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Can the Higgs sector contribute significantly to the muon anomalous magnetic moment?

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Abstract: A light CP-even Higgs boson with $m_h \sim 10$ GeV could explain the recent BNL measurement of the muon anomalous magnetic moment, in the framework of a general CP-conserving two-Higgs-doublet extension of the Standard Model with no tree-level flavor-changing neutral Higgs couplings. However, the allowed Higgs mass window is quite small and the corresponding model parameters are very constrained. The Higgs sector can contribute significantly to the observed BNL result for $g-2$ without violating known experimental constraints only if the hZZ coupling (approximately) vanishes and $M_\Upsilon \lesssim m_h \lesssim 2m_B$.

Keywords: Beyond Standard Model.

Contents

1. Introduction

A new experimental value of the muon anomalous magnetic moment, $a_\mu \equiv \frac{1}{2}(g-2)_\mu$, measured at BNL, was recently reported in ref. [1]. Comparing the measured value to its predicted value in the Standard Model (SM), ref. [1] reported that

$$a_\mu^{\rm exp} - a_\mu^{\rm SM} = 426 \pm 165 \times 10^{-11} \,. \tag{1.1}$$

Ref. [2] has reviewed the Standard Model computation of a_μ and concluded that if the deviation of eq. (1.1) can be attributed to new physics effects $[\delta a_\mu^{\rm NP}]$, then at 90% CL, $\delta a_\mu^{\rm NP}$ must lie in the range

$$215 \times 10^{-11} \lesssim \delta a_\mu^{\rm NP} \lesssim 637 \times 10^{-11} \,. \tag{1.2}$$

This contribution is positive, and is of the order of the electroweak corrections to a_μ. More precisely, the contribution needed from new physics effects has to be of the order of $G_F m_\mu^2/(4\pi^2\sqrt{2})$, where $G_F = 1.16637(1) \times 10^{-5}$ GeV^{-2} is Fermi's constant. In this paper, we consider the possibility that $\delta a_\mu^{\rm NP}$ arises entirely from the Higgs sector. In the SM, the Higgs boson contribution to a_μ is further suppressed (relative to the main electroweak contribution) by a factor of m_μ^2/m_h^2. In light of the recent SM Higgs mass limit, $m_h \gtrsim 113.5$ GeV obtained at the LEP collider [3], the SM Higgs contribution to a_μ is clearly negligible.

However, the Higgs sector contribution to a_μ could be considerably enhanced in a two-Higgs-doublet extension of the Standard Model (2HDM). The significance of the $(g-2)_\mu$ constraint for the 2HDM (in light of the LEP Higgs constraints) was emphasized in ref. [4], where the constraints of the previous BNL $(g-2)_\mu$ measurements were analyzed and the implications of future $(g-2)_\mu$ measurements were considered.[1] Now that we have the first possible indication of $\delta a_\mu^{\rm NP} \neq 0$, it is appropriate to revisit the question of the Higgs sector contribution to a_μ.

The enhancement of the Higgs sector contribution to a_μ relative to the SM result can arise from two different effects. First, an enhanced $h\mu^+\mu^-$ coupling proportional to the ratio of Higgs vacuum expectation values, $\tan\beta$, yields a Higgs contribution to $\delta a_\mu^{\rm NP}$ proportional to $\tan^2\beta$. Second, a suppressed hZZ coupling, proportional to $\sin(\beta-\alpha)$ [using notation reviewed below], can permit the existence of a CP-even Higgs boson mass substantially below the LEP SM Higgs mass limit. In units of $G_F m_\mu^2/(4\pi^2\sqrt{2})$, the overall enhancement is of order

$$\frac{m_\mu^2}{m_h^2} \times \tan^2\beta \times F\left(\frac{m_\mu^2}{m_h^2}\right) \simeq 1\text{—}10\ . \tag{1.3}$$

$F(x)$ is a loop factor which involves logarithms of the form $\ln(m_h^2/m_\mu^2) \sim \mathcal{O}(10)$. A light CP-even Higgs boson with $m_h \simeq 10$ GeV and $30 \lesssim \tan\beta \lesssim 50$, predicts a muon anomalous magnetic moment to lie in the 90% CL allowed range for new physics effects specified in eq. (1.2).

A 2HDM in which the Higgs sector contribution to $\delta a_\mu^{\rm NP}$ is significant is not compatible with the Higgs sector of the minimal supersymmetric extension of the Standard Model (MSSM). This is true because one cannot have a very light h with suppressed hZZ couplings without an observable rate for $Z \to hA$, in conflict with LEP data [6]. Moreover, the MSSM provides additional mechanisms for generating significant contributions to $\delta a_\mu^{\rm NP}$. A number of recent papers [7, 8, 9, 10, 11] have shown that the recent BNL measurement is compatible with supersymmetric contributions to $\delta a_\mu^{\rm NP}$ involving chargino and neutralino exchange, over an interesting region of MSSM parameter space.

