle.

Let us see what happens in the limit that $R \rightarrow 0$. Naively in this limit the operator 5.4 is just the identity so P becomes the type 0 projection T and we expect to get a type 0 model. This sloppy argument actually gives the correct answer. In this limit all the spacetime fermions become very massive because they can not have zero modes along the circle. Furthermore, given an integer w' , the successive winding sectors $w = 2w', 2w' + 1$ are almost degenerate when $R \ll \alpha' = 2$

- $w = 2w' : (NS+, NS+), (R+, R\pm) \longrightarrow m^2 = \left(\frac{2w'R}{2}\right)^2 - 1$
- $w = 2w' + 1 : (NS-, NS-), (R-, R\mp) \longrightarrow m^2 = \left(\frac{(2w'+1)R}{2}\right)^2 - 1.$

Neglecting the difference $\Delta m^2 = R^2/4$ we see that the spectrum in those two successive sectors combined is that of *the w' winding sector of the corresponding type 0 theory compactified on a circle with twice the radius $R' = 2R$* . In the limit $R \rightarrow 0$ this become exact and it makes more sense instead of talking about the type 0A/B model on a vanishing circle to describe the model as type 0B/A in uncompactified spacetime.

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