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A Light Higgs Boson Explanation for the $g-2$ Crisis [a]

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A light CP-even Higgs boson with a mass of around 10 GeV could explain the recent BNL measurement of the muon anomalous magnetic moment. This observation is based on a general CP-conserving two Higgs doublet extension of the Standard Model with no tree-level flavor changing neutral current couplings. The Higgs mass is constrained by experiments at CESR and LEP to be less than twice the lightest B-meson mass and greater than (roughly) the Upsilon mass. It may be possible to exclude or discover such a Higgs boson by fully analyzing the existing LEP data.

1 A possibility for New Physics from the $g-2$ measurement at BNL E821

Based on Davier-Höcker calculation [1] for the hadronic contributions to the muon anomalous magnetic moment, BNL [2] has reported a 2.6σ deviation from the Standard Model (SM) prediction. There are four possible interpretations of this discrepancy [3,4] :

- a statistical fluctuation with 0.9% probability.
- something is "wrong" with the experiment : this is going to be resolved within the coming year [3] since the data from the 2000 and 2001 runs have yet to be analyzed.
- something is "wrong" with the theoretical analysis of the hadronic contribution to the muon anomalous magnetic moment, $a_\mu^{\rm had}$. This analysis depends critically on the experimental data for $\sigma(e^+e^- \to {\rm hadrons})$ below $\sqrt{s} = 2$ GeV. One expects forthcoming data from Novorsibirsk and BES, and an intense theoretical effort from a number of groups [1,5,6,7,8] to reduce the systematic uncertainties that contribute to $a_\mu^{\rm had}$. [It is important to note here that the error on $a_\mu^{\rm had}$ is necessarily larger when not using tau data. Indeed, two [1,7] of the recent numbers have significantly been improved by the use of hadronic tau data (in the two and four pion final states) in conjunction with the conserved vector current. New results on $\tau^+ \to \pi^+\pi^0\nu$ decays are expected from ALEPH soon [9].]
- the by far most exciting possibility : there is a contribution due to New Physics beyond the SM, $\delta a_\mu^{\rm NP}$. Based on two different analyses of the hadronic contributions, one finds at 90% CL [1,7]

$$170 \times 10^{-11} \lesssim \quad \delta a_\mu^{\rm NP} \quad \lesssim 690 \times 10^{-11} \quad [{\rm Davier-Hocker(98)}]\,, \tag{1}$$

$$97 \times 10^{-11} \lesssim \quad \delta a_\mu^{\rm NP} \quad \lesssim 667 \times 10^{-11} \quad [{\rm Narison(01)}]\,. \tag{2}$$

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In what follows, we assume the fourth possibility as an explanation to the discrepancy under discussion. Two remarks concerning eqs. (1) and (2) are in order [4] : *(i)* the contribution from the New Physics is positive, $\delta a_\mu^{\rm NP} > 0$, and *(ii)*, is of the order of the Electroweak (EW) contributions, *i.e.,* $\delta a_\mu^{\rm NP} \propto G_F m_\mu^2/(4\pi^2\sqrt{2})$.

In the SM, the 1-loop Higgs boson contribution to the muon anomalous magnetic moment is suppressed by a factor of m_μ^2/m_h^2. This factor is particularly small given that the SM Higgs mass bound from LEP is $m_h > 113.5$ GeV. The situation is potentially different if we extend the SM by adding an extra Higgs doublet [10]. The idea that a light Higgs boson could enhanced the predicted value of a_μ has been recently employed in [11,12]. Here we strictly follow the discussion of [11] [b]. The enhancement arises from the Higgs sector is twofold: *(i)* the coupling of the muon to one of the CP-even Higgs bosons (h or H), CP-Odd Higgs (A) and charged higgs ($H^\pm$) is proportional to $\tan\beta$ (the ratio of the two vacuum expectation values), and hence $\delta a_\mu^{\rm NP} \propto \tan^2\beta$; *(ii)* the Higgs coupling to the Z-boson can be set to zero, *i.e.,* $\sin(\beta-\alpha) = 0$, (where α is the CP-even Higgs bosons mixing angle) and thus LEP constraints from $e^+e^- \to hZ$ do not apply [14]. In this case, as we show below, the Higgs mass can be as light as ~ 10 GeV.