In this paper, we focus on the possibility that the new physics contribution to a_μ arises solely from the Higgs sector. The two-doublet Higgs sector [12] contains eight scalar degrees of freedom. It is convenient to distinguish between the two doublets by employing one complex $Y=-1$ doublet, $\boldsymbol{\Phi_d}= (\Phi_d^0, \Phi_d^-)$ and one complex $Y=+1$ doublet, $\boldsymbol{\Phi_u}= (\Phi_u^+, \Phi_u^0)$. To avoid tree-level Higgs-mediated flavor changing neutral currents, we do not allow the most general Higgs–fermion interaction [13]. Instead,

[1] In ref. [4], it was assumed that the Higgs–fermion interaction was not the most general, but of a form that guarantees the absence of tree-level flavor–changing neutral Higgs couplings. Alternatively, one could assume the most general Higgs–fermion interaction (thereby generating tree-level Higgs-mediated flavor–changing neutral currents [FCNCs]), and choose the parameters of the model to avoid conflict with experimental limits on FCNCs. For example, such a model would possess a tree-level $h\mu^\pm\tau^\mp$ coupling, which could contribute significantly to $(g-2)_\mu$ [5]. We choose not to consider a 2HDM with flavor-changing neutral Higgs couplings in this paper.

we impose discrete symmetries (which may be softly-broken by mass terms), and consider two possible models [14]. In Model I, Φ_d^0 couples to both up-type and down-type quark and lepton pairs, while the coupling of Φ_u^0 to fermion pairs is absent.[2] In Model II, Φ_d^0 [Φ_u^0] couples exclusively to down-type [up-type] fermion pairs. When the Higgs potential is minimized, the neutral components of the Higgs fields acquire vacuum expectation values:[3]

$$\langle \mathbf{\Phi_d} \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_d \\ 0 \end{pmatrix}, \qquad \langle \mathbf{\Phi_u} \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_u \end{pmatrix}, \tag{1.4}$$

where the normalization has been chosen such that $v^2 \equiv v_d^2 + v_u^2 = (246 \text{ GeV})^2$, while the ratio $\tan\beta \equiv v_u/v_d$ is a free parameter of the model. The physical Higgs spectrum consists of a charged Higgs pair

$$H^\pm = \Phi_d^\pm \sin\beta + \Phi_u^\pm \cos\beta\,, \tag{1.5}$$

one CP-odd scalar

$$A = \sqrt{2}\left(\mathrm{Im}\,\Phi_d^0 \sin\beta + \mathrm{Im}\,\Phi_u^0 \cos\beta\right)\,, \tag{1.6}$$

and two CP-even scalars:

$$\begin{aligned} h &= -(\sqrt{2}\,\mathrm{Re}\,\Phi_d^0 - v_d)\sin\alpha + (\sqrt{2}\,\mathrm{Re}\,\Phi_u^0 - v_u)\cos\alpha\,, \\ H &= (\sqrt{2}\,\mathrm{Re}\,\Phi_d^0 - v_d)\cos\alpha + (\sqrt{2}\,\mathrm{Re}\,\Phi_u^0 - v_u)\sin\alpha\,, \end{aligned} \tag{1.7}$$

(with $m_h \leq m_H$). The angle α arises when the CP-even Higgs squared-mass matrix (in the Φ_d^0—Φ_u^0 basis) is diagonalized to obtain the physical CP-even Higgs states.

We briefly review the Higgs couplings relevant for our analysis. The tree-level h couplings to ZZ and AZ are given by

$$g_{hZZ} = \frac{g m_Z \sin(\beta-\alpha)}{\cos\theta_W}\,, \tag{1.8}$$

$$g_{hAZ} = \frac{g\cos(\beta-\alpha)}{2\cos\theta_W}\,. \tag{1.9}$$

For the corresponding couplings of H to ZZ and AZ, one must interchange $\sin(\beta-\alpha)$ and $\cos(\beta-\alpha)$ in the above formulae.

The pattern of couplings of the Higgs bosons to fermions depends on the choice of model. However, in this paper we are mainly concerned with the coupling of down-type fermions to Higgs bosons, which are the same in Model I and Model II.

[2] One can just as well assume that Φ_u^0 couples to both up-type and down-type quark and lepton pairs, while the coupling of Φ_d^0 to fermion pairs is absent. In this case, all the results of this paper would apply simply by replacing $\tan\beta$ with $\cot\beta$.

[3] In this paper, we neglect the possibility of significant CP-violation in the Higgs sector. In this case, the phases of the Higgs fields can be chosen such that the vacuum expectation values are real and positive.