2 Model II Higgs boson corrections to a_μ

The first calculation of the one-loop electroweak corrections to the muon anomalous magnetic moment was presented by Weinberg and Jackiw [15] and by Fujikawa, Lee and Sanda [16]. A very useful compendium of formulae for the one-loop corrections to $g-2$ in a general electroweak model was given in [17], and applied to the 2HDM in [18]. In the 2HDM, both neutral and charged Higgs bosons contribute to $g-2$. One can derive approximate results by expanding the loop integrals in terms of the parameters $m_\mu^2/m_{h,H,A,H^\pm}^2$.

At the end of section 1, we argued that the most significant Higgs contribution to $\delta a_\mu^{\rm NP}$ (consistent with the LEP SM Higgs search) arises in the parameter regime in which $\sin(\beta-\alpha) \simeq 0$ and $\tan\beta \gg 1$. Setting $\sin(\beta-\alpha) = 0$ and keeping only the leading terms in $m_\mu^2/m_{h,H,A,H^\pm}^2$ when evaluating the above integrals, the total Higgs sector contribution to a_μ at 1-loop[c] is given by:

$$\begin{aligned} \delta a_\mu^{\rm Higgs} &= \delta a_\mu^h + \delta a_\mu^H + \delta a_\mu^A + \delta a_\mu^{H^\pm} \\ &\simeq \frac{G_F m_\mu^2}{4\pi^2\sqrt{2}} \tan^2\beta \left\{ \frac{m_\mu^2}{m_h^2}\left[\ln\left(\frac{m_h^2}{m_\mu^2}\right) - \frac{7}{6}\right] - \frac{m_\mu^2}{m_A^2}\left[\ln\left(\frac{m_A^2}{m_\mu^2}\right) - \frac{11}{6}\right] - \frac{m_\mu^2}{6m_{H^\pm}^2} \right\}. \end{aligned} \tag{3}$$

Note that the logarithms appearing in eq.(3) always dominate the corresponding constant terms when the Higgs masses are larger than 1 GeV. It is then clear that A and $H^\pm$ exchange contribute a negative value to $\delta a_\mu^{\rm NP}$ and thus cannot explain the BNL $g-2$ measurement which suggests a positive value for $\delta a_\mu^{\rm NP}$. In addition, we should take m_A and $m_{H^\pm}$ large (masses above 100 GeV are sufficient) in order that the corresponding A and $H^\pm$ negative contributions are neglibly small. If $\delta a_\mu^{\rm NP}$ is to be a consequence of the Higgs sector, it must be entirely due to the contribution of the light CP-even Higgs boson. Note that the heavier CP-even Higgs, H, does not give a contribution proportional to $\tan\beta$; hence its contribution to $\delta a_\mu^{\rm NP}$ can be neglected in eq.(3). Thus, to a good approximation,

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[b] For the purposes of this presentation we refer to the Two Higgs Doublet Model (II) [2HDM(II)] [10]. For the Model I see the discussion in [11] and for Model III see [13].

[c] All the results given here are based on a one loop calculation. The authors of [19] argue that significant two-loop contributions contribute to $\delta a_\mu^{\rm Higgs}$ due primarily to the effect of one particular [Barr-Zee] diagram, which is of the same order as the corresponding one-loop contribution, but with opposite sign. Adding this contribution to that of eq. (3) therefore reduces the result for $\delta a_\mu^{\rm Higgs}$ at fixed m_h, implying that $m_h \lesssim m_b$ in order that the total Higgs contribution to a_μ lie within the range suggested by eqs. (1) and (2). However, this possibility is excluded by the non-observation of $\Upsilon \to h\gamma$ at CESR. By using the results of [20] for the two loop contributions from the CP-Odd Higgs boson to the g-2, reference [21] argues that the two-loop contribution of the Barr-Zee diagram could change the sign of the one-loop Higgs contribution, in which case a light CP-odd Higgs boson could generate an overall *positive* contribution to δa_μ of the required magnitude. However, there is at present no complete two-loop computation of the electroweak contribution to δa_μ in the 2HDM. Whether the dominance of the Barr-Zee diagram survives a more complete two-loop analysis remains to be seen.

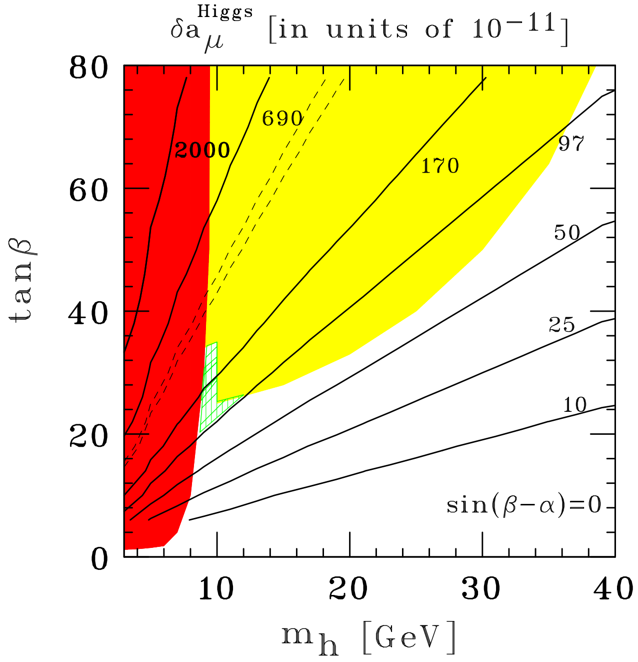

Figure 1: Contours of the predicted one-loop Higgs sector contribution to the muon anomalous magnetic moment, $\delta a_\mu^{\rm Higgs}$ (in units of 10^{-11}) in the 2HDM, assuming that $\sin(\beta-\alpha)=0$, and $m_H=m_A=m_{H^\pm}=200$ GeV (there is little sensitivity to the heavier Higgs masses). The dashed line contours correspond to the central value of $\delta a_\mu \equiv a_\mu^{\rm exp} - a_\mu^{\rm SM}$, as reported by Brown *et.al* and by Narison. The contour lines marked 170 and 690 correspond to 90% CL limits for the contribution of new physics to a_μ [eq. (1)]. The contour 97 corresponds to the lower bound on $\delta a_\mu^{\rm NP}$ given in eq.(2). The dark-shaded (red) region is excluded by the CUSB Collaboration search for $\Upsilon \to h\gamma$ at CESR. The light-shaded (yellow) region is excluded at 95% CL by the ALEPH and DELPHI searches for $e^+e^- \to hf\bar{f}$ ($f=b$ or τ) at LEP. In the small hatched region (green) nestled between the two experimentally excluded shaded regions, above the 97 contour line and centered around $m_h \simeq 10$ GeV, the Higgs sector contribution to $\delta a_\mu^{\rm NP}$ lies within the 90% CL allowed range.

$$\delta a_\mu^{\rm Higgs} \simeq \delta a_\mu^h \simeq \frac{G_F m_\mu^2}{4\pi^2\sqrt{2}} \left(\frac{m_\mu^2}{m_h^2}\right) \tan^2\beta \left[\ln\left(\frac{m_h^2}{m_\mu^2}\right) - \frac{7}{6}\right] . \tag{4}$$

One can check that a light Higgs boson with a mass of around 10 GeV and with $\tan\beta = 35$ gives $\delta a_\mu^{\rm Higgs} \simeq 280 \times 10^{-11}$, which is within the 90% CL allowed range for $\delta a_\mu^{\rm NP}$ quoted in eqs. (1) and (2). Contour lines corresponding to a full numerical evaluation of the Higgs sector one-loop contribution to $\delta a_\mu^{\rm Higgs}$ [in units of 10^{-11}] are exhibited in fig. 1, for $\sin(\beta-\alpha)=0$ and $m_H = m_A = m_{H^\pm} = 200$ GeV.[d] The relevant experimental bounds are also displayed in fig. 1; [see discussion below]. A careful inspection of the excluded region in the m_h *vs.* $\tan\beta$ parameter space shows that a light Higgs boson of around 10 GeV mass and $20 \lesssim \tan\beta \lesssim 35$ is permitted depending on the lower bound of eqs. (1) and (2). In this parameter regime, we obtain a value for $\delta a_\mu^{\rm NP}$ within the 90% CL allowed range of eq. (1). However, the central values of $\delta a_\mu^{\rm NP}$ given in [4,7] lie within the excluded regions of fig. 1.