For our analysis, the relevant couplings of the neutral Higgs bosons to $b\bar{b}$ or $\mu^+\mu^-$ relative to the SM value, m_f/v [$f=b$ or μ], are given by

$$h b\bar{b} \quad (\text{or } h\mu^+\mu^-): \qquad -\frac{\sin\alpha}{\cos\beta} = \sin(\beta-\alpha) - \tan\beta\cos(\beta-\alpha)\,, \tag{1.10}$$

$$H b\bar{b} \quad (\text{or } H\mu^+\mu^-): \qquad \frac{\cos\alpha}{\cos\beta} = \cos(\beta-\alpha) + \tan\beta\sin(\beta-\alpha)\,, \tag{1.11}$$

$$A b\bar{b} \quad (\text{or } A\mu^+\mu^-): \qquad \gamma_5\tan\beta\,, \tag{1.12}$$

(the γ_5 indicates a pseudoscalar coupling), and the charged Higgs boson couplings to muon pairs (with all particles pointing into the vertex) is given by

$$g_{H^-\mu^+\nu} = \frac{g m_\mu}{\sqrt{2} m_W} \ \tan\beta\, P_L\,, \tag{1.13}$$

where $P_L \equiv \frac{1}{2}(1-\gamma_5)$.

We have noted above that only light Higgs bosons with enhanced couplings to down-type fermions can contribute appreciably to $\delta a_\mu^{\rm NP}$. To avoid the LEP SM Higgs mass limit, such a light Higgs boson should be almost decoupled from ZZ. This implies that either h is light, with $|\sin(\beta-\alpha)| \ll 1$ [see eq. (1.8)] or A is light (since A has no tree-level coupling to vector boson pairs). In the next section, we will show that a light A makes a *negative* contribution to $\delta a_\mu^{\rm NP}$ and thus is not compatible with the recent BNL measurement. Hence, we focus on the 2HDM in which only h is light and $\sin(\beta-\alpha) \simeq 0$. From eq. (1.10), we see that if $\sin(\beta-\alpha) \simeq 0$, then the coupling of h to down-type fermions is proportional to $\tan\beta$. Thus, in the region of large $\tan\beta$ and small $\sin(\beta-\alpha)$, the contribution of a light CP-even Higgs boson of the 2HDM may yield a significant correction to $\delta a_\mu^{\rm NP}$ without being in conflict with the LEP SM Higgs search.

Although the considerations above apply to both Model I and Model II, it is important to note that the Higgs couplings to up-type fermions differ between the two models. The Model II $ht\bar{t}$ coupling relative to its SM value, m_t/v, is given by:

$$h t\bar{t} \ : \qquad \frac{\cos\alpha}{\sin\beta} = \sin(\beta-\alpha) + \cot\beta\cos(\beta-\alpha)\,, \tag{1.14}$$

whereas the Model I $ht\bar{t}$ coupling relative to m_t/v is the same as the Model II $hb\bar{b}$ coupling relavive to m_b/v. That is, for $\sin(\beta-\alpha)=0$, the Model II $ht\bar{t}$ coupling is proportional to $\cot\beta$ and is therefore suppressed at large $\tan\beta$, while in Model I, $|g_{ht\bar{t}}| = (m_t/v)\tan\beta \gg 1$. Thus, the $\tan\beta$ enhanced Model I Higgs couplings to $t\bar{t}$ are non-perturbative at large $\tan\beta$. Both theoretical and experimental considerations lead us to reject this possibility. Henceforth, we will assume that the 2HDM contains Model II Higgs–fermion couplings.

Finally, we note that in the parameter region cited above, the heavier Higgs bosons, H, A, $H^\pm$, cannot be arbitrarily heavy. If one attempts to take such a limit, one finds that there must be some Higgs quartic self-couplings that become significantly larger than 1 [15]. That is, this model does not possess a decoupling limit. However, the model stays weakly coupled as long as the heavier Higgs states are not too much larger than $v = 246$ GeV. In contrast, in the limit of $\cos(\beta - \alpha) = 0$, the couplings of h reduce to those of the SM Higgs boson. This decoupling limit can be formally reached by taking the masses of H, A, $H^\pm$ to be arbitrarily large, while keeping the quartic Higgs self-couplings $\lesssim \mathcal{O}(1)$ [15]. The resulting low-energy effective theory is just the SM with one Higgs doublet. Of course, as we have noted above, the contribution of SM Higgs boson to $\delta a_\mu^{\rm NP}$ is negligible. Thus, over an intermediate range of heavy Higgs masses, the contributions of H, A, $H^\pm$ (which are $\tan^2\beta$ enhanced) to $\delta a_\mu^{\rm NP}$ will be significantly larger than that of h even though $\cos(\beta - \alpha) \simeq 0$.

2. Model II Higgs boson corrections to the muon anomalous magnetic moment

The first calculation of the one-loop electroweak corrections to the muon anomalous magnetic moment was presented by Weinberg and Jackiw [16] and by Fujikawa, Lee and Sanda [17]. A very useful compendium of formulae for the one-loop corrections to $g-2$ in a general electroweak model was given in ref. [18], and applied to the 2HDM in ref. [19].[4] In the 2HDM, both neutral and charged Higgs bosons contribute to $g-2$. A convenient list of the relevant formulae can be found in ref. [4].