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[d] The results are insensitive to the values of the heavy Higgs masses above 100 GeV.

3 Experimental constraints from CESR and LEP

Let us consider the 2HDM in which $\sin(\beta-\alpha)=0$, $\tan\beta\gg 1$ and $m_h\sim\mathcal{O}(10\text{ GeV})$, which are necessary conditions if the Higgs sector is to be the source for $\delta a_\mu^{\rm NP}$ in the range given by eq.(1,2). The hAZ coupling is maximal , so we must assume that m_A is large enough so that $e^+e^-\to hA$ is not observed at LEP. The tree-level hZZ coupling is absent, which implies that the LEP SM Higgs search based on $e^+e^-\to Z\to Zh$ does not impose any significant constraints on m_h.[e] However, there are a number of constraints on light Higgs masses that do not rely on the hZZ coupling. For Higgs bosons with $m_h\lesssim 5$ GeV, the SM Higgs boson was ruled out by a variety of arguments that were summarized in [10]. For 5 GeV $\lesssim m_h\lesssim$ 10 GeV, the relevant Higgs boson constraint can be derived from the absence of Higgs production in $\Upsilon\to h\gamma$ [see dark-shaded (red) region in Fig.1]. A second bound on m_h can be derived from the non-observation of Higgs bosons at LEP via the process $e^+e^-\to hf\bar{f}$ ($f=b$, τ). However, only preliminary results have appeared so far by DELPHI [22] and ALEPH [23] [see light-shaded (yellow) region in Fig.1]. One noteworthy consequence of $m_h\sim 10$ GeV is the possibility of mixing between the h and the 0^{++} $b\bar{b}$ bound states $\chi_{b0}(1P)$ and $\chi_{b0}(2P)$, as discussed in [18,24]. As a result, the decay $\chi_{b0}\to\tau^+\tau^-$ should be prominent. For more details see [11].

4 Results

Using the experimental bounds on the Higgs mass discussed in section 3, we conclude that a light Higgs boson can be responsible for the observed 2.6σ deviation of the BNL measurement of the muon anomalous magnetic moment at the 90% CL in the framework of a CP-conserving [25] two-Higgs-doublet model with Model II Higgs-fermion Yukawa couplings only if the model parameters satisfy the following requirements:

$$\begin{aligned}
& m_\Upsilon \lesssim m_h \lesssim 2m_B\ , \\
& \sin(\beta-\alpha)\simeq 0\ , \\
& 27\lesssim \tan\beta\lesssim 35 \quad [\text{Davier}-\text{Hocker}(98)]\ , \\
& 20\lesssim \tan\beta\lesssim 35 \quad [\text{Narison}(01)]\ .
\end{aligned}\tag{5}$$

In addition, the other Higgs bosons (H, A and $H^\pm$) should be heavy enough to be consistent with the non-observation of HZ, hA and H^+H^+ production at LEP. Although, the parameter space is highly constrained, it is still useful to extend the LEP search for $f\bar{f}h$ production ($f=b$ or τ) to lower values of $\tan\beta$ and Higgs mass to either confirm or rule out the parameter range specified in eq. (5).

An extended Higgs sector provides one possible source of new physics that can be probed by a precision measurement of the muon anomalous magnetic moment. Many other new physics mechanisms for explaining the recent BNL measurement have also been explored [26,27,28,29,30]. If the $g-2$ "crisis" persists, it will be essential to find direct effects of the new physics at future colliders in order to establish the origin of a non-SM component to the muon anomalous magnetic moment.

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[e] Presumably, radiative corrections would lead to a small effective value for $\sin(\beta-\alpha)$. The LEP Higgs search yields an excluded region in the $\sin(\beta-\alpha)$ *vs.* m_h plane, and implies that for $m_h\sim 10$ GeV, $|\sin(\beta-\alpha)|\lesssim 0.06$ is not excluded at 95% CL [14].

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