$$\delta a_\mu^h = \frac{G_F m_\mu^2}{4\pi^2\sqrt{2}} \left(\frac{\sin\alpha}{\cos\beta}\right)^2 R_h \; F_h(R_h) \tag{2.1}$$

$$\delta a_\mu^H = \frac{G_F m_\mu^2}{4\pi^2\sqrt{2}} \left(\frac{\cos\alpha}{\cos\beta}\right)^2 R_H \; F_H(R_H) \tag{2.2}$$

$$\delta a_\mu^A = \frac{G_F m_\mu^2}{4\pi^2\sqrt{2}} \; \tan^2\beta \; R_A \; F_A(R_A) \tag{2.3}$$

$$\delta a_\mu^{H^\pm} = \frac{G_F m_\mu^2}{4\pi^2\sqrt{2}} \; \tan^2\beta \; R_{H^\pm} \; F_{H^\pm}(R_{H^\pm}, R_\nu) \tag{2.4}$$

where $R_{h,H,A,H^\pm} \equiv m_\mu^2/m_{h,H,A,H^\pm}^2$, $R_\nu \equiv m_\nu^2/m_{H^\pm}^2$ and

$$F_{h,H}(R_{h,H}) = \int_0^1 dx \, \frac{x^2(2-x)}{R_{h,H}x^2 - x + 1} \,, \tag{2.5}$$

$$F_A(R_A) = \int_0^1 dx \, \frac{-x^3}{R_A x^2 - x + 1} \,, \tag{2.6}$$

$$F_{H^\pm}(R_{H^\pm}, R_\nu) = \int_0^1 dx \, \frac{-x^2(1-x)}{R_{H^\pm}x^2 + (1 - R_{H^\pm} - R_\nu)x + R_\nu} \,, \tag{2.7}$$

[4]Here, we correct a small error in the expression in the $H^\pm$ contribution given in ref. [19].

The neutrino mass is negligible, so henceforth we set $R_\nu = 0$. Since $R_{h,H,A,H^\pm} \ll 1$, one can easily expand the above integrals in the corresponding small parameter. In the next two subsections, we write out the leading terms in this expansion, which are quite accurate in the Higgs mass range of interest.[5]

2.1 Non-decoupling limit: $\sin(\beta - \alpha) = 0$

In section 1, we argued that the most significant Higgs contribution to $\delta a_\mu^{\rm NP}$ (consistent with the LEP SM Higgs search) arises in the parameter regime in which $\sin(\beta-\alpha) \simeq 0$ and $\tan\beta \gg 1$. Setting $\sin(\beta-\alpha) = 0$ and keeping only the leading terms in R when evaluating the above integrals, the total Higgs sector contribution to a_μ is given by:

$$\begin{aligned} \delta a_\mu^{\rm Higgs} &= \delta a_\mu^h + \delta a_\mu^H + \delta a_\mu^A + \delta a_\mu^{H^\pm} \\ &\simeq \frac{G_F m_\mu^2}{4\pi^2\sqrt{2}} \tan^2\beta \left\{ \frac{m_\mu^2}{m_h^2}\left[\ln\left(\frac{m_h^2}{m_\mu^2}\right) - \frac{7}{6}\right] - \frac{m_\mu^2}{m_A^2}\left[\ln\left(\frac{m_A^2}{m_\mu^2}\right) - \frac{11}{6}\right] - \frac{m_\mu^2}{6m_{H^\pm}^2} \right\} . \end{aligned} \tag{2.8}$$

Note that the logarithms appearing in eq. (2.8) always dominate the corresponding constant terms when the Higgs masses are larger than 1 GeV. It is then clear that A and $H^\pm$ exchange contribute a negative value to $\delta a_\mu^{\rm NP}$. Since our goal is to explain the BNL $g-2$ measurement which suggests a positive value for $\delta a_\mu^{\rm NP}$, we should take m_A and $m_{H^\pm}$ large (masses above 100 GeV are sufficient) in order that the corresponding A and $H^\pm$ negative contributions are neglibly small.[6] If $\delta a_\mu^{\rm NP}$ is to be a consequence of the Higgs sector, it must be entirely due to the contribution of the light CP-even Higgs boson. Note that the heavier CP-even Higgs, H, does not give a contribution proportional to $\tan\beta$ (as shown in section 1); hence its contribution to $\delta a_\mu^{\rm NP}$ can be neglected in eq. (2.8). Thus, to a good approximation,

$$\delta a_\mu^{\rm Higgs} \simeq \delta a_\mu^h \simeq \frac{G_F m_\mu^2}{4\pi^2\sqrt{2}} \left(\frac{m_\mu^2}{m_h^2}\right) \tan^2\beta \left[\ln\left(\frac{m_h^2}{m_\mu^2}\right) - \frac{7}{6}\right] . \tag{2.10}$$

One can check that a light Higgs boson with a mass of around 10 GeV and with $\tan\beta = 35$ gives $\delta a_\mu^{\rm Higgs} \simeq 280 \times 10^{-11}$, which is within the 90% CL allowed range for $\delta a_\mu^{\rm NP}$ quoted in eq. (1.2). Contour lines corresponding to a full numerical

[5] The plot shown in this paper is based on the exact values of the above integrals.

[6] Grifols and Pascual [20] found that for a very light charged Higgs boson, the two-loop contribution to a_μ is positive and can be larger in magnitude than the one-loop result given in eq. (2.4):

$$\delta a_\mu^{H^\pm} = a_\mu^{H^\pm}(1\text{–loop}) + \frac{1}{180}\left(\frac{\alpha}{\pi}\right)^2 \left(\frac{m_\mu}{m_{H^\pm}}\right)^2 + \mathcal{O}\left[\left(\frac{m_\mu}{m_{H^\pm}}\right)^4 \ln\left(\frac{m_\mu}{m_{H^\pm}}\right)\right] . \tag{2.9}$$

However, the LEP bound on the charged Higgs mass ref. [21], $m_{H^\pm} > 78.7$ GeV, implies that both the one and two-loop charged Higgs contribution to $\delta a_\mu^{\rm NP}$ are negligible.

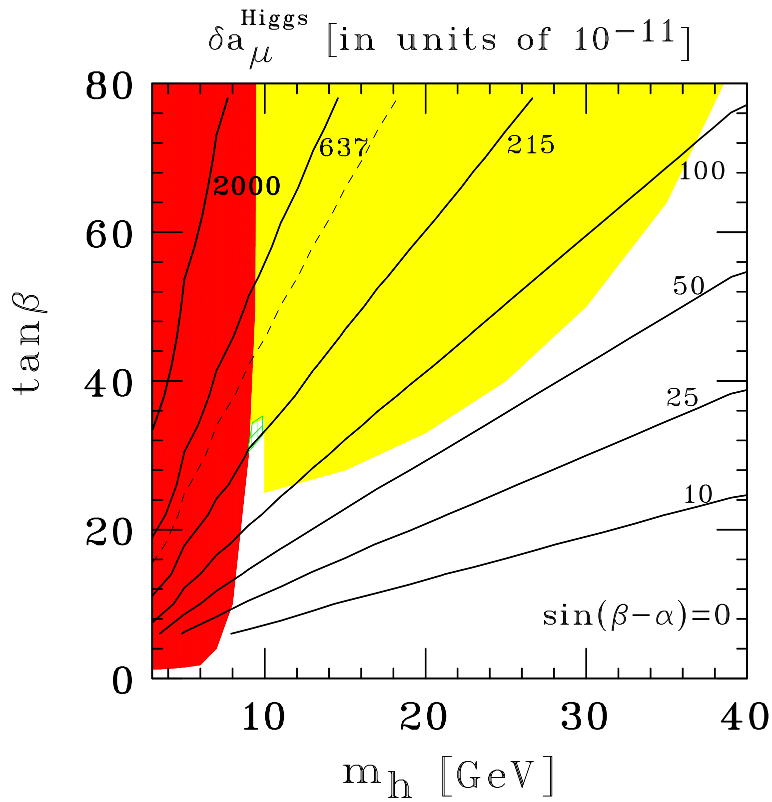

Figure 1: Contours of the predicted one-loop Higgs sector contribution to the muon anomalous magnetic moment, $\delta\alpha_\mu^{\rm Higgs}$ (in units of 10^{-11}) in the 2HDM, assuming that $\sin(\beta-\alpha)=0$, and $m_H=m_A=m_{H^\pm}=200$ GeV (there is little sensitivity to the heavier Higgs masses). The dashed line contour corresponds to the central value of $\delta a_\mu \equiv a_\mu^{\rm exp}-a_\mu^{\rm SM}$, as reported in ref. [1]. The contour lines marked 215 and 637 correspond to 90% CL limits for the contribution of new physics to a_μ [eq. (1.2)]. The dark-shaded (red) region is excluded by the CUSB Collaboration search for $\Upsilon \to h\gamma$ at CESR [22]. The light-shaded (yellow) region is excluded at 95% CL by the ALEPH and DELPHI searches for $e^+e^- \to hf\bar{f}$ ($f=b$ or τ) at LEP [23, 24]. In the small hatched region (green) nestled between the two experimentally excluded shaded regions, above the 215 contour line and centered around $m_h \simeq 10$ GeV, the Higgs sector contribution to $\delta a_\mu^{\rm NP}$ lies within the 90% CL allowed range [eq. (1.2)].

evaluation of the Higgs sector one-loop contribution to $\delta a_\mu^{\rm Higgs}$ [in units of 10^{-11}] are exhibited in fig. 1, for $\sin(\beta-\alpha)=0$ and $m_H=m_A=m_{H^\pm}=200$ GeV.[7] The relevant experimental bounds are also displayed in fig. 1; these limits are reviewed in section 3. A careful inspection of the excluded region in the m_h *vs.* $\tan\beta$ parameter space shows that a light Higgs boson of around 10 GeV mass and $30 \lesssim \tan\beta \lesssim 35$ is permitted. In this parameter regime, we obtain a value for $\delta a_\mu^{\rm NP}$ within the 90% CL

[7] The results are insensitive to the values of the heavy Higgs masses above 100 GeV.

allowed range of eq. (1.2). However, the central value of $\delta a_\mu^{\rm NP}$ given in eq. (1.2) lies within the excluded regions of fig. 1.

2.2 Decoupling limit: $\cos(\beta-\alpha)=0$

In the decoupling limit, where $\cos(\beta-\alpha)\simeq 0$ and $m_A \gg m_Z$, the couplings of the light Higgs boson, h, are (nearly) identical to those of the SM Higgs boson. As a result, the LEP SM Higgs mass bound of $m_h \gtrsim 113.5$ GeV applies. For $\cos(\beta-\alpha)=0$, the H couplings to down-type fermion pairs are enhanced by $\tan\beta$ [see eq. (1.10)]. Thus, the Higgs sector contribution to $\delta a_\mu^{\rm NP}$ is given by eq. (2.8), with m_h replaced by m_H. In the decoupling limit, $m_H \simeq m_A \simeq m_{H^\pm}$ [the mass differences are of $\mathcal{O}(m_Z^2/m_A)$]. Setting $\cos(\beta-\alpha)=0$ and $m_H=m_A=m_{H^\pm}$, we find

$$\delta a_\mu^{\rm Higgs} \simeq \frac{G_F m_\mu^2}{4\pi^2\sqrt{2}}\left(\frac{m_\mu^2}{m_A^2}\right)\tan^2\beta\left[\frac{1}{2}-\left(\frac{2m_\mu^2}{m_A^2}\right)\ln\left(\frac{m_A^2}{m_\mu^2}\right)\right]. \qquad (2.11)$$

The contribution of h is not $\tan\beta$–enhanced and is thus negligible. In is interesting to note that for values of $m_A \lesssim m_h\tan\beta$, the heavier ("decoupled") Higgs bosons actually dominate in the Higgs sector contribution to $\delta a_\mu^{\rm NP}$.[8] However, for 100 GeV $< m_A <$ 1000 GeV, and $30 < \tan\beta < 100$, the Higgs sector contribution to a_μ ranges from about 5×10^{-12} to 5×10^{-14}, which is three to five orders of magnitude below what is needed to explain the BNL measurement of a_μ.

3. CESR and LEP constraints on a light Higgs boson

Let us consider the 2HDM in which $\sin(\beta-\alpha)=0$, $\tan\beta\gg 1$ and $m_h\sim\mathcal{O}(10~\text{GeV})$, which are necessary conditions if the Higgs sector is to be the source for $\delta a_\mu^{\rm NP}$ in the range given by eq. (1.2). The hAZ coupling is maximal [eq. (1.9)], so we must assume that m_A is large enough so that $e^+e^-\to hA$ is not observed at LEP. The tree-level hZZ coupling is absent, which implies that the LEP SM Higgs search based on $e^+e^-\to Z\to Zh$ does not impose any significant constraints on m_h.[9] However, there are a number of constraints on light Higgs masses that do not rely on the hZZ coupling. For Higgs bosons with $m_h\lesssim 5$ GeV, the SM Higgs boson was ruled out by a variety of arguments that were summarized in ref. [12]. For 5 GeV $\lesssim m_h \lesssim 10$ GeV, the relevant Higgs boson constraint can be derived from the absence of Higgs production in $\Upsilon\to h\gamma$.

An experimental search for $\Upsilon\to h\gamma$ by the CUSB Collaboration at CESR [22] found no candidates. The Higgs mass limit obtained from this result depends on

[8] If we formally take $m_A\to\infty$, we recover the Standard Model Higgs contribution to a_μ.

[9] Presumably, radiative corrections would lead to a small effective value for $\sin(\beta-\alpha)$. The LEP Higgs search yields an excluded region in the $\sin(\beta-\alpha)$ *vs.* m_h plane, and implies that for $m_h\sim 10$ GeV, $|\sin(\beta-\alpha)|\lesssim 0.06$ is not excluded at 95% CL [25, 26].

the theoretical prediction. In addition to the non-relativistic, tree-level prediction of ref. [27], there are three classes of corrections that have been explored in the literature: $\mathcal{O}(\alpha_s)$ hard QCD corrections [28, 29], relativistic corrections to the non-relativistic treatment of the $b\bar{b}$ bound state [30, 31], and bound state threshold corrections [32]. The theoretical picture that emerges is uncertain. The hard QCD corrections are large and suggest that $\mathcal{O}(\alpha_s^2)$ corrections could be significant. In addition, relativistic effects enter at the same order as the $\mathcal{O}(\alpha_s)$ corrections; both are of $\mathcal{O}(v^2/c^2)$ and the two must be treated consistently. Finally, ref. [32] argued that strong cancellations can occur among various contributions in the threshold region, leading to an additional suppression in rate of about 14 for $m_h = 8.5$ GeV (and even a larger suppression as $m_h \to M_\Upsilon$). The application of the theoretical analysis of $\Gamma(\Upsilon \to h\gamma)$ to the CUSB data suggests that values of $m_h \lesssim 5$—7 GeV can be ruled out at 95% CL, although a precise upper limit cannot be obtained due to the theoretical uncertainties outlined above.

The above discussion was relevant for obtaining a limit on the mass of the SM Higgs boson. In the 2HDM considered here, $\tan\beta \gg 1$, and the prediction for $\Gamma(\Upsilon \to h\gamma)$ is enhanced by a factor of $\tan^2\beta$. For values of $\tan\beta \gtrsim 10$, the CUSB data can reliably rule out Higgs masses up to about 8 GeV. As $m_h \to M_\Upsilon$, the precise experimental limit is not very well known due to the theoretical uncertainties near threshold mentioned above. Our estimate for the excluded region for $m_h \lesssim M_\Upsilon$ is indicated by the dark (red) shaded region in fig. 1. Note that for Higgs masses above 8 GeV, $\tan\beta \gtrsim 30$ if the Higgs sector contribution to $\delta a_\mu^{\rm NP}$ lies in the 90% CL range specified in eq. (1.2). For such large values of $\tan\beta$, the predicted rate for $\Upsilon \to h\gamma$ is increased by at least three orders of magnitude relative to the SM. This factor should dwarf the theoretical uncertainties discussed above except for values of m_h very close to M_Υ. Thus, in the 2HDM parameter regime of interest, we obtain a lower bound of $m_h \gtrsim M_\Upsilon$.

A second bound on m_h can be derived from the non-observation of Higgs bosons at LEP via the process $e^+e^- \to hf\bar{f}$ ($f = b,\, \tau$). The cross-section for this process depends on the h Yukawa couplings to down-type fermions. In the 2HDM with $\sin(\beta-\alpha) = 0$, these Yukawa couplings are enhanced (relative to the corresponding SM value) by $\tan\beta$. Preliminary analyses by the ALEPH and DELPHI Collaborations at LEP based on the search for $e^+e^- \to hf\bar{f}$ ($f = b,\, \tau$), where $h \to \tau^+\tau^-$, $b\bar{b}$, find no evidence for light Higgs boson production [23, 24]. Combining the two analyses, we exclude at 95% CL the light-shaded (yellow) region of fig. 1. Note that the lower limit on $\tan\beta$ changes discontinuously at $2m_B$, where B is the lightest B-meson [$m_B = 5.279$ GeV]. For Higgs masses that lie in the range $2m_\tau \lesssim m_h \lesssim 2m_B$, the dominant Higgs decay mode is $h \to \tau^+\tau^-$.[10] In this mass range, the ALEPH limit on $\tan\beta$ is better than the corresponding DELPHI limit. In particular, for

[10] By assumption, $\tan\beta \gg 1$ and the rate for $h \to c\bar{c}$ is suppressed by a factor of $\cot^2\beta$.

$M_\Upsilon \lesssim m_h \lesssim 2m_B$, the ALEPH excluded region implies that $\tan\beta \lesssim 35$. For values of $m_h > 2m_B$, the Higgs decays primarily into $b\bar{b}$, and the DELPHI limit (which is more powerful than the ALEPH limit in this mass range) completely excludes the region of parameter space in which the Higgs sector contribution to $\delta a_\mu^{\rm NP}$ lies in the 90% CL range specified in eq. (1.2).

One other light Higgs process observable at LEP that is sensitive to the Higgs–fermion Yukawa couplings, even in the absence of the ZZh and W^+W^-h couplings, is the one-loop process $Z \to h\gamma$. Both up-type and down-type fermions contribute in the loop, so the decay rate in Model I and Model II differs. Ref. [33] analyzes the implication of this process for the general 2HDM with Model II couplings and shows that the LEP experimental constraints in the m_h *vs.* $\tan\beta$ plane for $\tan\beta > 1$ are weaker than the ones obtained from $e^+e^- \to hf\bar{f}$ discussed above. In Model I, we can can use the results of ref. [33] simply by interchanging $\tan\beta$ and $\cot\beta$. For $m_h \sim 10$ GeV, the LEP experimental constraints imply that $\tan\beta < 10$. Thus, we have an independent reason to conclude that the Model I 2HDM cannot provide an explanation for the BNL measurement of a_μ.

Finally, one must check the implications of the precision electroweak data for constraining the Type II 2HDM with a light Higgs boson. This data is known to provide an excellent fit to the Standard Model with one Higgs doublet and $m_h = 86^{+48}_{-32}$ GeV [34]. Nevertheless, ref. [35] demonstrates that even with a light Higgs mass below 20 GeV, the CP-conserving Type II 2HDM provides an equally good fit to the precision electroweak data.

One byproduct of this analysis is the potential for an improved exclusion limit on the CP-odd Higgs boson mass in the region of light m_A. In the m_A *vs.* $\tan\beta$ plane, the experimentally excluded region in a general 2HDM is essentially the same as the shaded regions of fig. 1, based on the absence of $e^+e^- \to Af\bar{f}$ ($f = b$ or τ) and $\Upsilon \to A\gamma$. If $m_A \ll m_h$, m_H, $m_{H^\pm}$, then eq. (2.10) is replaced by:

$$\delta a_\mu^{\rm Higgs} \simeq \delta a_\mu^A \simeq \frac{-G_F m_\mu^2}{4\pi^2\sqrt{2}} \left(\frac{m_\mu^2}{m_A^2}\right) \tan^2\beta \left[\ln\left(\frac{m_A^2}{m_\mu^2}\right) - \frac{11}{6}\right] . \tag{3.1}$$

The $\delta a_\mu^{\rm Higgs}$ contours shown in fig. 1 would apply in this case [independent of the value of $\sin(\beta-\alpha)$] if each number accompanying the contours is multiplied by -0.9 (approximately). Technically, one cannot use this to exclude any region of m_A *vs.* $\tan\beta$ parameter space, since the negative contribution of eq. (3.1) can be canceled by some positive contribution (which by the assumption of eq. (1.2) must exist). However, if a future measurement were to establish that $\delta a_\mu^{\rm NP} \simeq 0$, then barring an accidental cancellation from more than one source of new physics, it would be possible to significantly extend the present excluded region in the m_A *vs.* $\tan\beta$ plane.

4. Final Results and Conclusions

If we combine the experimental bounds on the Higgs mass discussed in section 3, we conclude that a light Higgs boson can be responsible for the observed 2.6σ deviation of the BNL measurement of the muon anomalous magnetic moment at the 90% CL in the framework of a two-Higgs-doublet model with Model II Higgs-fermion Yukawa couplings only if the model parameters satisfy the following requirements:

$$\begin{aligned} & m_\Upsilon \lesssim m_h \lesssim 2m_B\,, \\ & \sin(\beta-\alpha) \simeq 0\,, \\ & 30 \lesssim \tan\beta \lesssim 35\,. \end{aligned} \tag{4.1}$$

In addition, H, A and $H^\pm$ must be sufficiently heavy to satisfy the LEP experimental constraints. In the model specified above, the SM Higgs mass bound applies to H so that $m_H \gtrsim 113.5$ GeV. The constraint on m_A is deduced from the absence of $Z \to hA$ (either by direct observation or as inferred from the measured width of the Z), which implies that $m_A \gtrsim 80$ GeV.[11] Finally, in a general 2HDM, $m_{H^\pm} \gtrsim 78.7$ GeV [21].

One noteworthy consequence of $m_h \sim 10$ GeV is the possibility of mixing between the h and the 0^{++} $b\bar{b}$ bound states $\chi_{b0}(1P)$ and $\chi_{b0}(2P)$, as discussed in refs. [19] and [36]. As a result, the decay $\chi_{b0} \to \tau^+\tau^-$ should be prominent. The predicted rate is roughly

$$\frac{\Gamma(\chi_{b0} \to \tau^+\tau^-)}{\Gamma(\chi_{b0} \to \text{ hadrons})} \simeq \frac{2.5 \times 10^{-7}\ \text{GeV}^2}{(m_\chi - m_h)^2} \tan^4\beta\,, \tag{4.2}$$

which is valid for m_h near m_χ but separated by a few Higgs widths.[12] Due to the large $\tan^4\beta$ enhancement, the predicted branching ratio for $\chi_{b0} \to \tau^+\tau^-$ can be substantial. Remarkably, the Particle Data Group [37] provides no data on possible decay modes of the χ_{b0} other than the radiative decays, $\chi_{b0} \to \Upsilon\gamma$, $\Upsilon'\gamma$.

Apart from a careful study of χ_{b0} decays, the 2HDM specified by eq. (4.1) could be confirmed or ruled out by a more complete analysis by the LEP Collaborations of their data in search of $e^+e^- \to hf\bar{f}$ ($f = b$ or τ). We note that the ALEPH and DELPHI exclusion plots used in fig. 1 are based on a preliminary analyses and have not formally appeared in the literature. Without employing these LEP limits, the allowed 2HDM parameter space in which h contributes significantly to $\delta a_\mu^{\rm NP}$ is substantially larger. As advocated in ref. [38], the $\tan\beta$ exclusion limit could

[11] With further LEP analysis, it might be possible to push the limit on m_A higher. The large $\tan\beta$ MSSM Higgs analysis implies that $m_h + m_A \gtrsim 180$ GeV due to the non-observation of $e^+e^- \to hA$. However, this analysis, which searches for hA via a four jet topology, is highly inefficient for a very light h and is thus not applicable to the present model.

[12] If the two masses are within a Higgs width, then the mixing of the two states will be close to maximal [36], and the corresponding $\tau^+\tau^-$ branching ratio of both eigenstates would be close to 100% due to the large $\tan^4\beta$ enhancement.

be lowered if a complete analysis were performed using all of the LEP data. The potential significance of such a result should be clear from fig. 1.

In the absence of additional information from the LEP collider, one must wait for a further improvement of the BNL measurement of the muon anomalous magnetic moment. A factor of four increase in data is expected when the data sets from the 2000 and 2001 runs are fully analyzed . If the significance of a nonzero result for $\delta a_\mu^{\rm NP}$ increases, it will be crucial to discover the source of the new physics. To further constrain the Higgs sector contribution to $\delta a_\mu^{\rm NP}$, a high energy e^+e^- linear collider that can perform precision studies of Higgs processes is required [39]. One must either discover a light Higgs boson with $m_h \sim 10$ GeV or improve the present constraints in the m_h *vs.* $\tan\beta$ plane.

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